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## 1. INTRODUCTION

Proper censoring of the weather radar data on the National Weather Surveillance Radar – 1988 Doppler (i.e., WSR-88D) is essential for the forecasters and automated algorithms. Presently, spectral moments at each range location are censored (i.e., labeled not useful) if the Signal-to-Noise Ratio (SNR) is insufficient, or the echoes from the subsequent trips are overlaid. Current censoring uses power measurements to determine if the SNR is above predetermined threshold relative to the noise power (e.g., 2 dB for reflectivity, and 3.5 dB for velocity measurements). As part of the NPI (NEXRAD Product Improvement) the network of WSR-88D weather surveillance radars (i.e., NEXRAD) is expected to be upgraded to include polarimetric capability. The dual-polarization research WSR-88D (KOUN) radar was upgraded to simultaneously transmit and receive horizontally and vertically polarized waves effectively sharing the available power from the transmitter between the two channels. Consequently, the power of the returned echoes in each channel is twice less than in the single-polarization system, thus resulting in the 3 dB smaller SNR (Scharfenberg et al., 2005). This loss has two effects on the spectral moments. First is that more data fall below display and processing thresholds and hence are lost. Second is the inevitable increase in the errors of estimates. It is quite clear that the 3 dB SNR loss has significant impact on the data and it becomes imperative to find the alternative censoring algorithm which would mitigate these adverse effects.

## 2. CLASSICAL METHODS FOR HYPOTHESES TESTING

One of the most commonly used approaches applicable to the situation at hand is the likelihood-ratio test. A likelihood-ratio test is a statistical test in which the ratio is computed between the maximum of the likelihood function under the null hypothesis and the maximum of the likelihood function under the alternative hypotheses. This ratio is then compared against some threshold to decide which of the two hypotheses should be accepted. The threshold value is tied to some constraint, usually a requirement that the probability of rejecting the null/alternative hypothesis when null/alternative hypothesis is true be acceptable. The likelihood ratio test criterion for hypotheses testing is given as (Neyman and Pearson, 1933):

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$$\frac{\sup_{\theta_0 \in \Omega} p_{\theta} (x_1, x_2, \dots, x_n | \theta_0 \in \Omega)}{\sup_{\theta_1 \in \Theta \setminus \Omega} p_{\theta} (x_1, x_2, \dots, x_n | \theta_1 \in \Theta \setminus \Omega)} \geq T(\theta_0, \theta_1) \quad (1)$$

where  $[x_1, \dots, x_n]^T$  is a vector of observations and  $p_{\theta}(x_1, \dots, x_n | \theta_1 \in \Omega)$  is the distribution function having parameters belonging to the  $\Omega$  subset of  $\Theta$ . To apply the likelihood-ratio principle we must know the joint *pdf* of the elements of the observations vector  $\mathbf{V} = [V_h(0), \dots, V_h(M-1), V_v(0), \dots, V_v(M-1)]^T$  from the horizontal and vertical channels. It is known that each element is a complex Gaussian random variable, thus the *pdf* is:

$$pdf(\mathbf{V}) = \pi^{-M} |\det \mathbf{C}^{-1}| \exp(-\mathbf{V}^H \mathbf{C}^{-1} \mathbf{V}), \quad (2)$$

where  $\mathbf{C}$  is the covariance matrix of size  $2M \times 2M$  defined as:

$$\mathbf{C} = E\{\mathbf{V}\mathbf{V}^H\} = \begin{bmatrix} E\{\mathbf{V}_h \mathbf{V}_h^H\} & E\{\mathbf{V}_h \mathbf{V}_v^H\} \\ E\{\mathbf{V}_v \mathbf{V}_h^H\} & E\{\mathbf{V}_v \mathbf{V}_v^H\} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_h & \mathbf{C}_{vh} \\ \mathbf{C}_{hv} & \mathbf{C}_v \end{bmatrix}. \quad (3)$$

Obviously, the parameters of interest are SNR,  $v$ ,  $\sigma_v$ ,  $Z_{DR}$ ,  $\rho_{hv}$ , and  $\phi_{dp}$ . If both signal and noise are present these parameters are unknown and the maximum of the likelihood function must be found by varying these parameters. In the absence of signal, though, the parameters are known hence the likelihood function can be computed. After a bit of algebra the likelihood ratio turns out to be:

$$\frac{1}{N_h} \sum_{m=0}^{M-1} |V_h(m)|^2 + \frac{1}{N_v} \sum_{m=0}^{M-1} |V_v(m)|^2 - \inf_{SNR, v, \sigma_v, Z_{DR}, \rho_{hv}, \phi_{dp}} \mathbf{V}^H \mathbf{C}^{-1} \mathbf{V} \geq T. \quad (4)$$

Because of the Neyman-Pearson lemma this approach achieves the maximum Probability Of Detection (POD) subject to the constraint that Probability of False Alarm (PFA) is less than or equal to  $\epsilon$ .

Apparently, the likelihood-ratio presents a quite appealing candidate for signal detection on the NEXRAD network. The important aspect however, is the computational intensity of this advanced detection scheme. It turns out that calculation of (4) for each set of parameters requires 0.33 GFLOP. Current NEXRAD signal processor is based on the Simgnet RVP8 which features "Dual SMP Pentium processors easily upgradeable as faster processors become available" ([RVP8 specifications](#)). To be on the safe side assume these two processors are running at 4 GHz each. At peak performance (1 FLOP per each cycle) this processor would deliver total of 8 GFLOP of computing power. Because numerous calculations of (4) would need to be performed (i.e., to find *inf*), we can readily see that the likelihood-ratio approach would be extremely demanding on the NEXRAD computing

resources to the point that its applicability would not be feasible. Consequently, we proceed by pursuing an alternative approaches based partly on our knowledge of the signal features and partly on intuition. This is done having in mind the constraints imposed by the current NEXRAD signal processing capabilities.

### 3. METHOD EVALUATIONS

Main idea behind the approaches investigated in the subsequent text is to utilize the weather signal coherency in sample-time and across channels to improve its detection. Each approach essentially consists of comparing the output of some  $f(V_h(0, \tau_s), \dots, V_h(M-1, \tau_s), V_v(0, \tau_s), \dots, V_v(M-1, \tau_s))$  against a threshold to decide if signal is present at range location  $\tau_s$ . Complex random variables  $V_h(m, \tau_s)$ , and  $V_v(m, \tau_s)$  are obtained by sampling voltage echoes in horizontal and vertical channels, respectively. Each function  $f$  is a combination of several core functions that are actually estimates of powers, autocorrelations, and cross-correlation in horizontal and vertical channel. Because the calculation of these functions is needed for generation of spectral moments and polarimetric variables, computational impact on existing resources is minimal.

Numerous combinations of the core functions were evaluated through Monte Carlo simulations and the one that yielded the best POD vs. PFA is:

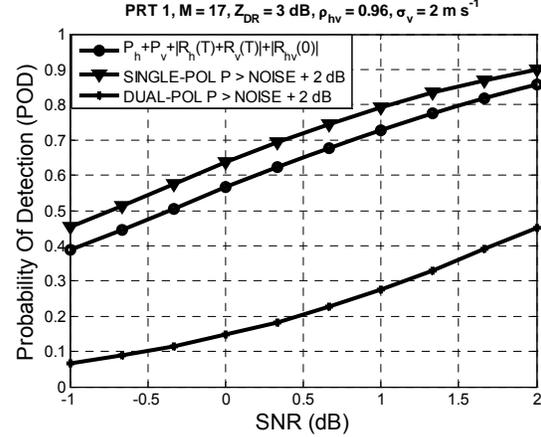
$$\left( \hat{P}_h + \hat{P}_v + \left| \hat{R}_h(T) + \hat{R}_v(T) \right| + \left| \hat{R}_{hv}(0) \right| \right) > THR \quad (5)$$

It is dubbed the *optimal sum*. To quantify the improvement in signal detection PODs are examined for varied SNR,  $\sigma_v$  and  $Z_{DR}$ . Monte Carlo simulations were used and the detection threshold was set so that the PFA is  $10^{-5}$ . The rationale for choosing this value for the PFA stems from the fact that on the average a PPI has  $360 \times 1000 = 360000$  points, if PFA is taken to be  $10^{-5}$  that would yield only 3.6 false detections per sweep. It is important to note, though, that in case of NEXRAD surveillance mode (i.e., PRT 1,  $M = 17$ ) the power detection threshold is set to 2 dB above the noise power level which yields the PFA of  $1.17 \times 10^{-6}$ . Nonetheless, this increase in the PFA, as will be shown later, does not obscure the significant weather returns.

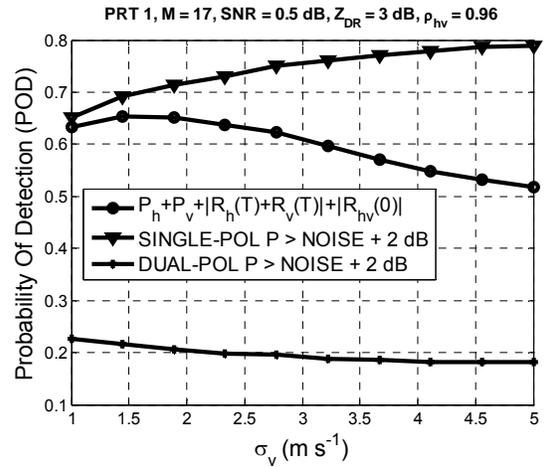
### 4. COMPARISON WITH THE LIKELIHOOD-RATIO HYPOTHESIS TESTING

So far, it has been shown that the optimal sum yields significant improvement in signal detection compared to the standard power based approach. The question which naturally arises is that of the maximum POD that can be achieved subject to the condition of the predetermined PFA.

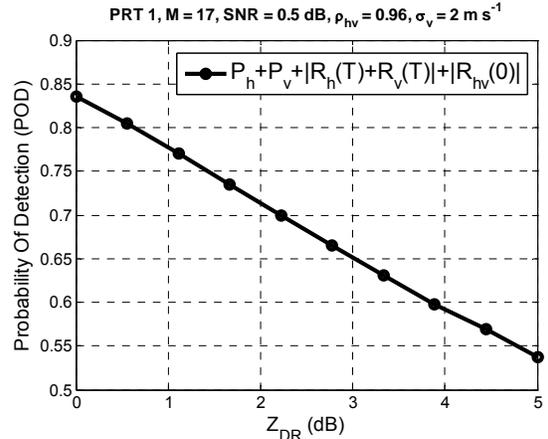
When the distribution parameters are known, hypothesis testing using the likelihood-ratio provides the most powerful test according to the Neyman-Pearson lemma. Consequently, the likelihood-ratio can be used as a good benchmark test to evaluate how close the detection scheme approaches the maximum detection rate. This comparison is shown in Figure 4. Apparently, the optimal sum in the current setting does not achieve the maximum possible detection rate. Thus, there is a



**Figure 1.** POD for varied SNR. For comparison, the optimal sum curve is plotted against the case with no 3 dB loss, and the 3 dB loss with the threshold 2 dB above noise.



**Figure 2.** POD for varied  $\sigma_v$ . For comparison, the optimal sum curve is plotted against the case with no 3 dB loss, and the 3 dB loss with the threshold 2 dB above noise.



**Figure 3.** POD for varied  $Z_{DR}$ .

possibility for further improvement by, for example, multiplying the terms with weights (to be found) that depend on signal parameters, that is

$$\left( \hat{P}_h + \alpha \hat{P}_v + \beta \left| \hat{R}_h(T) + \hat{R}_v(T) \right| + \gamma \left| \hat{R}_{hv}(0) \right| \right) > THR. \quad (6)$$

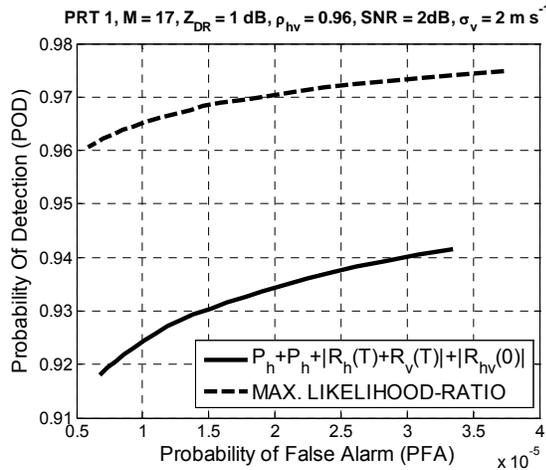


Figure 4. ML vs optimal sum comparison.

## 5. TIME-SERIES IMPLEMENTATION

Real data evaluation of the optimal sum was performed using the set of dual-polarization time series. This set was collected at a PRT of 3.1 ms, with  $M = 17$ , and at elevation of 0.48 deg. In standard operation, this corresponds to surveillance scan with the threshold set to 2 dB above noise power. Consequently, this threshold will be used in subsequent analysis. To simulate the effect of the 3 dB power loss, the noise power was doubled in each channel by adding the noise samples as follows:

$$V_h(n) = V_h(n) + N_h(n) \text{ and } V_v(n) = V_v(n) + N_v(n). \quad (7)$$

Threshold for each approach was set so that the PFA is  $10^{-5}$ . The original “single-polarization” reflectivity field is plotted in Figure 5.

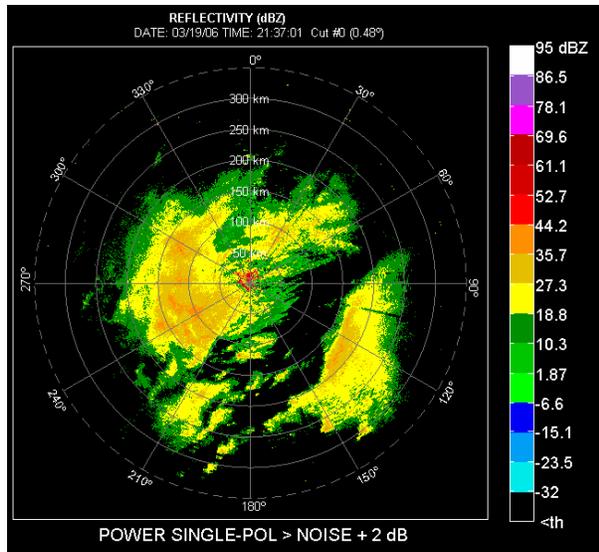


Figure 5. The original “single-pol” reflectivity field.

Adding the additional noise (i.e., doubling the noise power), but not changing the threshold, yields the

reflectivity field as shown in Figure 6. This case is equivalent to setting the threshold to -1 dB below the total noise power. It is noticeable that many spurious speckles have emerged. This is because the PFA has increased from  $1.17 \times 10^{-6}$  to 0.003 (which amounts to 1080 false detections per scan on the average).

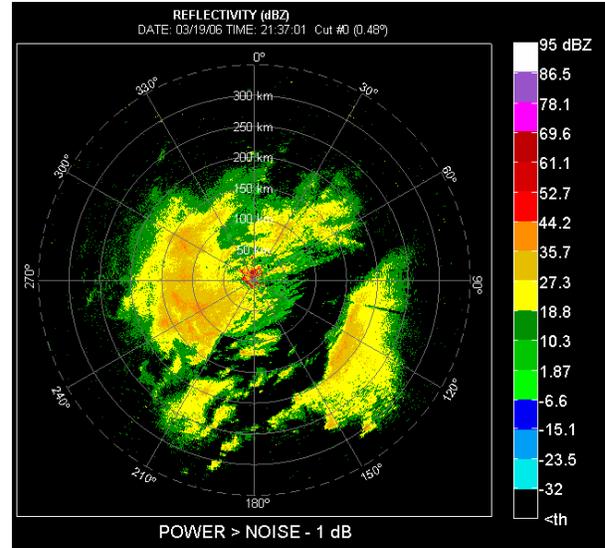


Figure 6. Reflectivity field obtained after doubling the noise power, but keeping the same threshold level with respect to the original noise power.

In the reflectivity field for which the threshold is elevated to 2 dB above the composite noise level (Figure 7) a significant portion of the features on the rim of the phenomena has been lost compared to the original field (Figure 5).

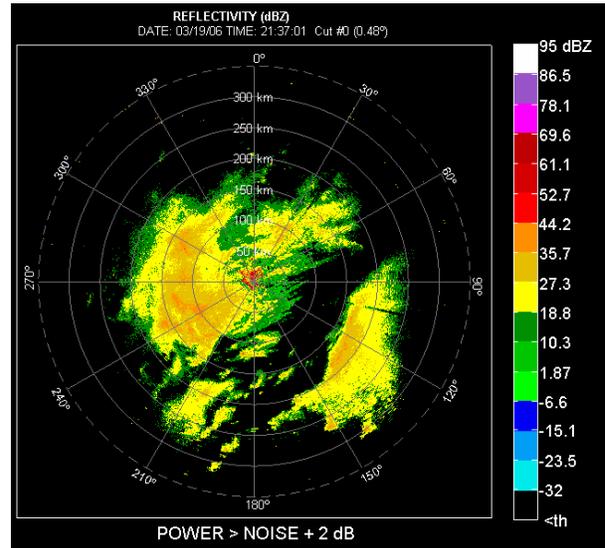
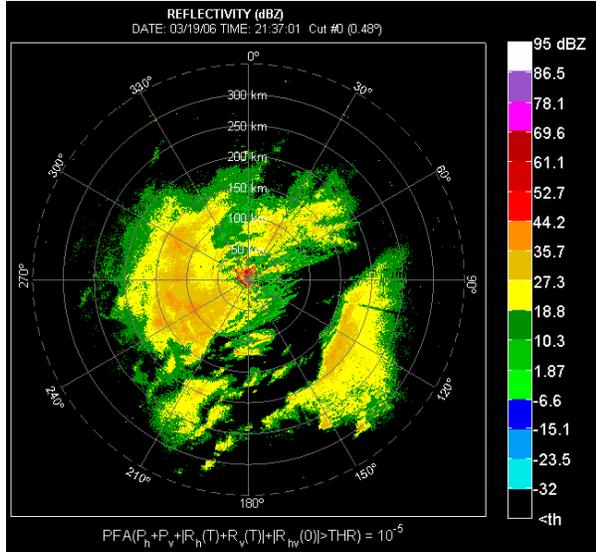


Figure 7. Reflectivity field obtained after doubling the noise power, and increasing the threshold to 2 dB above the doubled noise power.

Application of the optimal sum produces the field in Figure 8. Note that most of the low reflectivity perimeter features have been recovered.



**Figure 8.** Reflectivity field obtained after doubling the noise power, and using the optimal sum for detection.

To quantify the performance, various ratios of detections are compared in Table 1. These ratios are explained next. Let us view the field (in polar coordinates) as a matrix of size  $360 \times NRL$ , where  $NRL$  stands for the *Number of Range Locations*. Let  $MZ$  stand for the original reflectivity matrix where each matrix entry is power value at a given location. Let  $MN(D)$  stand for the matrix with doubled noise power in which each matrix entry is 1 if the decision that the signal is present is positive, otherwise it is 0.  $D$  in the brackets denotes the decision test used in determining the matrix entries. Then the values in the row termed as the *Ratio of total detections* are calculated as:

$$\frac{\text{num}([MZ > NOISE + 2dB] .* MN(D))}{\text{num}([MZ > NOISE + 2dB])} \quad (8)$$

*Greater than* operator is binary (1 if true, and 0 if false) and is applied to each matrix entry yielding new matrix with 0, 1 entries. The operator  $.*$  acts as an element-wise matrix multiplication (same as in MATLAB). The  $\text{num}$  operator gives the total number of 1s in the matrix. The *Ratio of bounded detections* is obtained as:

$$\frac{\text{num}([NOISE + 5dB > MZ > NOISE + 2dB] .* MN(D))}{\text{num}(NOISE + 5dB > MZ > NOISE + 2dB)} \quad (9)$$

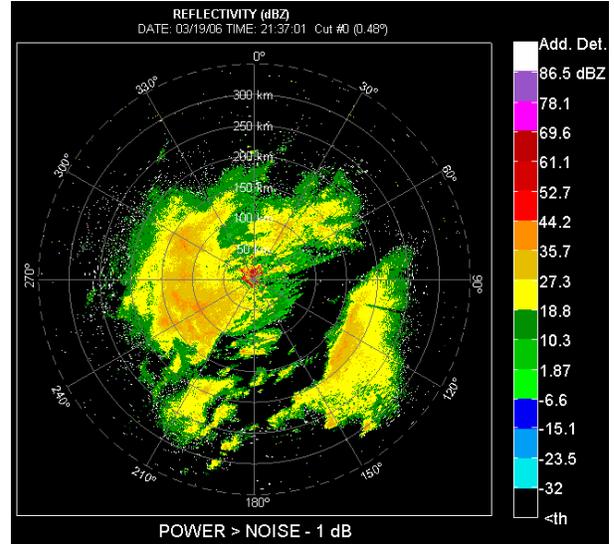
This ratio essentially gives the portion of locations that fall below the 2 dB threshold due to the 3 dB loss in power, but are still detected using each of the three evaluated approaches. The *Ratio of additional detections* is:

$$\frac{\text{num}([MZ < NOISE + 2dB] .* MN(D))}{\text{num}(MZ < NOISE + 2dB)} \quad (10)$$

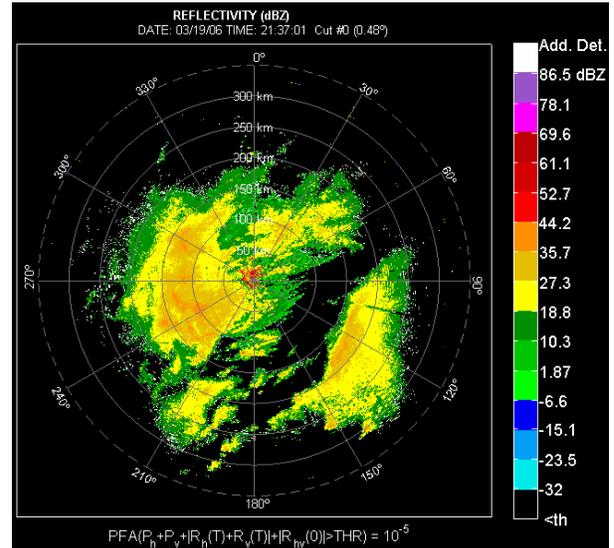
This ratio gives the portion of bins that are originally censored, but are detected as signals using each of the three approaches.

$D$	$P \geq N_h - 1\text{dB}$	$P \geq N_h + 2\text{dB}$	OPTIMAL SUM
<i>Ratio of total detections</i>	0.982045	0.912165	0.989966
<i>The Ratio of bounded detections</i>	0.800269	0.141154	0.884036
<i>Ratio of additional detections</i>	0.029869	0.000143	0.032996

**Table 1.** Time-series statistics.



**Figure 9.** Reflectivity field when the threshold is lowered to -1 dB of the noise power with data previously censored in the original field highlighted in white.



**Figure 10.** Reflectivity field when the optimal sum is used for detection with data previously censored in the original field highlighted in white.

Notice that if the power threshold is -1 dB below noise and the optimal sum is used approximately 3% of the data that initially were censored are detected as

signals. In Figure 9 and Figure 10 these additional detections are highlighted in white for both cases. Observe that additional detections made by the optimal sum are predominantly located at the rim of the weather system. This gives an additional assurance that the majority of these detections are indeed valid weather returns.

## 6. SUMMARY

Methods to threshold polarimetric weather radar data were investigated. Motivation comes from the 3 dB SNR loss in radars that transmit (and receive) simultaneously electro magnetic waves at horizontal and vertical polarizations. It is very likely that the forthcoming dual polarization upgrade of the WSR-88D network will employ this technique. Thus it is desirable to mitigate the effects of this loss in sensitivity. It was established that the classical likelihood-ratio test (which yields the best possible detection rate) is not practical for implementation in real-time due to excessive computational requirements on the existing signal processor. Therefore alternative approaches based on combinations of terms needed for estimation of Doppler spectral moments and polarimetric variables were investigated. A promising one that linearly combines powers and auto and cross correlations was chosen for evaluation. It is termed the "optimum sum" and its performance was compared to the likelihood-ratio and was evaluated on radar data. The analysis shows satisfactory performance. In addition, it implies that better results could possibly be achieved by appropriately weighting the terms in the optimal sum.

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