8B.5 SIMULATION AND INTERPRETATION OF NOISE IN THE REFRACTIVITY RETRIEVAL

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1. INTRODUCTION

The index of air refraction (*n*) or refractivity $N = (n-1)\times 10^6$ related to atmospheric pressure, temperature, and water vapor pressure affects microwave propagation in the lower altitudes of the troposphere (Bean and Dutton 1968) and is known to be most sensitive to moisture changes under warm weather conditions (Fabry et al. 1997). Hence, the refractivity retrieval by a S-band scanning radar has been welcomed to provide high resolution near-surface moisture estimates in time and space for the study of severe convection (Weckwerth et al. 2005).

The refractivity retrieval algorithm (Fabry et al. 1997; Fabry 2004) is based on the variation of the measured radar phase (ϕ_i) returned particularly from ground targets such as power poles, buildings, mountains etc. used to be eliminated as ground clutter interference (Steiner and Smith 2002). Although these ground targets are supposed to be stationary, the radar backscattered signals from the targets fluctuate and are related to the radar-pulse traveling time (*t*) that is the function of path-averaged refractive index and range (r_i) to the fixed ground targets for a given frequency (*f*);

$$\phi_i = 2\pi f t = \frac{4\pi f r_i}{c}\,\overline{n(0 \rightarrow r_i)}\,, \qquad [1]$$

where *c* is the speed of light in vacuum. Since an accurate range to each individual target is difficult to know within the accuracy of sub-mm, the algorithm introduces a phase field at reference time assuming a homogeneous refractivity over the field. This reference phase can be obtained when the measured refractivity gradients over the area are expected to be least varied such as the moment after several hours of stratiform rain with windy condition. Then, using the known reference values of phase and refractivity over the field, the path integrated refractivity at the current time can be derived from the difference between the reference phase field and the current phase field over the short paths along the radial.

However in reality, [1] is more complicated because of uncertainties related to: 1) propagation delay caused by the structure of the refractive index along the beam trajectory to a target, and 2) the target properties such as a target's shape, range to a target as well as the illuminated area of a target by radar beam, caused by the combination of precipitation, swaying vegetation, or propagation (Fabry 2004). To mitigate these uncertainties, the area of swaying vegetation or precipitation can be recognized during the calibration process, whereas propagation delay part is not easy to correct except smoothing the phase change fields.

For reliable smoothing of the phase change fields over small regions as well as for minimizing the phase aliasing when a phase change exceeds ±180˚, the current algorithm assumes that these ground targets and radars are perfectly aligned to each other along the azimuth on the horizontal and vertical plane in flat terrain. Consequently, if the terrain is complex, or if the target distribution is not certainly known, or if strong refractivity gradients exist, the computed phase difference field under assumptions above becomes quite noisy resulting in low quality of the refractivity retrieval field.

Therefore, for better quality of the refractivity retrieval, it is crucial to assess the characteristics of ground targets and to understand phase measured from these targets associated with atmospheric propagation. Furthermore, it would be useful to quantify some expected uncertainties in the phase measurement for the calibration of the retrieval. To achieve this task, we expand, in section 2, the idea of predicting error sources (Fabry 2005) by building a simple phase simulator with a set of statistically generated target heights over the region of the S-Pol radar from the IHOP_2002 field experiment. Section 3 presents the simulated results obtained for a selected case and its evaluation with the observed results. The discussion and conclusion is followed in section 4.

2. PHASE SIMULATOR

To quantify the noisiness of the phase changes between a reference time and the current time, it is required to know target heights and how these are illuminated by the radar beam. One possible way to obtain these within the range resolution is to generate statistical random target heights based on an empirical probability density function of target heights using measured ground echo intensity and terrain data from the USGS Digital Elevation Model (DEM) with a grid resolution of 3 arc second (approx. 100 meter raster data).

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2.1 Generation of random ground target heights and numbers

Since the target quality matters in the generation of statistical targets for the phase simulation, two sources of practical information are used here to determine “good” solid ground targets. One is the average signal to noise ratio fields (SNR in dB) obtained with the power received from the coherent radar. If strong SNR above a certain threshold is observed in a region, there would be high possibility to have numerous ground target structures over the region. The other is the map of visible target heights between the lowest ray and the terrain height at each range bin in the polar coordinate with the resolution of 150 meters in range and 1 degree in azimuth (*hv*: Fig.1). Using both SNR values and the visible target heights on a clear daytime, it is possible to obtain a cumulative probability of exceeding a SNR value for the region of each different depth below the radar horizon (*hv*). According to Doviak and Zrnic (1993), *hv* can be obtained with the lowest elevation angle (θ) that intercept the ground at each range grid under the assumption of a constant *dN/dh* in the vertical.

$$hv = \sqrt{r^2 + (\mathrm{Re})^2 + 2r\,\mathrm{Re}\sin\theta} - \mathrm{Re} + H_r\,, \qquad [3]$$

where *r* is radar measurable range with the resolution of 150 meters, and H_r is the S-Pol radar height about 900 meters above the sea-level located on the Oklahoma Panhandle, and Re is the equivalent earth radius considering the curvature of the earth radius (*R*) associated with the gradient of the refractive index:

$$\mathrm{Re} = \frac{(R+H_r)}{1+(R+H_r)\left(\dfrac{dn}{dh}\right)}. \qquad [4]$$

For a given SNR threshold (e.g., approx. larger than 12 dB in our case) and the reliable target quality index provided by Fabry (2004) to avoid contamination due to moving ground targets, the cumulative probability can be converted to the probability density function (PDF) of having at least more than one target at a specific height by calculating its slope within each height interval (Fig. 2). Based on the smoothed PDF, the Poisson distribution (Kalbfleisch 1985) is used to obtain the number of targets per range bin. Regarding to this number of targets, each different target height can be randomly generated. As a result of this low probability in the PDF, the generated number and height of targets within a range bin is small and low respectively, e.g. one or two at the height of less than 10 meters. Considering the environment near the S-Pol site as being flat farm land, it is expected not to have many tall targets around the radar. Therefore, we can use above target information to build the phase simulator over 2-D fields.

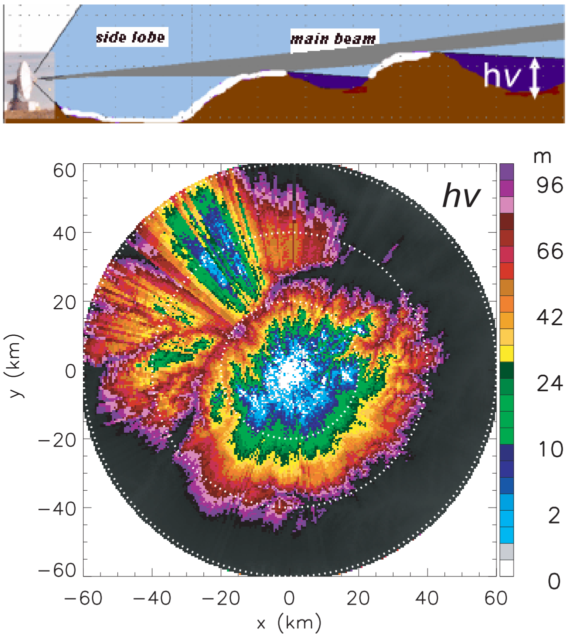

Fig. 1: Top: the illustration of the beam height between the lowest beam and the ground. Bottom: the map of beam heights along the azimuth (i.e., visible target height) over the S-Pol radar region generated with 0 ppm km^{-1}, sub-refracted case. The area of white represents the ground intercepted by the ray, and the area of colors may be seen if the heights of the targets exceed the corresponding heights of colors.

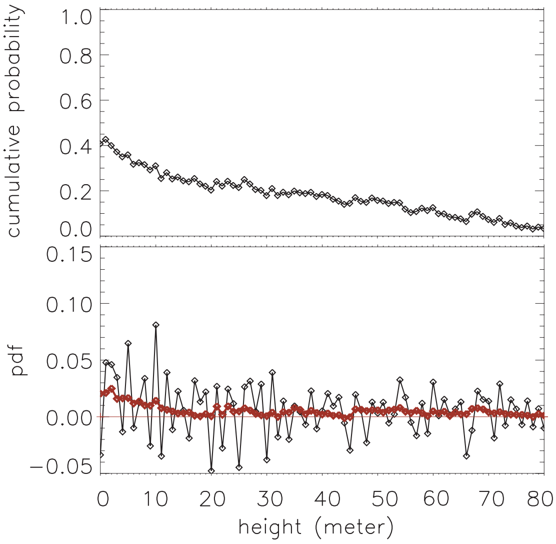

Fig. 2: Top: cumulative probability exceeding 12 dB of SNR. Bottom: the probability density function (in black) that at least more than one target would exist at each height, and the smoothed PDF (in red) to avoid negative probability.

2.2 Building a phase simulator

Once a target height over the terrain is known from above statistics, the target height sampled by the radar ray, *h*(*r*), can be geometrically expressed at a given constant *dN/dh* as

$$h(r) = H_r + \frac{H_i - H_r}{H_i} + \frac{1 + R\dfrac{dN}{dh}}{2R}(r^2 - r_i\, r), \qquad [5]$$

which is equivalent to [3] and can be used to rewrite [1] (Fabry 2005):

$$\varphi_i = \frac{4\pi f}{c}\int_0^{r_i}\left(n(r) + \frac{dn}{dh}h(r)\right)dr \qquad [6]$$

$$= \frac{4\pi f}{c}\left\{\left(1 + 10^{-6}\overline{N}\right)r_i + \frac{dN}{dh}\left(\left(\frac{H_i - H_r}{2}\right)r_i - \frac{1 + R\dfrac{dN}{dh}}{12R}r_i^3\right)\right\},$$

where r_i is an optical distance (or simply just “range”) along the ray trajectory affected also by the condition of *dN/dh* between the radar and a target at height H_i:

$$r_i = \frac{(\mathrm{Re} + H_i - H_r)\sin\left(\dfrac{r}{\mathrm{Re}}\right)}{\sqrt{1 - \left(\dfrac{(-r)}{2\,\mathrm{Re}} + (H_i - H_r)\dfrac{1}{r}\right)^2}}, \qquad [7]$$

As we can see in [6], the phase is now affected not only by $\overline{N}$ variation, the first term that we expect to retrieve, but also by additional terms considered to be unwanted “noise”. Therefore, to examine this noisiness depending on the vertical gradient of refractivity and the target heights, the phase simulation can be done by computing 1) differences in [6] between the reference time and the interesting time for each point target within the range bin and 2) the sum of each target’s phase differences as the representative for each range bin.

3 RESULTS FROM A SELECTED CASE

3.1 Case selected

Since the phase simulation needs a constant *dN/dh* over the fields near the ground, availability of sounding data is considered to choose the case. Unfortunately, only one site over our radar coverage, 8 km away from the radar in range, provided a limited number of soundings per day. Once low-level *dN/dh* at the reference time is determined, the phase change simulation can be done with different *dN/dh* of the current time. To evaluate the effect of dN/dh on the phase change fields obtained from this simulator, the time of interest selected here is also considered when N variability itself was expected to be small in order to avoid another source of uncertainty.

3.2 Comparison between simulated and observed phase difference fields

After the phase changes over the field were computed between the reference time and the current time of interest, the field looked as noisy as salt and pepper for both simulated and observed cases. To quantify this variability, the local standard deviations of these phase changes were computed within areas of 2.4 km in range by 10 degree in azimuth.

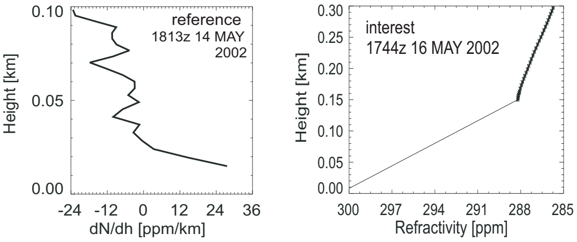

Fig. 3: An example of dN/dh in the vertical measured from soundings near the ground at the reference time (left). Although the values are not really constant up to 100 meters in the vertical, we choose 0 ppm km^{-1} *(sub-refraction) as the representative value in [6]. Since no data was recorded up to 100 meters at the time of interest (right), the vertical gradient of N at this time is computed with interpolated N, providing approx. -80 ppm km*$^{-1}$ *in this layer.*

Figure 4 shows the simulated results with two different vertical gradients of refractivity. The coverage computed here was mainly determined by the beam blockage related to terrain heights and the *dN/dh* of the reference time. Less variability appears at near range rather than at far range for both different *dN/dh* cases, which is expected to see because of increasing range term in [6] and of poor target visibility. Since faster bending of a ray toward the ground occurs for the larger magnitude of *dN/dh*, we can also see higher uncertainty in super-refraction (-80 ppm km^{-1}) than near normal condition (-44 ppm km^{-1}).

Meanwhile, the standard deviation field of observed phase changes was obtained as shown in Fig. 5, using phase data scanned at the reference time of 18:07z 14 May 2002 and at the interesting time of 17:44z 17 May 2002. Since the average refractivity at interesting time was about 30 ppm higher than at the reference time, N correction was also performed in the phase changes along the range. For the same coverage as the simulated fields, the resulting variability in the observation looks quite different; especially at near ranges, the value appears much higher than at far range as well as than at near range of the simulation. This means that the simulation could not reproduce the reality well even though the scanning time was chosen carefully with the sounding data providing *dN/dh* for the simulation.

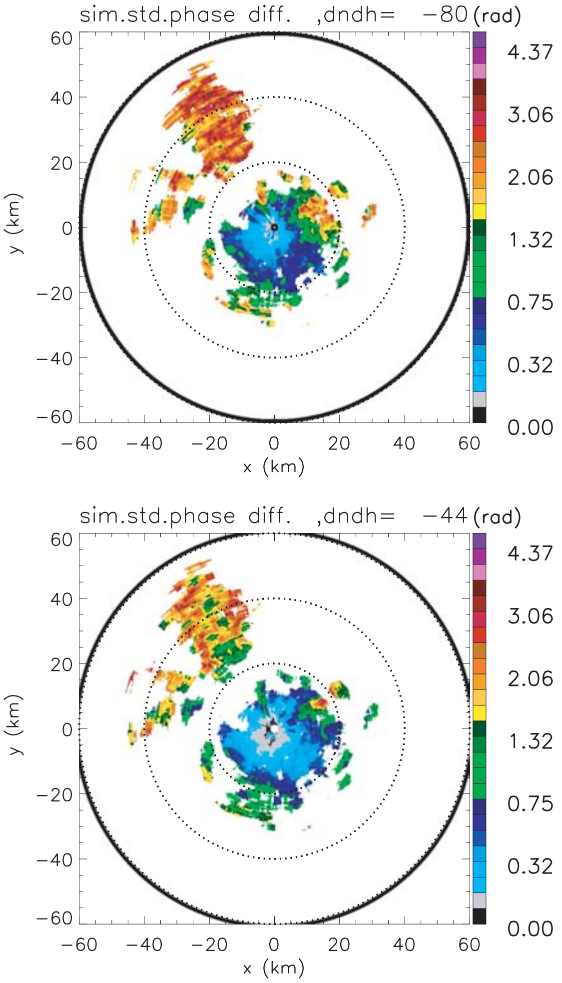

Fig. 4: Local standard deviation fields of simulated phase differences in radian between reference time with 0 ppm km^{-1} and interesting times with -80 ppm km^{-1} (up: super-refraction) as well as with -44 ppm km^{-1} (down: close to normal condition).

4. DISCUSSION AND CONCLUSION

Our phase simulator was built to understand the sources of uncertainty in the phase measurement in order to retrieve more reliable refractivity. Since the phase is affected by atmospheric propagation factor due to the vertical refractivity gradients associated with target heights illuminated by radar rays, we used the simulator to quantify the noisiness in the phase changes due to this effect over the complex terrain. As a result, the simulation did show high variability with increasing both range and the slope of *dN/dh.* However, when this simulated result at near range was compared with the observed one for its validation, the simulated one showed much less variability than the observed one, meaning that factors above did not seem to be enough to fully understand the real variability in the phase changes. In other words, the variability at near range may be simply caused by unknown moving targets at near range since we could not obtain or expect such variability shown in the observation from following trials with the simulator: 1) for different constant N values considering the uncertain small scale structures in N itself, 2) for different *dN/dh* along the azimuth regarding to the inappropriate assumption of one representative value of *dN/dh* over the radar coverage. In conclusion, more careful speculation on this target configuration as well as its movement will be required to understand the noisiness better in the phase changes resulting in the refractivity retrieval.

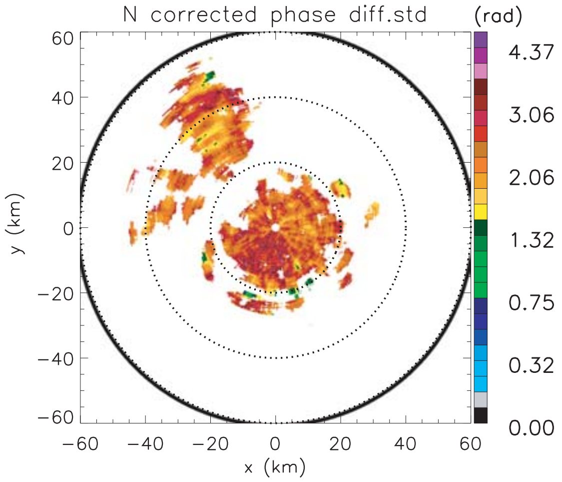

Fig. 5: Local standard deviation field of observed phase differences in radian between the reference time of 1807z 14 May 2002 and at the interesting time of 1744z 17 May 2002.

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