INVESTIGATION OF DOPPLER SPECTRA FROM A TORNADIC SUPERCELL THUNDERSTORM: ARE THEY GAUSSIAN?

R. Reinoso-Rondinel[1,*], T. Y. Yu[1] and R. D. Palmer[2]

[1] School of Electrical and Computer Engineering, University of Oklahoma, Norman, Oklahoma, USA
[2] School of Meteorology, University of Oklahoma, Norman, Oklahoma, USA

Abstract

The Doppler spectrum of weather signals reveals the power-weighted radial velocity distribution within the radar resolution volume and has gained increasing attention in recent years. The three fundamental radar parameters, reflectivity, mean radial velocity and spectrum width, are defined from the three spectral moments and can be estimated either by spectral methods in the frequency domain or the autocovariance method in the time domain. For a Gaussian-shaped Doppler spectrum, the first three moments (zero, first and second moments) are sufficient to fully characterize the spectrum. However, to our knowledge, there are only limited studies to systematically investigate and verify the Gaussian assumption using modern Doppler weather radar. For non-Gaussian or asymmetric spectra, the mean radial velocity and spectrum width estimated by the autocovariance method can be biased. In this work, Doppler spectra from a tornadic supercell thunderstorm collected by the research WSR-88D (KOUN) in Norman, Oklahoma are examined. Many interesting spectra (ex, non-Gaussian, skewed, flat top, etc.) are observed and will be shown. Simple models in relation to the storm dynamics are proposed to explain the observed spectrum shape. Moreover, mean squared error between the observed and fitted spectra and higher moments such as skewness and kurtosis are introduced to verify the Gaussian assumption within the storm.

1. INTRODUCTION

The Doppler spectrum can be used to estimate power, mean radial velocity and spectrum width in the frequency domain. It is often assumed that the spectrum has a Gaussian spectral shape, which is supported by the "Central Limit Theorem". The Autocovariance Method exploits this assumption to estimate the mean radial velocity and spectrum width from the time series data. It have been observed that in many weather events the Doppler spectral shape is not Gaussian. Then, moment estimation can be biased because of this Gaussian assumption [Sirmans and Bumgarner, 1975; Di Vito et al., 1992]. The method used here to estimate a Doppler spectrum was the nonparametric Periodogram method [Doviak and Zrnić, 1993].

To verify the Gaussian assumption, we have studied a case of supercell tornadic from the research WSR-88D (KOUN) radar operated by the National Severe Storms Laboratory (NSSL). Interesting Doppler spectra were found that can provide important weather information. In this paper, we investigate the shape of the Doppler spectra using two techniques. The first technique is based on higher moment estimation from the Doppler spectrum, including Skewness and Kurtosis, which are a form of the third and fourth order moment, respectively. Statistics based on simulations have been calculated to study these higher-order moments. Also, a threshold for each moment was chosen to quantify the Gaussian assumption within the storm. The second technique is the mean square error between the Doppler spectrum and a parabolic fit, since a Gaussian spectrum has a parabolic form when the spectrum is converted from linear to logarithm scale. From Janssen and Spek [1985] this mean squared error is defined as Variability and provides the deviation of the Doppler spectrum from a Gaussian shape.

A criterion of SNR was used to insure good spectral estimation. The results are analyzed for measurements of reflectivity, mean radial velocity, spectrum width, skewness, kurtosis and the mean squared error within a supercell storm of May 10, 2003. Specific spectrum examples are provided. Finally, we provide a Gaussian percentage for the studied case and some relationships between estimation moments and storm dynamics.

2. DATA SET

A tornadic supercell thunderstorm took place on May 10, 2003 in Oklahoma and Level-I data from 0343 UTC time are processed. Table 1 shows a brief description of the

* *Corresponding author address:* Ricardo Reinoso-Rondinel, University of Oklahoma, School of Meteorology, 120 David L. Boren Blvd., Rm 5900, Norman, OK 73072-7307; e-mail: rein3@ou.edu

radar parameters which are important for the Doppler spectrum processing and estimation.

Table 1: Radar Parameters

Quantity	Value
Frequency	2.99 GHz
Half-power beam width	1°
Polarization	Horizontal
Range resolution	250 m
Pulse repetition time	0.78 ms
Number of gates	468
Elevation angle	1.45°

To visualize the supercell thunderstorm, Figure 1 shows the reflectivity estimated from the Level-I data. Regions where reflectivity is lower than the 35 dBZ threshold were not processed for spectral shape.

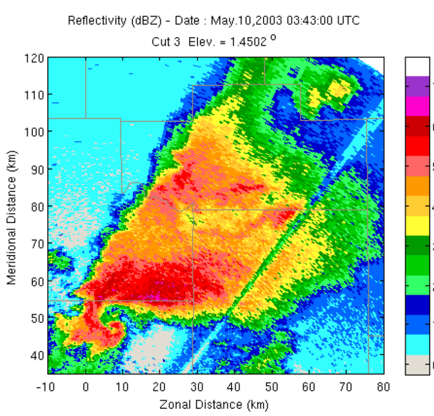

Figure 1: Reflectivity PPI

3. POWER SPECTRAL DENSITY (PSD) ESTIMATION

The power spectrum density was calculated using the Periodogram method with 32 pulses and a Gaussian time window. Also, a spatial moving window is applied to smooth the periodogram. This window is formed by 16 gates consists of 2 radials and 8 gates in each radial. To measure the deviation from a Gaussian shape, only the spectral data above 5 dB of the noise level is taken for further processing. Then, noise level for each spectrum is estimated by the average of 15 percent of the number of the lowest spectral values. Figure 2 shows a single periodogram with its averaged periodogram Doppler spectrum as well as black dots which represent the spectral values 5 dB higher than the estimated noise level.

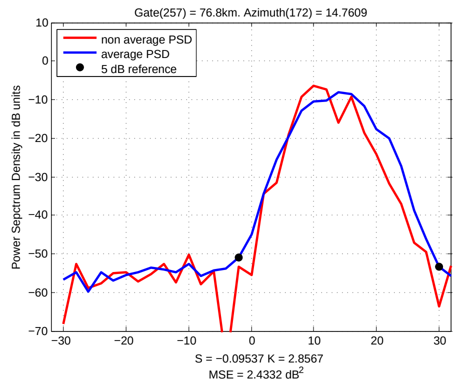

Figure 2: PSD averaged

4. MOMENTS ESTIMATION

The estimation of the first, second, third and fourth moments are the mean radial velocity, spectrum width, skewness and kurtosis, respectively. For our work, the moments are calculated from the Doppler spectrum using moment method. The third and fourth moments are used here to quantify the shape of the spectrum. Since those higher order moments are not typically estimated from the Doppler spectrum, introductory information is provided.

4.1. Skewness

Skewness is the normalized third central moment. Under probability theory, skewness is a measure of asymmetry of a probability density function. According to P. Z. Peebles [2001], skewness is defined by the following equation.

$$S = \frac{\mu_3}{\sigma^3}. \quad (1)$$

where μ_3 and σ are the third central moment and the standard deviation, respectively. The estimation of skewness is calculated from the spectrum as

$$\hat{S} = \frac{\frac{\sum_{l=l_0}^{l_0+L-1}(\nu_l - \hat{V}_r)^3 \cdot PSD_l}{\sum_{l=l_0}^{l_0+L-1} PSD_l}}{\left(\frac{\sum_{l=l_0}^{l_0+L-1}(\nu_l - \hat{V}_r)^2 \cdot PSD_l}{\sum_{l=l_0}^{l_0+L-1} PSD_l}\right)^{1.5}}. \quad (2)$$

L is the number of PSD samples, PSD_l, above 5 dB of the estimated noise level, ν_l denotes the radial velocity samples and $\hat{V}_r$ is the estimated mean radial velocity. For a symmetric Doppler spectrum, skewness should be equal to 0. If the value of skewness smaller or larger than zero, the Doppler spectrum is said to left or right skewed, respectively. For the verification of the Gaussian assumption, a skewness range of -0.4 to 0.4 was used. Figure 3 shows the histogram of skewness for some reasonable spectrum width values using simulations.

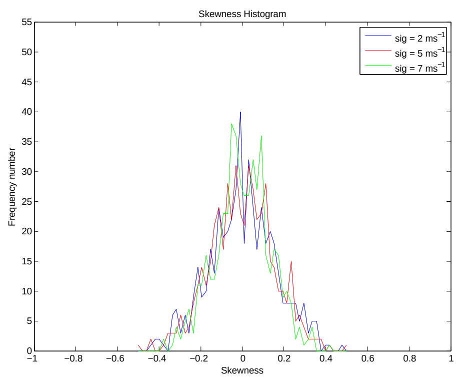

Figure 3: Histrogram of Skewness for assumed Gaussian-shaped spectra

4.2. Kurtosis

Kurtosis is the normalized fourth central moment. From probability theory, kurtosis is a measure of peakedness of the probability density function. Equation 3 defines kurtosis [Ding and Nguyen, 2000].

$$K = \frac{\mu_4}{\sigma^4}. \quad (3)$$

Similar to equation 2, kurtosis is estimated from the Doppler spectrum.

For an ideal Gaussian spectrum, kurtosis should be equal to 3. We have used a valid interval from 2.5 to 3.5 as Figure 4 suggests.

5. MEAN SQUARED ERROR

Thus, all moments have been computed from the Doppler spectrum in linear scale. Because of Gaussian

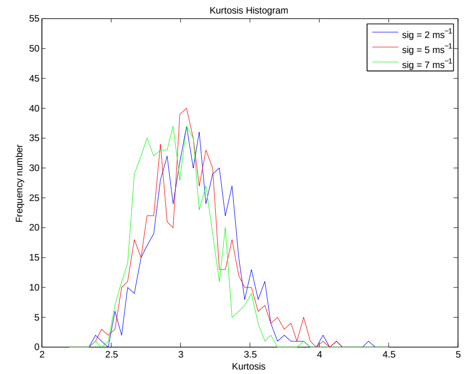

Figure 4: Histrogram of Kurtosis for assumed Gaussian-shaped spectra

spectrum should correspond to a parabola in logarithmic scale, the mean square error can be estimated in the logarithmic scale. A term named Variability is defined by Janssen and Spek [1985] as the mean squared difference between 10 times the logarithm of the estimated spectrum and the least squares fitted parabola, G, as presented below.

$$Variability = \frac{1}{L} \sum_{l=l_0}^{l_0+L-1} (G_l - 10 \cdot \log PSD_l)^2 \quad (4)$$

where L is defined as before. It have been shown by Welch [1967] that the PSD has a Chi-square distribution with an equivalent number of degrees of freedom, m, described by equation 5 where A is equal to the number of gates used to smooth the PSD, E and Var denote the expected value and the variance, respectively. A is equal to 16 which gives the number of degrees of freedom equal to 32.

$$m = \frac{2E^2[PSD_l]}{Var[PSD_l]} = 2A \quad (5)$$

According to the appendix of Janssen and Spek [1985], the expected value of Variability is equal to the variance of the PSD in logarithm scale. Then, for m equal to 32 the theoretical expected Variability value is equal to 1.21 dB^2. Figure 5 shows the Variability histogram of simulated averaged PSD for different spectrum width. For narrow PSD, the expected value might be a little larger [Welch, 1967] and this is why we considered a minimum of 3 ms^{-1} for spectrum width. Moreover, to set a threshold for the Variability we choose 8 dB^2 based on Figure 5.

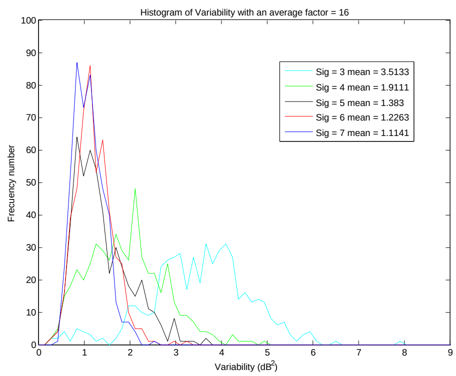

Figure 5: Histrogram of Variability

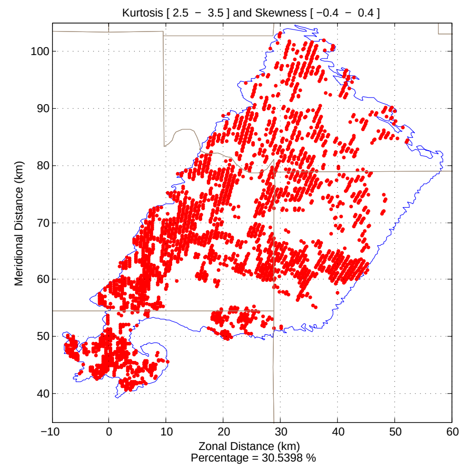

Figure 6: Locating Gaussian-shaped spectrum based on moments estimation

6. RESULTS

Estimation of the mean radial velocity, spectrum width, skewness and kurtosis over the tornadic supercell thunderstorm are shown in Figure 8. Note that a 35 dBZ threshold has been set to filter out spectra with relative low signal to noise ratio. In addition, regions where the spectrum width is larger than 7 ms^{-1} have been removed because of shape distortion due to aliasing, especially near the hook echo area. The skewness and kurtosis seem to be correlated in the southern region of the storm. Kurtosis shows values smaller than 3 in areas which has a relative wide spectrum width. In addition, those wide spectral regions are retrieved by skewness in the north and south west supercell areas where skewness departs from zero either positively or negatively. For both moments, the south central and north east regions show a predominant non-Gaussian characteristics which correspond to high reflectivity and medium to high spectrum width. The north west central and south west regions show agreement with the Gaussian assumption.

From the range where a Gaussian characteristic can be accepted based on the intervals for both skewness and kurtosis, Figure 6 is a map where the Gaussian-shaped spectrum predominant. From this result, only 30 percentage of the spectra hold to the Gaussian assumption.

From the mean square error point of view, Figure 7 shows locations where the Variability is equal or lower than 8 dB^2. North west, small central and a south west area are identified as the Gaussian-shaped regions, which show a correlation between the normalized third and fourth central moments. The Gaussian degree percentage given by variability is 35 percent considering only spectra with spectrum widths between 3 and 7 ms^{-1}.

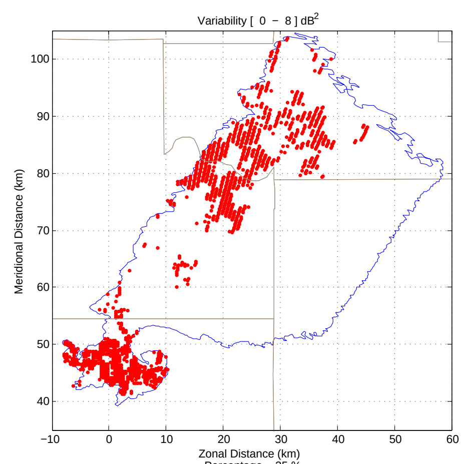

Figure 7: Locating Gaussian-shaped spectrum based on MSE estimation

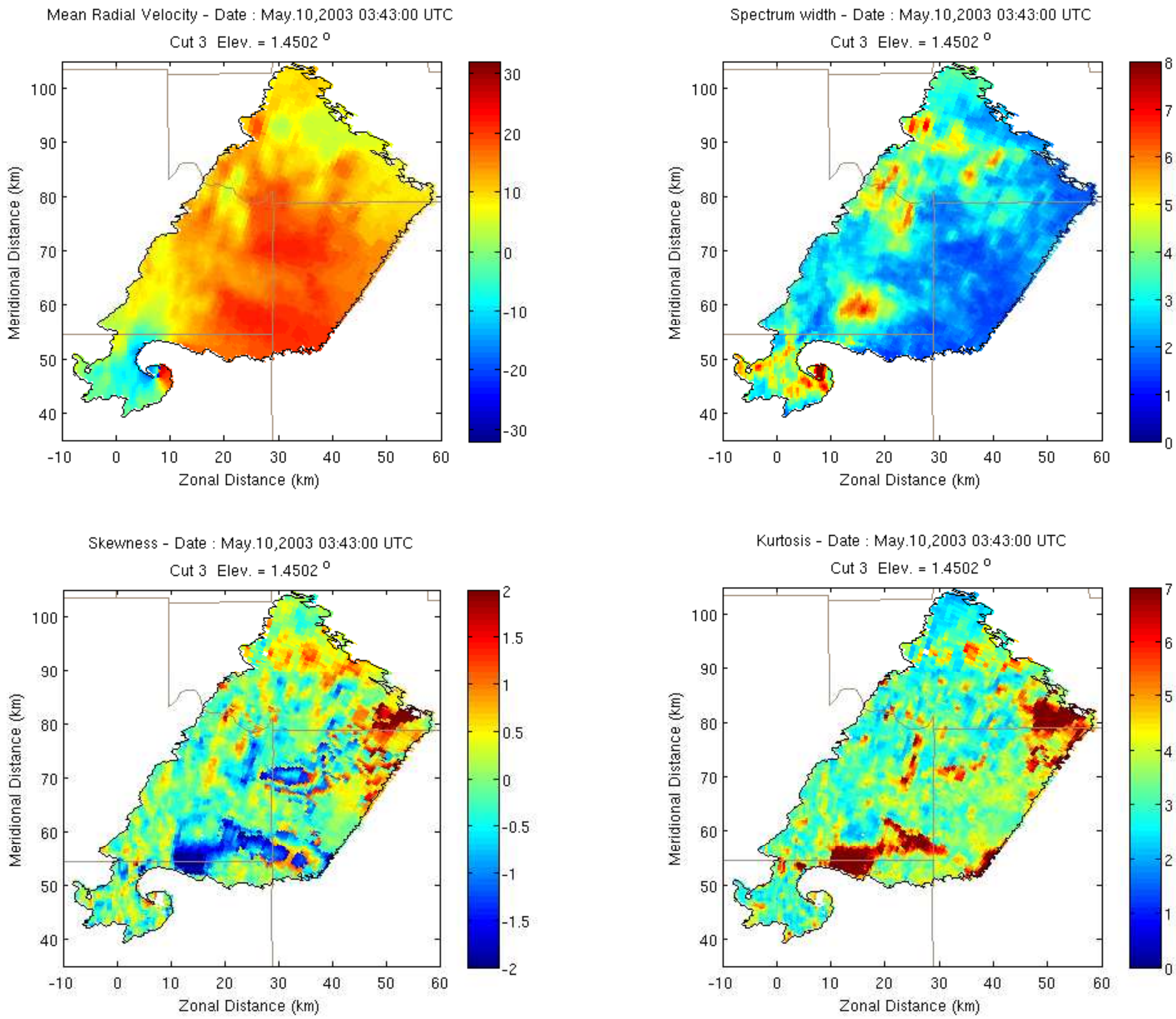

Figure 8: Moments estimation

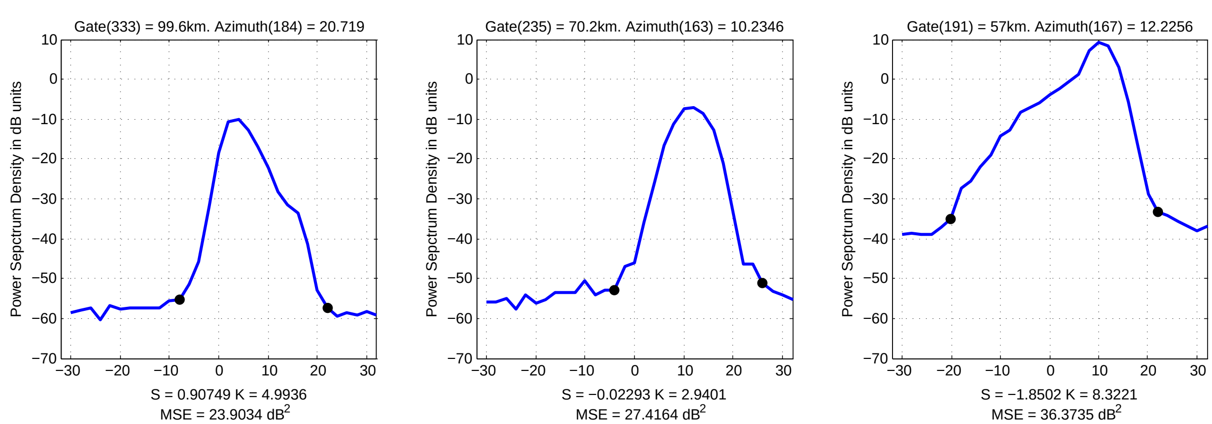

Figure 9: Skewness values for different shapes

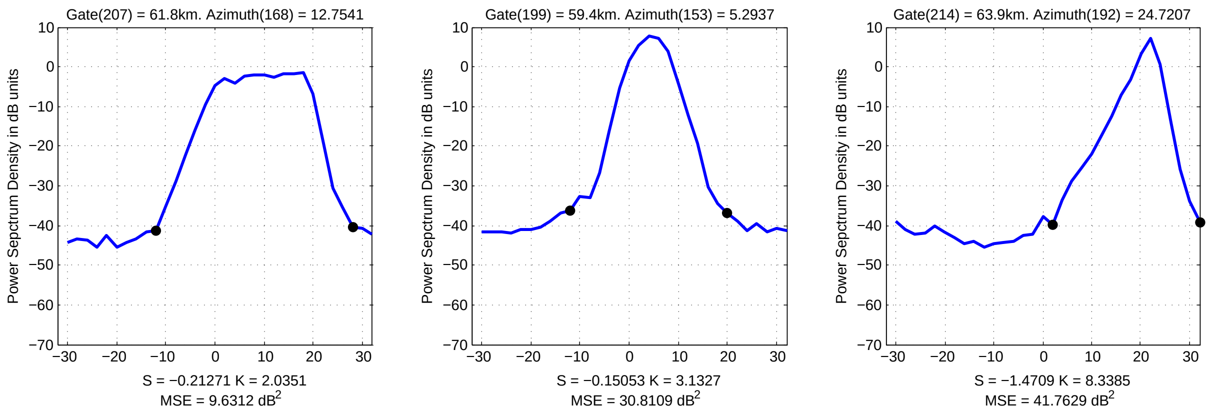

Figure 10: Kurtosis values for different shapes

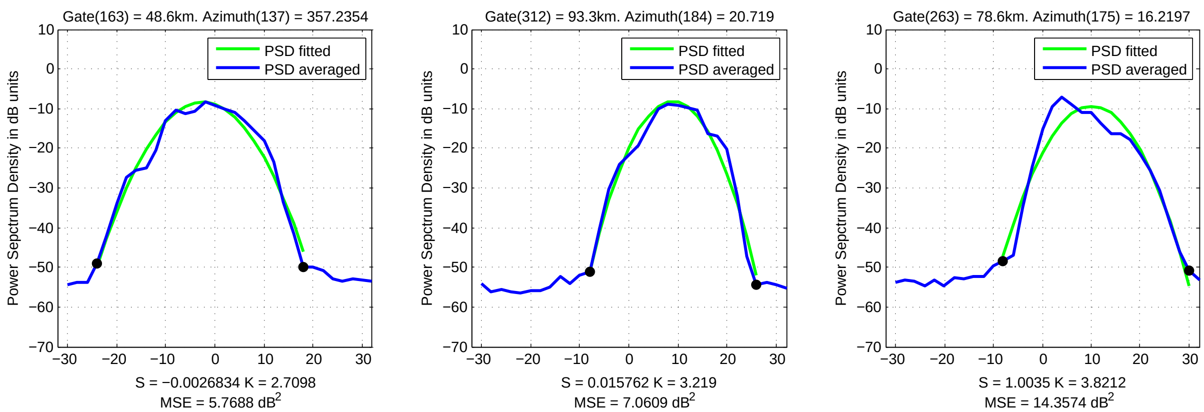

Figure 11: Doppler spectra and parabola fitted

For better visualization, different shaped spectra focused in skewness and kurtosis are shown in Figures 9 and 10 with their respective moments. On the other hand, Variability performance is showed in Figure 11, where the Doppler spectrum is fitted by a parabola.

7. RELATION BETWEEN STORM DYNAMICS AND SPECTRAL SHAPE

The supercell thunderstorm studied here has the characteristics of high precipitation type, matured evolution with a strong vertical shear that provide separation between updraft and downdraft. In addition, the storm is characterized by a remarkable boundary rain reflectivity, bounded weak echo region and hooklike features as showed by Figure 1 and 8. The location of the forward-flank (FFD) and rear-flank (RFD) downdrafts, north central and south west respectively, can be seen from the mean radial velocity, Figure 8, showing velocities near 0 ms^{-1}. The reflectivity figure shows strong precipitation and the spectrum width estimates with values close to 5 ms^{-1}. Such conditions enhance a Doppler spectra Gaussian-shaped which can be seen from Figures 6 and 7. Now, in the central supercell region, Figure 1 shows a strip shape where reflectivity precipitation is approximately 45 dBZ. This strip is also observed by a uniform flow of about 15 ms^{-1} and 3 ms^{-1} of mean radial velocity and spectrum width, respectively, in Figure 8. Under this condition, Figure 6 reflects another area with dominant Gaussian-shaped spectra. Figure 7 does not indicate this due to the threshold made for spectrum width when the spectra are not fitted.

Moving to the south central region we find a non Gaussian-shaped dominant area which is remarked by negative skewness and high kurtosis. One possible explanation for the Doppler spectrum shape on the right of Figure 9 could be a low-level updraft rotation process. Because the RFD forms a flanking line gust front forming horizontal rotation and the FFD enters the mesocyclone region, tilting and stretching of the horizontal vorticity creating low-level rotation form. Doppler spectra shapes are characterized by strong positive velocity and many negatives velocity components giving and environment where possible low-level rotation can be produced. In the north east region, Figure 6 symbolize a Gaussian assumption disagreement, skewness and kurtosis has not a uniform distribution and it is still an open question as to what makes this region happen to be non-Gaussian.

8. CONCLUSIONS

Skewness, kurtosis and a parabola fit have been proposed to quantify the deviation from Gaussian-shaped spectra in a tornadic supercell thunderstorm. Several examples Doppler spectra were shown to appreciate values given by the proposed Gaussian measures.

Simulations allowed to set intervals where a Doppler spectra could be classified as Gaussian-shaped according to skewness, kurtosis and mean square error.

The results show three regions where Gaussian-shaped spectra are dominant and two opposites cases. It is important to make clear that only spectra having a spectrum width between 3 and 7 ms^{-1} were used to fit the PSD with a parabola. Therefore, the percentage given by variability has not been processed under the same conditions where skewness and kurtosis were computed for spectra width smaller or equal to 7 ms^{-1}. Finally, explanations were given to provide a possible connection between spectra shape and storm dynamics.

Analysis of more Level-I data are desired for further Gaussian assumption evaluation under different weather characteristic and radar parameters that would provide more information for a better storm dynamics understanding.

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