1. INTRODUCTION

During periods of precipitation, vertically pointed wind profiling radars can be used to directly measure the drop size distribution (DSD) through the Doppler velocity spectrum. This is achieved by directly mapping the Doppler spectrum from velocity space into diameter space. In the absence of vertical ambient air motion and if Rayleigh scatter from discrete particles is the dominant contribution to the radar signal, the velocities detected by the profiler are primarily due to falling hydrometeors. Under these conditions, the DSD is retrieved from the Doppler spectrum by applying an appropriate expression that relates drop diameter to terminal fall speed. Precipitation parameters such as rainfall rate, radar reflectivity factor, liquid water content, mass-weighted mean drop diameter, and median volume drop diameter can be calculated from the retrieved DSD. Unlike in-situ instruments located at the surface, measurements from the profiler can be used to investigate the evolution of these parameters with height.

Several factors complicate the DSD retrieval process. First, the retrieval method relies on the assumption that there is no significant vertical ambient air motion. The presence of undetected updrafts and downdrafts will bias the retrieved number concentration. Second, there are many choices for fall speed relationships. Since the retrieval method assumes a single fall speed relationship, different expressions will result in different retrievals. Third, an inherent artifact of the DSD retrieval process prohibits accurate retrievals of the number concentration of very small drops, but the smallest diameter at which number concentration information should be included in DSD calculations is somewhat subjective. Ground clutter at the lowest sampling heights exacerbates this issue. Each of these considerations introduces errors into DSD retrievals that propagate through the precipitation estimations. These factors are examined and error analysis is presented.

The present study focuses on precipitation systems passing over central Oklahoma. The principal measurements are made using a 915-MHz boundary layer radar (BLR) and a two-dimensional video disdrometer (2DVD) located near the BLR. Emphasis is placed on non-convective systems due to the assumption of a quiescent environment. Time-height development of several parameters associated with the DSD are presented. In particular, transitions across the melting layer are examined. Rainfall parameters including rainfall rate, radar reflectivity factor, mass-weighted mean diameter, and median volume diameter are compared between the lowest sampled height from the BLR and the 2DVD. This study is motivated by ongoing comparisons with the the NOAA National Severe Storms Laboratory polarimetric S-band weather radar KOUN.

2. RETRIEVAL PROCEDURE

A DSD can be retrieved from each Doppler spectrum at each sampled height. Two main parts comprise the retrieval process. The first step is to map the Doppler spectrum (which is a distribution of power-weighted radial velocities [Doviak and Zrnić, 1993]) to a distribution of drop diameters. This is accomplished by assuming a fall speed relationship so that a value of velocity may be calculated for any value of diameter. The distribution of velocities can then be recast as a distribution of diameters by applying that equation in reverse. One such relationship is given by Atlas et al. [1973]:

\[
    v(D) = \begin{cases} 
    3.78D^{0.67}, & D < 3\text{mm} \\
    9.65 - 10.3e^{-0.6D}, & D > 3\text{mm} 
    \end{cases} 
\]

where \( D \) is the drop diameter in millimeters and \( v \) is the terminal velocity in m s\(^{-1}\). This hybrid relationship spans two diameter regimes. Another relationship is given by Brandes et al. [2002]:

\[
    v(D) = -0.1021 + 4.932D - 0.951D^2 + 0.07934D^3 - 0.002362D^4 
\]

Unless otherwise stated, the fall speed relationship used in this study is Eq. 2.
After transforming the set of velocity measurements into a set of assumed diameters, the second step is to use the Doppler spectrum to obtain information about the number of drops \( N(D) \) at each diameter. This is done by using the Doppler spectrum to find the power, which in turn is related to the equivalent radar reflectivity factor \( Z_e \). Additionally, the radar reflectivity factor is known in terms of drop diameters:

\[
Z_e = \int_0^{\infty} N(D_e) D_e^6 dD_e
\]

where \( D_e \) is the equivalent drop diameter and \( N(D_e) \) is the number density of hydrometeors per unit diameter per unit volume of air. When a calibrated profiler observes Rayleigh scattering from raindrops in the absence of noise, vertical motion, and turbulence, \( Z_e \) calculated from a Doppler spectrum should equal \( Z \) calculated from the DSD. Therefore, under the assumption that the Doppler spectrum due to falling hydrometeors is associated only with velocities greater than zero and using the convention that \( v > 0 \) indicates downwards motion, the contribution of \( N \) drops of size \( D \) to the Doppler spectrum \( S \) at speed \( v \) is

\[
S_{rc}(v)dv = N(D)D^6 dv
\]

where \( S_{rc}(v) \) is the range-corrected and calibrated (normalized) Doppler spectrum, \( D \) is the drop diameter, \( N(D) \) is the number of drops of diameter \( D \) per unit volume, and \( \frac{dv}{dD} \) comes from the relationship between drop diameter and terminal velocity [Atlas et al., 1973; Gos-sard, 1988]. Since all of the quantities except \( N(D) \) are known, rearranging this equation yields:

\[
N(D) = \frac{S_{rc}(v) dv}{D^6 \frac{dv}{dD}}
\]

In this manner, \( N(D) \) can be retrieved from a Doppler spectrum.

In order to compare retrieved DSDs to those measured by the 2DVD, this study focuses on time averaging and integral parameters such as reflectivity factor \( Z \), rainrate \( R \), and median volume diameter \( D_0 \). The reflectivity factor \( Z \) is given in Eq. 3. The rainrate \( R \) is given by:

\[
R = \frac{\pi}{6} \int_{D_{min}}^{D_{max}} N(D) D^3 v(D) dD
\]

where \( D_{min} \) and \( D_{max} \) are the minimum and maximum diameters respectively, \( N(D) \) has units of m\(^{-3}\) mm\(^{-1}\), and \( v(D) \) is the terminal velocity of the drop. In the presence of ambient air motion, \( v(D) \) is replaced by \( (v(D) - w) \), where \( w \) is the vertical velocity of the ambient air. The median volume diameter \( D_0 \) is given by:

\[
\int_0^{D_0} D^3 N(D) dD = \int_{D_0}^{\infty} D^3 N(D) dD
\]

where \( N(D) \) is the number concentration and \( D \) is the diameter. For measured DSDs rather than analytical models, the minimum diameter is \( D_{min} \) instead of zero and the maximum diameter is \( D_{max} \) instead of infinity.

3. INSTRUMENTATION

3.1. 2DVD

The two-dimensional video disdrometer (2DVD) directly measures DSDs by creating a virtual measuring area with two orthogonal light sheets. Each light sheet is monitored by a line-scan camera, and drops that pass through the virtual measuring area create shadows that are detected by the cameras. This information is then processed to determine rain drop properties such as diameter, oblateness, and the number of drops of each size that fell through the measuring area. The 2DVD collects data continuously and reports these properties for consecutive one minute periods. For a more complete description of 2DVD operation, see Kruger and Krajewski [2002].

3.2. Wind profiler

The UHF wind profiler used in this investigation operates at 915 MHz. Table 1 lists typical operating parameters of the vertical beam mode used during this study.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>915 MHz</td>
</tr>
<tr>
<td>pulse width</td>
<td>700 ns</td>
</tr>
<tr>
<td>range resolution</td>
<td>105 m</td>
</tr>
<tr>
<td>number of coherent integrations</td>
<td>100</td>
</tr>
<tr>
<td>number of FFT points</td>
<td>128</td>
</tr>
<tr>
<td>interpulse period</td>
<td>60 µs</td>
</tr>
<tr>
<td>Nyquist velocity</td>
<td>13.6 m s(^{-1})</td>
</tr>
<tr>
<td>effective dwell time</td>
<td>23 s</td>
</tr>
<tr>
<td>number of height gates</td>
<td>60</td>
</tr>
<tr>
<td>lowest sampled height</td>
<td>142.5 m</td>
</tr>
<tr>
<td>highest sampled height</td>
<td>6.3 km</td>
</tr>
</tbody>
</table>

Table 1: Typical operating parameters for the UHF profiler used in this study.

Typical scanning Doppler radars use the convention that \( v_r < 0 \) indicates motion toward the radar. This study employs the opposite convention: \( v_r > 0 \) indicates motion toward the radar. Since the profiler antenna points vertically, motion towards the antenna indicates falling (downward motion of) hydrometeors. During periods of
precipitation and in the absence of significant vertical ambient air motion and turbulence, the range of velocities comprising the Doppler spectrum is due to a range of sizes of hydrometeors within that resolution volume.

4. ERROR ANALYSIS

4.1. Vertical air motion

This DSD retrieval method assumes that all contributions to the Doppler spectrum are due to hydrometeors falling through quiescent air. If the air itself is also moving, the fall speeds measured by the profiler will be the combined fall speeds of the hydrometeors and the air rather than the hydrometeors alone. Ideally, this effect would be reduced by using the clear air component of the Doppler spectrum to estimate the ambient air motion and correct for the bias [Ulbrich, 1983]. It was found during this study that the precipitation signal overwhelmed the clear air signal. Additionally, significant ground clutter also tended to mask the clear air signal. In the absence of external information about vertical air motion, this retrieval method is restricted to cases with limited vertical air motion (e.g., stratiform rain).

In stratiform rain, typical vertical velocities for ambient air are about 20–60 cm s\(^{-1}\) [Rutledge et al., 1988]. Figure 1 shows a graph of rain drop fall speed \(v(D)\) (Eq. 2) under quiescent conditions and fall speed \(v'(D) = v(D) + w\) in descending air.

![Figure 1: Comparison of rain drop terminal velocity in quiescent conditions and in descending air.](image)

To gain insight into how a hidden bias in the Doppler spectrum will affect integral parameters for retrieved DSDs, it is useful to examine the effect of vertical air motion on integral parameters for analytical DSDs. Table 2 lists the true rain rate for two analytical DSDs along with the rain rates that result from applying an updraft \((w < 0, \text{ updraft})\) and a downdraft \((w > 0, \text{ downdraft})\).

Table 2 shows the expected result that for a given DSD, a hidden updraft decreases \(R\) while a hidden downdraft increases \(R\). The general problem presented by this retrieval method, however, is that the DSD itself changes based on the values of \(v(D)\). In other words, the Doppler spectrum represents \(v'(D)\) rather than \(v(D)\). One way to study this problem is to take a known DSD, assume the drops fall with velocity \(v(D)\) in calm air, uniformly bias the fall speeds by an updraft/downdraft to obtain \(v'(D) = v(D) + w, w > 0\) for a downdraft and \(w < 0\) for an updraft, retrieve the DSD corresponding to \(v'(D)\), calculate integral parameters for the retrieved DSD, and compare the original integral parameters to those from the retrieved DSD. The difference between the two sets of integral parameters shows how the presence of an unaccounted-for updraft/downdraft affects the retrieved DSD. Table 3 lists integral parameters for a Marshall-Palmer DSD \((R = 10 \text{ mm h}^{-1})\) as well as integral parameters from the DSD retrieved for nonzero vertical ambient air motions. Although only one DSD (Marshall-Palmer) at one rainrate \((R = 10 \text{ mm h}^{-1})\) is shown in Table 3, other rainrates and analytical DSDs produce similar results.

Table 3 shows a counterintuitive result: rainrates from retrieved DSDs decrease in downdrafts and increase in updrafts. To understand this result, consider the idealized Doppler spectrum shown in Figure 2. In the presence of an updraft (or, equivalently, after removing a downdraft), all drops fall more slowly, so the entire spectrum shifts to the left. The contribution to the spectrum \(S(v)dv\) at low \(v\) is higher than it was before because even though \(S\) itself has not changed, every \(S\) is now located at a lower \(v\). Since \(D \propto v\), there is higher \(S\) at lower \(D\). The contribution to the spectrum \(S(v)\) is directly proportional to \(N(D)D^3\), so if \(S\) remains constant and \(D\) decreases, then \(N(D)\) must increase. In other words, it takes far more small drops than large drops to produce a given spectral contribution \(S\). The rainrate \(R\) from retrieved DSDs increases in an updraft (or after removing a downdraft) because \(R\) is proportional to \(N(D)D^3\). Similarly, in the presence of a downdraft (or after removing an updraft), \(R\) from retrieved DSDs will decrease.

4.2. Fall speed relationships

The DSD retrieval process relies on the existence of a relationship between a drop's diameter and its fall speed. Since a variety of fall speed relationships have
Table 2: Effect of ambient air velocity on rain rate. For both cases, $|w| = 0.6$ m s$^{-1}$, representing a worst-case scenario for stratiform rain. The distributions are given by Marshall et al. [1947] and Laws and Parsons [1943].

<table>
<thead>
<tr>
<th>DSD</th>
<th>true R</th>
<th>updraft R</th>
<th>downdraft R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshall-Palmer 5 mm h$^{-1}$</td>
<td>4.37 mm h$^{-1}$</td>
<td>5.63 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Marshall-Palmer 10 mm h$^{-1}$</td>
<td>8.86 mm h$^{-1}$</td>
<td>11.14 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Marshall-Palmer 20 mm h$^{-1}$</td>
<td>17.91 mm h$^{-1}$</td>
<td>22.09 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Marshall-Palmer 50 mm h$^{-1}$</td>
<td>45.31 mm h$^{-1}$</td>
<td>54.69 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Laws and Parsons 5 mm h$^{-1}$</td>
<td>4.48 mm h$^{-1}$</td>
<td>5.52 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Laws and Parsons 10 mm h$^{-1}$</td>
<td>9.03 mm h$^{-1}$</td>
<td>10.97 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Laws and Parsons 20 mm h$^{-1}$</td>
<td>18.20 mm h$^{-1}$</td>
<td>21.80 mm h$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Laws and Parsons 50 mm h$^{-1}$</td>
<td>45.86 mm h$^{-1}$</td>
<td>54.14 mm h$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Effect of ambient air velocity on integral parameters for retrieved DSDs. The original DSD follows Marshall-Palmer with R = 10 mm h$^{-1}$. Fallspeeds for the drops were calculated for the range of drop diameters, biased with an updraft/downdraft, and used to construct a Doppler spectrum. The DSD was then retrieved from the constructed spectrum, and all integral parameters were calculated from the retrieved DSD. Note that only NaNs and Infs were excluded and no thresholding was applied.

<table>
<thead>
<tr>
<th>Marshall-Palmer R = 10 mm h$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>original $w = -0.6$ m s$^{-1}$ $w = 0.6$ m s$^{-1}$</td>
</tr>
<tr>
<td>R 10 mm h$^{-1}$</td>
</tr>
<tr>
<td>Z 39.4 dB</td>
</tr>
<tr>
<td>$D_{0i}$ 1.40 mm</td>
</tr>
<tr>
<td>$D_{\text{rms}}$ 1.58 mm</td>
</tr>
</tbody>
</table>

been published to cover a wide range of drop diameters, one source of error in retrieved rainfall parameters may be due to the selection of a particular fall speed relationship. Figure 3 shows a time history of rainfall rate from 2 May 2005 based on retrieved DSDs using Eq. 1–2. Since the differences are quite small, it is reasonable to conclude that any appropriate fall speed relationship valid over the range of diameters is sufficient for DSD retrievals.

4.3. Minimum included diameter

Since this retrieval method assumes that only hydrometeors contribute to the Doppler spectrum, clutter contamination near 0 m s$^{-1}$ is erroneously interpreted as the result of very large numbers of very small drops. This is an artifact of the retrieval method. Applying velocity thresholding to a Doppler spectrum prior to retrieving the DSD ensures that clutter contamination is excluded, but it also imposes a lower bound on the retrieved drop diameters. Applying a minimum threshold in velocity is equivalent to truncating the DSD, which introduces errors in the integral parameters. To estimate this error, it is useful to examine analytical DSDs because the rainrate is known. Figure 4 shows the effect of velocity thresholding on rainfall rate calculation for four DSDs: Marshall-Palmer [Marshall et al., 1947], Laws and Parsons [Laws and Parsons, 1943], Joss-Drizzle [Joss and Gori, 1978], and Joss-Thunderstorm [Joss and Gori, 1978]. As the velocity threshold is raised, more of the DSD is truncated and the rainrate decreases. It was found in this study that a threshold of 2.5–2.7 m s$^{-1}$ was appropriate for most of the precipitation events studied. A threshold of 2.5–2.7 m s$^{-1}$ corresponds to drop diameters of 0.5–0.7 mm using Eq. 2.

4.4. Air density

Since air density decreases with height, it is necessary to correct for this effect prior to retrieving DSDs. An outside source of air density information is required to calculate the correction factor. Two options for air density information are the U. S. Standard Atmosphere and environmental soundings. Although the differences between them are small, it is prudent to check whether or not those small differences will greatly affect DSD retrievals and subsequent calculations. Figure 5 shows rainfall rates from two DSD retrievals for the same rain
5. RESULTS

5.1. Case study: 17 Sept 2006

The rain event on 17 Sept 2006 in central Oklahoma was selected as a case study due to prolonged periods of stratiform rain and the existence of a 2DVD data set for comparison even though the precipitation was often mixed-type rather than purely stratiform. Storms initiated ahead of a southeastward-moving cold front and moved to the northeast, eventually merging with remnants from an earlier line. Based on the 12z (0700 local) sounding, the freezing level was at approximately 4387 m (595 mb).

Figure 6 shows a range-time-intensity plot of SNR (top), Doppler velocity (middle), and spectrum width (bottom) from the profiler for 17 Sept 2006. The melting layer is most visible on the Doppler velocity plot just below 4000 m where the velocities suddenly increase from near 0 m s$^{-1}$ (shown in yellow and green) to 2–5 m s$^{-1}$ (shown in orange and red). The location of the radar bright band/melting layer in this figure (below the freezing level) is consistent with the typical melting layer structure described by Stewart et al. [1984].

Figure 7 shows a comparison of $Z$ between the profiler and 2DVD on 17 Sept 2006. A time offset is clearly visible, but since there is some question as to whether or not the radar control computer time offset is a constant, the time history curve for the profiler is simply presented “as-is” without shifting. Agreement in amplitude is excellent because this is the data set used for calibration of the profiler. The very good agreement in features (peaks and valleys) demonstrates that this retrieval method works. The four rain-free periods appear in the 2DVD curve as flat lines with occasional sharp peaks. SNR thresholding would be a way to eliminate these areas of the plot where it is not raining and thus makes no sense to do DSD retrievals.

Figure 8 shows a comparison of rainrate between DSDs retrieved from the profiler, DSDs measured by the 2DVD, and rainfall measured over 5-minute periods by the Oklahoma Mesonet station in Washington (co-located with the profiler and 2DVD). Since Mesonet stations record the accumulated rainfall over each 5-minute period of the day, a conversion is required to obtain a rainfall rate representing the average rainfall rate over the 5 minute period. Figure 8 shows that the profiler and 2DVD are generally in good agreement with each other and often in good agreement with the Mesonet station. Disagreements with the Mesonet station tend to occur during periods of low rain rates. During low rain rate periods, the Mesonet tipping bucket rain gauge may require several 5-minute periods to register a single tip (each tip is 0.254 mm, or 0.01 inch). Table 4 lists the total accumulated rain measured by each instrument.
Comparison of rainfall rate based on retrieved DSDs using different fall speed relationships

![Graph showing rainfall rate over time with two curves for different fall speed relationships.](image)

**Figure 3:** Time history of rainfall rate for 2 May 2005 based on DSDs retrieved with different fall speed relationships. The total accumulated rain for each retrieval is listed beneath the legend.

<table>
<thead>
<tr>
<th>profiler gate 2</th>
<th>2DVD</th>
<th>Mesonet</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>27.61 mm</td>
<td>38.19 mm</td>
</tr>
</tbody>
</table>

**Table 4:** Total accumulated rainfall for 17 Sept 2006.

Height profiles of \(a\) and \(b\) from Z-R relationships (both \(Z\) and \(R\) obtained from retrieved DSDs) for all gates are shown in Figure 9. The calculations excluded rain-free regions of the data set as well as a few strongly convective regions, which were excluded only because this method incorrectly retrieves DSDs in convective regions. Below the melting layer, \(a\) and \(b\) are nearly constant, although \(b\) slightly decreases with increasing height and \(a\) slightly increases with increasing height. Directly below the freezing level, \(b\) increases and then decreases.

### 5.2. Discussion

The agreement in total rainfall rate between the profiler and any other instrument is rather poor. Based on other work showing much better agreement for a known stratiform case [Kanofsky, 2007], it is believed that the poor agreement for 17 Sept 2006 is largely due to the mixed nature of the precipitation.

Since this DSD retrieval method relies on the presence of liquid water drops and the assumption of quiescent ambient air, DSD retrievals are restricted to regions below the melting layer in stratiform rain. It is necessary to exclude periods of convective rain that may be present in the data sets from any DSD retrievals.

It was found that the choice of fall speed relationship produces only minimal errors in rainrate estimates, provided that the selected relationship is valid over an appropriate range of diameters. Using two different fall speed relationships to retrieve DSDs and calculate rainfall rate resulted in a difference of approximately 1 mm over an 8 hour data set.

It was found that an appropriate velocity threshold for these radar parameters was approximately 2.6–2.7 m s\(^{-1}\), corresponding to a diameter threshold of 0.6–0.8 mm with Eq. 2. Simulations with analytical DSDs suggest a \(-4\)% to \(-10\)% error in rainrate due to trun-
Figure 4: Effect of velocity thresholding on rainrate. “D_{min}” refers to the minimum diameter that is included in the rainrate calculation. Higher values of D_{min} correspond to higher velocity thresholds and more truncation. The arrows point to the intersection of the −10% and −20% error lines with the rainrate curves for the four DSDs. The four numbers refer to the diameters at the intersections. For example, for a given acceptable error of −10%, the cutoff diameter must be no larger than ≈ 0.59 mm for a Joss-Drizzle DSD with R = 5 mm h⁻¹ and no larger than ≈ 1.41 mm for a Laws and Parsons DSD with R = 20 mm h⁻¹.

It was found that the error in rainrate due to the difference between actual air density (calculated from radionsondes) and air density given by the U. S. Standard Atmosphere was less than 0.3 mm h⁻¹ for stratiform rain near the ground. In regions where this retrieval method is valid, the U. S. Standard Atmosphere is an appropriate source of air density information.

6. FUTURE WORK

A method is currently being explored to retrieve vertical air motions from combined polarimetric weather radar and profiler measurements during precipitation periods [Teshiba et al., 2005]. These ambient air velocity estimates could be used to correct for non-quiescent conditions.

Another area for future work involves KOUN, the dual-pol research radar at NSSL. KOUN data sets can be used for DSD retrievals by assuming that the DSD follows a modified gamma distribution and then calculating the parameters for the distribution [Zhang et al., 2001; Ryzhkov et al., 2005]. DSD retrievals from RHI scans above the instrumentation site would provide additional observations of DSDs aloft, which could be compared to those obtained with the profiler. Some avenues for exploration include assessing the degree of agreement between instruments, determining whether KOUN DSD retrievals produce accurate estimates of rainfall rate at
Figure 5: DSDs were retrieved for the same rain event using two different air density sources. One retrieval used density information from the U. S. Standard Atmosphere while the other retrieval used density information from a sounding that was released near the profiler site.

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References


Figure 6: Range-time-intensity plot of SNR (top), Doppler velocity (middle), and spectrum width (bottom) from the profiler for the precipitation event on 17 Sept 2006. All times shown are local.


Figure 7: Comparison of $Z$ between profiler and 2DVD for 17 Sept 2006.

Figure 8: Comparison of rain rate between 3 instruments for 17 Sept 2006. For more information about the slight variations in Mesonet rain rate, see Appendix ??.
Figure 9: Height profile of $a$, $b$, and goodness of fit for 17 Sept 2006. Also shown is the height of the freezing level obtained from the 12Z sounding.