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SINGLE SCATTERING FROM FROZEN HYDROMETEORS FOR CLOUD RADAR APPLICATIONS

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1. INTRODUCTION

In order to observe solid precipitation by ground-based as well as space-borne cloud radars, precise knowledge of the scattering behavior of frozen hydrometeors at centimeter- and millimeter-wavelengths is required. Only for homogeneous spherical particles, the scattering of electromagnetic waves can be computed in a mathematically exact way using the conventional Mie-theory. However, frozen hydrometeors exist in a huge variety of non-spherical shapes.

In this study single scattering parameters (radar cross section, scattering cross section, absorption cross section) for simplified shape classes of frozen hydrometeors were calculated using the discrete-dipole approximation (DDA) method. The particles were modeled as hexagonal plates, columns, needles and dendrites as well as rather spherical graupel particles. The calculations were carried out over a wide range of centimeter- and millimeter-wavelengths.

2. THE DDA

The discrete-dipole approximation (DDA) is a powerful technique for computing scattering and absorption by targets of arbitrary geometry (Draine and Flatau, 1994). The DDA models the actual particle by an array of dipoles. Each dipole may be thought of as representing a particular subvolume of the particle. Each dipole is subject to the incident wave and is also subject to the electric fields of all other dipoles.

In this study the applicability of the DDA method to calculate scattering of radar waves by liquid and frozen precipitation particles is tested. By employing the DDA to water and ice spheres with a diameter *D* comparable with the size of natural cloud and precipitation particles and comparing the results to the Mie-solution, it was evaluated how many dipoles have to be used to reach a certain accuracy for given refractive indices and wavelengths. This number of dipoles can be a guiding value when modeling non-spherical precipitation particles with no exact scattering solution.

The scattering parameters were simulated for incoming plane waves from 2.8 GHz to 150 GHz. The frequency and temperature dependant refractive indices of water and ice were chosen according to Ray (1972). The calculations were carried out with the ADDA software package in the version 0.76 (Yurkin and Hoekstra, 2006). Initially the spheres were modeled with a grid of 16 (2176 dipoles) as shown in Figure 1. The separation *d* between the dipoles was slightly altered so that the volume of the dipole representation of the sphere is exactly correct with the volume of the sphere. Instead of $d = D/16 = D * 0.0625$, the value $d = D * 0.0622$ was used.

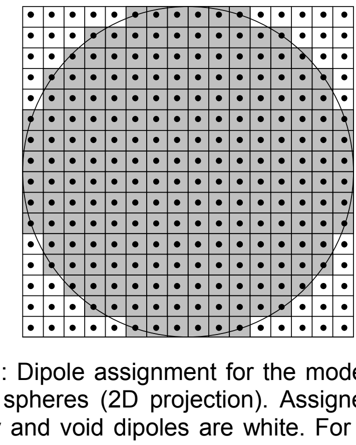

Figure 1: Dipole assignment for the modeled water and ice spheres (2D projection). Assigned dipoles are gray and void dipoles are white. For the whole sphere 2176 dipoles were assigned.

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A principal criterion for the applicability of the DDA is that the separation d between the dipoles is small compared to the wavelength λ. Numerical studies e.g. Draine and Flatau (1994) indicate that this is adequately satisfied if

$$|m|\ k\ d < 1 \qquad (1)$$

where m is the complex refractive index of the target material and $k = 2\pi / \lambda$.

However, for accurate calculations of the scattering phase function (e.g., for radar cross-sections), a more conservative criterion

$$|m|\ k\ d < 0.5 \qquad (2)$$

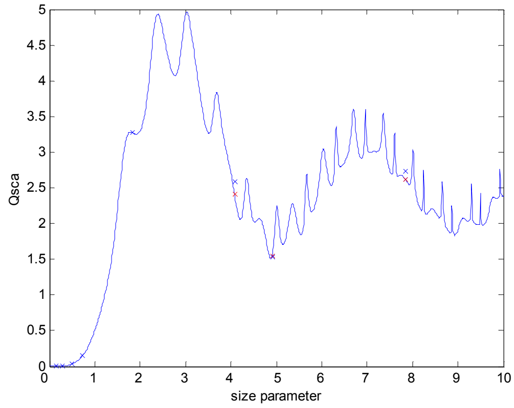
is suggested by Draine (2000).

The calculations of this study comply with the more conservative criterion (2), and often go considerably below. Figure 2 compares the scattering parameters of an ice sphere (D = 5 mm) determined with the DDA with the correct Mie solutions. Table 1 summarizes characteristic figures for this example. The sphere was initially modeled with a grid of 16. It can be seen that the results show a tendency to differ if $|m| * k * d > 0.5$. When using a grid of 64 however $|m| * k * d < 0.5$ and the calculation is again quite exact.

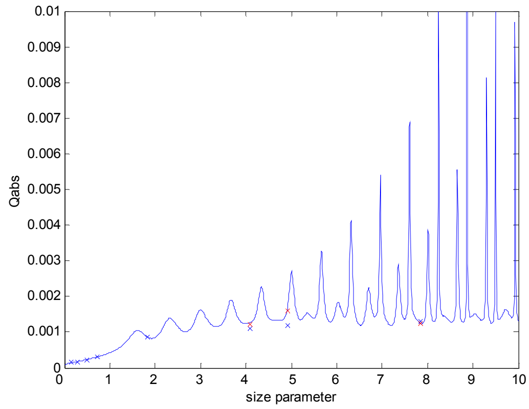
When modeling water spheres with the DDA it was found that $|m|\ k\ d$ should be smaller then 0.05 in order to get reliable scattering results.

Table 1: Characteristic size figures of a modeled ice sphere with D = 5 mm for frequencies up to 150 GHz. For frequencies higher than 43 GHz $|m| * k * d > 0.5$. For these frequencies therefore the sphere were modeled with a grid of 64.

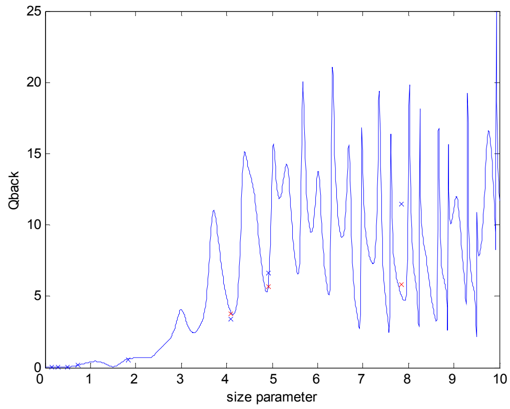
Frequency [GHz]	Grid	Dipoles/λ	$\|m\| * k * d$
2.8	16	342.9	0.033
5.6	16	171.4	0.065
9.6	16	100.0	0.112
14	16	68.6	0.163
35	16	27.4	0.408
78	16	12.3	0.909
94	16	10.2	1.095
150	16	6.4	1.748
78	64	49.2	0.227
94	64	40.9	0.274
150	64	25.6	0.437



(a)



(b)



(c)

Figure 2: Efficiency factors for scattering Qsca (a) absorption Qabs (b) and backscattering Qback (c) for an ice sphere D = 5 mm as a function of the size parameter circumference of the sphere over wavelength. The solid blue lines represent the correct Mie solution. The blue crosses represent the results of the DDA with a grid of 16 (2176 dipoles). The red crosses those with a grid of 64 (137376 dipoles).

3. SCATTERING PARAMETERS OF SIMPLE ICE CRYSTALS

With the DDA-method also the scattering parameters of several simple ice crystal shapes as they were classified by Magono and Lee (1966) were calculated. The particles were represented by dipoles in a way that at every frequency the model at least complies with criterion (2). The ice crystals were assumed to consist of pure ice at -5° C. The refractive index of ice at this temperature and at the calculated frequencies were chosen according to Ray (1972).

In Figure 4 the scattering parameters of a hexagonal prism (Figure 3) are presented. The prism has a volume of 1 mm³. The incident wave is assumed to propagate along the principal axis c.

Table 2 gives the size parameters for the regarded prism.

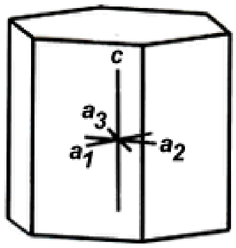
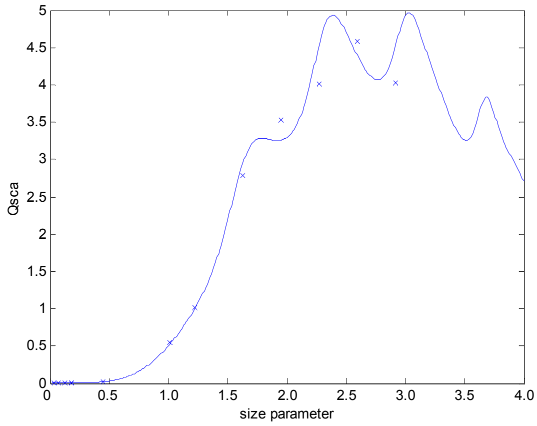


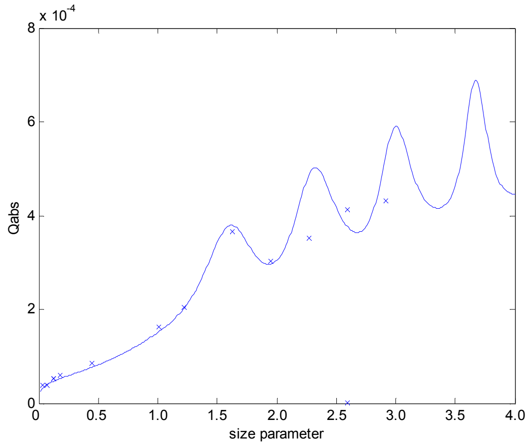
Figure 3: Hexagonal prism with principal axis c and secondary axes a_1, a_2, a_3.

Table 2: Observed frequencies and corresponding size parameters $2\pi r_{ev} / \lambda$ for a hexagonal prism with a volume of 1 mm³. r_{ev} is the radius of a equal volume sphere and λ is the wavelength.

Frequency [GHz]	Size parameter $2\pi r_{ev} / \lambda$
2.8	0.04
5.6	0.07
9.6	0.12
14	0.18
35	0.45
78	1.01
94	1.22
125	1.62
150	1.95
175	2.27
200	2.60
225	2.92



(a)



(b)

Figure 4: The blue crosses show the efficiency factors for scattering Qsca (a) and absorption Qabs (b) for a hexagonal prism with a volume of 1 mm³. The solid lines show the exact Mie solution of the equal volume ice sphere (r_{ev} = 0.62 mm).

Up to a size parameter of about 2 the scattering parameters of the ice crystal fit with that of the equal volume sphere. For size parameters higher than 2 the specific shape of the particle becomes important.

4. SCATTERING PARAMETERS OF GRAUPEL PARTICLES

Graupel occurs nearly spherical in shape and therefore their scattering behavior can simply be approximated by that of spheres. They can reach diameters up to 5 mm. For the following calculation a rimed graupel particle (5 mm in diameter) was regarded. A rimed graupel particle has many interstices between the droplets that instantly freeze on the surface. The density of graupel particles varies greatly – from about 0.05 g/cm to as high as 0.89 g/cm (Pruppacher and Klett, 1978). For the modeling of the inhomogeneous material of such a particle, the Maxwell-Garnett (1904) approximation was used. Figure 5 shows the backscattering or radar cross section (RCS) of a rimed graupel particle with 5 mm in diameter under the assumption that 80 % of the particle consist of ice while the remaining 20 % represent air-locks. The RCS of the rimed graupel particle is on the average about 3 dB lower in the plotted frequency range than that of a solid ice sphere with the same size.

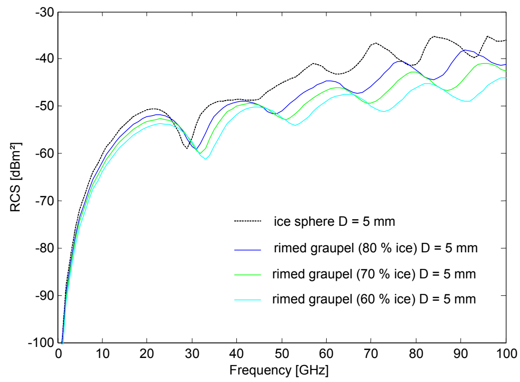


Figure 5: RCS of a rimed graupel particle, 5 mm in diameter. The inhomogeneous material was approximated by the formula of Maxwell-Garnett. For comparison: RCS of a solid ice sphere with the same size.

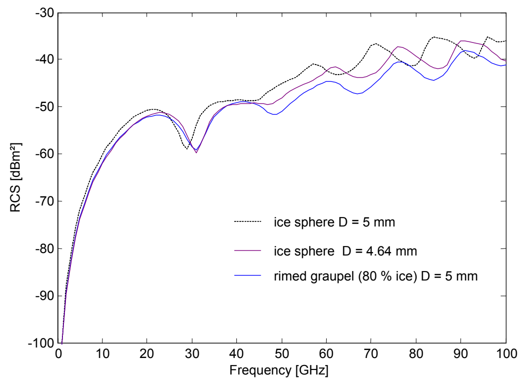


Figure 6: RCS of a rimed graupel particle with 5 mm in diameter and the approximation with a solid ice sphere with the same liquid volume.

In Figure 6 the RCS of a spherical rimed graupel particle is compared with that of an equal volume solid ice sphere. If we assume that the rimed graupel particle has a diameter of 5 mm and consists of 80 % ice, the equal volume ice sphere has a diameter of 4.64 mm. Up to 40 GHz, exceeding the characteristic notch of the RCS function of ice spheres, the RCS of the two particles are almost identical. This again would correspond with a size parameter of about 2.

5. CONCLUSION

In this study the scattering parameters of non-spherical ice crystals and nearly spherical graupel particles were observed. The results show that if the size parameter $2 \pi r_{ev} / \lambda < 2$ exclusively the ice volume is crucial for scattering and absorption. For such small sizes the actual shape of the particle needs not to be modeled - just the Mie solution of an equal volume ice sphere can be used to calculate the scattering parameters.

The results also indicate that the DDA is an applicable tool for calculating scattering and absorption of centimeter- and millimeter wavelengths from of non-spherical ice particles. For particles that consist of water (e.g. melting particles) however, the DDA does not seem applicable because of the higher refractive index of water than that of ice. In addition it was found, that for water $|m| * k * d$ should be smaller than 0.05 which makes the scattering calculation especially at millimeter wavelengths a time consuming task.

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