1. INTRODUCTION

Radar-based extrapolation utilizes observations in an optimal way to generate accurate precipitation forecast. Recent advance in data assimilation techniques facilitates the use of observations in numerical models and leads to the improved precipitation forecast. However, they still suffer from phase and intensity errors, possibly due to imperfect parameterization of various physics. Small phase errors can easily undermine the use of the traditional point-by-point skill scores (Ebert and McBride 2000). Thus, the separate evaluation of phase and intensity errors is extremely important to better represent the model performance. In addition, their consistency should be evaluated to better understand dynamics and parameterization in numerical models. In this work, we demonstrate the systematic evaluation of model errors (phase and intensity errors) from a meso-scale model and their consistency in space and time. Furthermore, we apply a correction algorithm of these errors to improve the accuracy of model precipitation forecast. This correction algorithm assumes the consistency of model errors in time and utilizes radar extrapolation techniques to advect model errors into the future. The applicability of this correction technique at the scale of North America is already shown (Lee and Zawadzki 2006).

2. Data and model used

A meso-scale, real-time, four-dimensional data assimilation and short-term forecasting system (RTFDDA) has been built upon a high-resolution MM5 and the Newtonian Relaxation (nudging) scheme. This MM5-RTFDDA incorporates three-dimensional mosaic radar data to modify the latent heat and is cycled every three-hours (Cram et al. 2001; Liu et al. 2002; Xu et al. 2005). The mosaic reflectivity is first converted into precipitation fields and is interpolated into the model grid. Then, the precipitation field is nudged into the model and the latent heat is modified. Various other observations, such as those from the traditional surface and upper air sounding network, mesonet, profilers, aircraft reports and satellites, are assimilated using an observational nudging approach.

A field demonstration of MM5-RTFDDA was conducted around Illinois and Indiana areas during May 15 – August 31, 2006. This uses three nested domains with 45 km, 15 km, and 5 km horizontal resolutions. The insertion of radar data is performed every 15 minutes up to $t_0 - 0.25$ hours and the forecast is made from $t_0$ hours to $t_0 + 12$ hours. We have used the precipitation forecast at the temporal resolution of five minutes and at the grid spacing of 5 km by 5 km in the domain of about 1000 km by 750 km.

Three-dimensional mosaic radar data is obtained from the National Mosaic and Next Generation QPE project (NMQ; Zhang et al. 2005; http://www.nmq.nssl.noaa.gov/). Data from the individual radars are first quality controlled and then are analyzed into eight tiles from which three-dimensional continental US grid data are formed at the resolution of 0.01 degree in horizontal, 0.25 km ~ 2 km in vertical, and 5 minutes in time. These three-dimensional radar mosaic data are used in the nudging. The hybrid surface rain rate maps (HSR) are derived from the mosaic by taking the lowest non-missing values at each grid column and then by applying convective and stratiform R-Z relationships. The convective and stratiform regions are identified from the derived vertical profiles of reflectivity. The HSR is used to quantify the model errors and to verify the correction results.

3. Methodology

a. Overall procedure

The following describes the general procedure of correcting phase and intensity errors of model precipitation forecasts:
1) Derive the local initial phase errors at the current time, \( t_c \), using the variational echo tracking (VET) between HSR and model precipitation fields.

2) Derive the local phase errors at previous time steps \( (t_c – n\Delta t, \cdots, t_c – \Delta t) \). This could be done by applying the VET between HSR and model fields at each time step. The derived phase errors should rely on the performance of models. Thus, this procedure does not guarantee the temporal continuity of phase errors when the model performance is especially poor. Instead, we have applied the semi-Lagrangian advection scheme to derive the phase errors. The two matching grids from HSR and model in which the local initial phase errors are corrected are advected backward to each previous time step using model and radar echo motions. Then, the phase errors at each time step are derived from the locational difference between the two advected grids. That is, we derive n phase errors at each model grid and these errors follow the echo motions.

3) These \( (n + 1) \) time-dependent phase errors [including initial phase errors derived from the procedure 1)] are fitted with a linear regression as a function of time. Thus, each grid has the linear time-tendency equation of phase errors. The procedure 1) to 3) refers to as the derivation of Lagrangian time-dependent phase errors.

4) In addition to the phase errors, we can derive the intensity (or amplitude) errors and their tendency at each grid. Here, we have derived the intensity errors by calculating the difference between HSR and phase error corrected model precipitation field at \( t_c \). The tendency of the intensity errors could be derived in a similar way as in the phase errors.

5) The next step is to advect the Lagrangian time-dependent phase and intensity errors to the model forecast lead time, \( t_c + n\Delta t \). First, we have derived the model echo motions with VET between two-successive model precipitation fields. Then, the model errors are advected to the forecast time with the derived motion vectors and the linear tendency is taken into account.

6) Finally, these advected errors from the procedure 4) are applied to modify the model forecast.

We refer all these procedure as “Adjustment of Rain from MOdel with Radar data (ARMOR)” ARMOR correct the linearly varying phase errors along the echo motions. Thus, its performance relies on the persistence of the tendency of model errors. The false alarms, misses, and intensity difference are corrected. The new storm development and dissipation that are predicted by models are incorporated in the corrected forecasts. The falsely predicted precipitation fields at \( t_c \) are suppressed throughout the forecast time.

b. Derivation of phase errors at \( t_c \)

The variational echo tracking (VET) is applied to derive the radar and model echo motions that are used to derive and to advect the phase and intensity errors. This method minimizes the cost function that is composed of the difference \( J_{\Psi_R} \) between radar reflectivity maps at different time steps and the smoothness term \( J_{\Psi_M} \) of motion vectors (Germann and Zawadzki 2002). The model echo motion vectors are derived in a similar way with converted reflectivity maps from model precipitation forecasts. The phase errors can be derived from the same procedure but using HSR and model fields at a given time \( t \). Then, we can write the cost function as below:

\[
J = J_{\Psi_R} + J_{\Psi_M}
\]

where

\[
J_{\Psi_R} = \int_{\Omega} a|\psi_R(x + \alpha x + \beta y, t) - \psi_M(x, y, t)|^2 \, dx \, dy
\]

\[
J_{\Psi_M} = b\int_{\Omega} \left( \frac{\partial^2 \alpha}{\partial x^2} + \frac{\partial^2 \beta}{\partial y^2} \right) + \left( \frac{\partial^2 \beta}{\partial x^2} + \frac{\partial^2 \alpha}{\partial y^2} \right) \, dx \, dy
\]

Here \( \Psi_R \) and \( \Psi_M \) are the radar reflectivity (in dBZ), as a function of space and time, converted from HSR and model precipitation forecast with \( Z = 210 \) \( \text{R}^{1.47} \), respectively; \( \Omega \) is the domain over which ARMOR is applied. \( \alpha \) and \( \beta \) are the control variables of the minimization problem.

The solution of the minimization gives, for each pixel, a vector \((\alpha, \beta)^T\). Here, we have assumed the time phase errors of model are zero. However, to account for time phase errors the cost function can be modified. When \( t = t_c \) of the initiation of the correction, the ensemble of these vectors gives the \( x \) and \( y \) components of initial spatial phase errors within a domain \( \Omega \), that is, the matrix of errors \( \tau_p(x, t_c) \). This matrix represents the full two dimensional field of vectors necessary to produce the displacements and deformations of the model output to match the observations at \( t_c \).

The minimization of the cost function is performed by a conjugate-gradient method. In this manner a field of vectors \( (\tau_p) \) is determined over the domain, one vector per each resolution pixel of the NWP precipitation output. The parameters \( a \) and \( b \) are adjustable weights, with a representing the uncertainty in radar measurements and \( b \) chosen as an empirical compromise between
eliminating noise in the retrieved spatial phase error vectors and the spatial variability in the phase error vectors.

4. Results

The correction method is applied for the MM5 11z run \((t_0 = 11z)\) in which the observational nudging is performed from 08z, July 3, 2006 up to 11z July 3. The precipitation forecasts are made from 11z. We have used 13 precipitation forecasts and HSRs five minutes apart between 11z and 12z to derive initial phase errors, tendency of phase errors, and intensity errors. That is, \(n = 12\), \(t_c = 12z\), and \(\Delta t = 5\) minutes. Different to the previous study in which hourly precipitation accumulations are used, the frequent model outputs \((\Delta t = 5\) minutes\) make possible to use only 1-h window to derive the tendency of model errors. Then, the correction of model errors is applied for the precipitation forecasts from 12z \((t_c = t_0 + 1h: 1\ h\ forecast)\) every five minutes \((\Delta t = 5\) minutes\).

Fig. 1: Precipitation rate from radar (HSR) and MM5 forecast at \(t = t_c = 12z (t_0 + 1\ h)\). MM5 forecasts are generated from 11z by incorporating radar data from 08z. (a) HSR, (b) MM5 precipitation forecast, (c) Initial phase errors overlaid with MM5 forecast, (d) MM5 forecast after correcting initial phase errors shown in (c).

Fig. 1 shows HSR and MM5 forecast at \(t = 12z\) in which the model correction is initiated. The comparison of HSR (Fig. 1a) and original 1-h forecast (Fig. 1b) demonstrates the model precipitation is predicted at the wrong locations with intensity difference. The model precipitation areas are much less than those of HSR, leading to significant misses. However, the general features are captured except for area A where the model completely misses precipitation band. Model
precipitation at areas A and B should be shifted to south-east to match with HSR while that at area C to east direction. The derived initial phase errors \( \tau_p(x, t_0) \) (Fig. 1c) illustrates well these mis-matching.

We have imposed the spatial smoothness with weight \( b \) in eq. (2) to avoid the spatial dis-continuity of phase errors. The phase error correction (Fig. 1d) displaces the model precipitation forecast at proper position and expands precipitation areas to match with HSR. This illustrates the efficiency of the simple phase correction to improve model skills. However, the intensity difference and significant misses are still present. In addition to the phase error correction, the correction of intensity errors ensures that the corrected field is nearly identical to HSR in Fig. 1a (not shown in here).

Fig. 2 illustrates the effectiveness of the correction at 14z (3-h forecast and 2-h after initiating correction). The original forecast (Fig. 2b) shows the positional errors which in general are increased. The predicted Lagrangian phase errors (vectors in Fig. 2b) using their tendency between 11z and 12z are quite different to the initial phase errors shown in Fig. 1c. The significant increase is noticeable at west regions. The corrected model precipitation fields (Fig. 2c) better matches with HSR. The overall position is well corrected with the predicted Lagrangian phase errors.

**Fig.2:** Precipitation rate from radar (HSR) and MM5 forecast at \( t = 14z \) (\( t_0 + 3 \) h). (a) HSR, (b) original MM5 precipitation forecast with Lagrangian phase errors (vectors) predicted from the tendency between 11z and 12z and then advected to 14z, (c) MM5 forecast after correcting Lagrangian phase errors, and (d) MM5 forecast with the correction of Lagrangian phase errors and intensity errors.
The precipitation band at the area A is not properly predicted. In fact, this band is intensified (see Fig. 1a and Fig. 2a). In addition to the Lagrangian phase error correction, the intensity correction generates this precipitation band. Although the model completely misses this band between 12z and 14z, the intensity error correction maintains this missed band. In such a case (miss case), the intensity correction is equivalent to radar extrapolation based on model echo motions. In addition, the intensity correction better predicts the shape of precipitation fields. The precipitation fields at area B are properly located with the phase error correction although the trend of the intensity from HSR and model is opposite. It is interesting to note that the strong precipitation at the area C is properly predicted with the model although the locational errors exist. This proper prediction is well reproduced in the correction (Fig. 2d). In the area D, the correction eliminates the wrong strong precipitation by identifying it as false alarms. In general, the intensity correction shows overall significant improvement. The precipitation at the area E is from the outside of domain at t = t_c. Thus, no information for the correction is available.

A quantitative comparison is shown in Fig. 3 in terms of critical success index (CSI) and cross-correlation (r). Unlike the typical trend of model forecast skills (Lin et al. 2005), in general, the forecast skills of original forecasts decrease with time and extremely low (CSI < 0.3 and r < 0.2). The correction of Lagrangian phase errors that are predicted from their tendency between 11z and 12z significantly improves skills up to CSI ~ 0.4 and r ~ 0.45, illustrating the importance of positional errors in model forecasts. Instead of applying predicted Lagrangian phase errors, the correction with the initial phase errors $\tau_p(x, t_c)$ that assumes the geographical dependence of the model positional errors provides the similar performance as the Lagrangian phase correction only up to 3-h forecast time (not shown). The additional intensity correction that takes into account false alarms, misses, and intensity difference further improves the skill scores and performs better than radar extrapolation after 2-h forecast time. However, it should be noted that the skill scores after correction decrease rapidly with time. This is partially due to the drop of model skill scores with time and more importantly due to the nature of the correction algorithm that projects the past information into the future assuming the Lagrangian persistence of the model errors. Thus, all corrections are bounded to the persistence of errors.

Fig. 3 Critical success index (CSI) and cross-correlation of model forecasts (the black line: original model forecast, blue line: correction with predicted Lagrangian phase errors, red line: intensity correction in addition to the Lagrangian phase correction, and green line: radar extrapolation).
5. Summary

We have demonstrated a way of determining and correcting model phase (or positional) and intensity errors in a meso-scale model run. We have used the variational echo tracking to derive motions vectors and model errors. From the model forecasts for 1-h, we have derived the tendency of phase errors and intensity errors. These derived errors are projected into the model forecast time from 1-h to 12 h. Then, the model forecasts are corrected with these projected model errors. In general, similar to the results in the previous studies at a continental scale, the correction method significantly improves the performance.

The model verification shows the existence of significant phase errors even at the model initial time, indicating that MM5-RTFDDA with frequent cycling is not sufficient to correct the model background phase errors. These initial phase errors increase with time. The correction of model errors significantly improves the accuracy up to 9 hours. The corrected forecast is comparable to radar extrapolation up to 2 h and performs better afterward.

The correction method assumes the persistence of the model errors along the motion of precipitation fields. Thus, the performance of the correction is still bounded into the Lagrangian persistence of model errors, shown by the rapid drop of skill scores with time. As the model performance improves, so does the consistency of model errors. Subsequently, the correction method should further improve the model forecast.

References


