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2.4 A CALCULATION OF LYAPUNOV EXPONENTS FOR ATMOSPHERIC BLOCKING

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1. INTRODUCTION

The development of a mainly meridional circulation pattern in the midtroposphere is commonly referred to as blocking. This recurrent obstruction and/or stagnation of basic zonal atmospheric flow gives rise to difficulties in obtaining a more confident and meaningful local and regional weather forecast (Elliott and Smith 1949).

Considerable theoretical efforts have been made to mimic and simulate the phenomena of atmospheric blocking, particularly those emanating from the model solutions of quasi-geostrophic potential vorticity equation. For a review and summary, see De Swart 1988. A common theme there is to address various aspects of stability of the atmospheric flow.

The restricted nature of above model solutions do limit their applicability range. Therefore, an empirically oriented approach seems to be in place to quantify the various stability regimes of atmospheric flow. In this context, blocking may be viewed as an unstable but stationary regime in barotropic atmospheric circulation. A diagnostic/stability study of atmospheric blocking may thus be performed by partitioning the atmospheric flow into planetary and synoptic scale components.

In recent years, several studies have examined the relative role of each scale and their interactions as well as the nature of the interactions themselves (e.g., Tracton 1990; Lupo et al. 2007 and references cited therein). In addition to these studies, and from those of earlier pioneers (e.g., Kalnay-Rivas and Merkine 1981; Frederiksen 1982; Shutts 1983; Mullen 1986, 1987), a consistent picture emerges that the synoptic-scale plays an important role in the lifecycle of blocking events (a necessary, but not sufficient condition). Many of the studies represented above show that the magnitude of the synoptic-scale forcing is large compared to that of the planetary-scale forcing.

However, others have shown that the planetary-scale is very influential in the lifecycle of the blocking events (e.g., Haines and Holland 1998; Colucci and Baumhefner 1998). While the studies referenced in the above paragraph do not downplay the role of the planetary-scale, they do focus more on the role of synoptic-scale contributions. For instance, in their model study, Haines and Holland suggest that blocking regimes will break down when there is a substantial change in the planetary-scale flow regime. Colucci and Baumhefner (1998) focus on the role of planetary-scale deformation as a pre-conditioned environment for the formation of blocking events (Colucci 2001). These two studies together support the notion that while the planetary-scale may not itself lead to block formation and maintenance, nevertheless this scale may provide a favorable environment in the interaction with the synoptic-scale environment. Thus, a substantial change in the large-scale flow regime would not support blocking and these events would decay fairly quickly.

The main goal of this work is to demonstrate that abrupt changes in the planetary-scale environment can lead to the rapid decay of blocking. This work will thus look at the utility of Lyapunov exponents (Lyapunov 1966) as a diagnostic tool in blocking studies. In this context, time variability of planetary-scale geopotential height and of a stability index (the Lyapunov exponents) will be studied. After discussing some details of the data to be used in our analysis, the methodology of our analysis is presented in Section 2. Using the method presented in Section 2, the synoptic and dynamic analysis of a chosen blocking event is performed in Section 3. The results are summarized in Section 4.

2. DATA AND METHOD

2.1 Data

The data set used here was the National Center for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) gridded re-analyses data (Kalnay et al. 1996). This data was archived at NCAR and was obtained from the mass-store facility in Boulder, CO in netCDF format. This re-analyzed data was the 2.5° by 2.5° latitude-longitude analyses

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available on 17 mandatory levels from 1000 hPa to 10 hPa at 6-h intervals on daily basis. These analyses include the standard atmospheric variables relevant for determination of physical properties of the atmosphere such as the geopotential height, temperature, relative humidity, vertical motion, *u* and *v* wind components and surface information. The mandatory level data were interpolated quadratically in ln*p* to 50 hPa level-increments, and these more closely resemble raw sounding information (Lupo and Bosart 1999).

2.2. Methodology

The blocking criterion of Lupo and Smith (1995a) was used here to determine the onset and termination times for the blocking event studied here. Details regarding this criterion and it's application can be found in the references in Section 1. Basically, these studies employ an extended set of conditions set forth earlier by Rex (1950a, 1950b) where a climatological study of 16 year data (1933 –1949) was performed under that set of conditions.

This study will demonstrate that changes in the planetary-scale flow regimes can be correlated to the onset and, more importantly, to the decay of blocking events. The techniques used here to extract planetary-scale variability have been used to extract interannual variability from a one-dimensional time series recently by Mokhov et al. (2000, 2004, and references therein) or Federov et al. (2003) and will be only briefly presented here with modifications.

The techniques used in these references are based on standard dynamic analysis techniques for physical systems (e.g., Lorenz 1963). In particular, the planetary-scale height fields were averaged over a 40^o degree latitude by 60^o degree longitude box within the blocking sector to produce one number for each time period. This process is analogous to the procedure used by Hansen (1986) in deriving the wave amplitude index, with the exception that we filtered the fields first and then averaged them within a box. They averaged the entire mid-latitude height field into a band and then filtered to obtain a single number for the time period.

A second-order, two-dimensional Shapiro (1970) filter was used on the variables in the data set in order to separate the planetary-scale wavelengths from the synoptic-scale wavelengths. Applying this filter results in a response function, which retains 2%, 44%, 80% of the signal for waves having a wavelength of 3000 km, 4500 km, and 6000 km at 45^o N (or S) latitude, respectively. More details regarding the use of the filtering procedure can be found in Lupo and Smith (1995b).

Historically, the planetary-scale flow in both hemispheres has been assumed to behave as, or treated like, an oscillating pendulum (e.g., Lorenz 1963; Hansen 1986; Nese et al. 1987; Hansen and Sutera 1988). More recently, this type of physical behavior has been discussed by Lynch (2003) who also extended the analogy to describe the large-scale flow as a swinging "spring" in describing the behavior of Rossby wave triads.

Let us note that if the planetary-scale flow were steady-state and geostrophically balanced over a long period of time, the phase plot would approximate the harmonic behavior. However, it is apparent that the planetary-scale flow may have more than one stable state (e.g., Charney and DeVore 1979; Yoden 1985; Nese et al. 1987; Nitsche et al. 1994). Whether there are two such states or more is not the focus of this work. Here the focus is to identify a change in flow regime character and associate this with the growth or decay rate of the blocking events.

Lyapunov exponents can quantify the leading order stability properties of a dynamical system. Lyapunov exponents for a dynamical system can in principle be computed by several methods. Here we take the point of view that they are calculable from the linear stability analysis of the barotropic atmospheric flow under the assumption of quasi-geostrophy (Dymnikov et al. 1992). We thus discuss a source of diagnostic tools for blocking onset and decay in the mathematical theory of stability in infinite dimensions. The basic premise here is that *atmospheric blocking should be thought of as an unstable atmospheric circulation* whose state is best analyzed by its stability characteristics, as we've stated above.

The dynamic equation of viscous incompressible barotropic fluid on sphere with respect to orographic inhomogeneity h for the stream function ψ with prescribed forcing f is given by

$$\frac{\partial \Delta\psi}{\partial t} + \rho J(\psi, \Delta\psi + h + l) + \sigma\Delta\psi - \nu\Delta^2\psi = f. \quad (1)$$

Δ is the Laplacian operator on the unit sphere. ρ is the dimensionless non-linearity parameter. To take into account non-linear effects, in the following analysis, we take ρ = 1. J is the Jacobian incorporating the non-linear interactions. l is the Coriolis parameter. The term $\sigma\Delta\psi$ describes the dissipation in planetary atmospheric boundary layer. The term $\nu\Delta^2\psi$ describes the turbulent viscous dissipation.

Expanding ψ as $\psi = \overline{\psi} + \psi'$, where $\psi' = \psi'(t)$ and $\overline{\psi} \neq \overline{\psi}(t)$. The equation of motion for linearization operator A is $\partial\Delta\psi' / \partial t + A\psi' = 0$, where

$$A\psi' = \rho J(\psi', \Delta\overline{\psi} + h + l) + \rho J(\overline{\psi}, \Delta\psi') + \sigma\Delta\psi' - \nu\Delta^2\psi'. \qquad (2)$$

The disturbance energy equation in terms of scalar product $(A\psi', \psi')$ is $\partial E'/\partial t = (A\psi', \psi')$. Since $A=S+K$, where S is the skew-symmetric operator and K is the symmetric operator, the disturbance energy equation may be rewritten with $A \to S$. Note that the stationary solution will be stable if all the eigenvalues of the operator S with respect to stationary solution are negative. We shall take the sum of positive eigenvalues of the operator S as the characteristics of the instability of the stationary point.

Passing to finite dimensional notation for convenience in Cartesian coordinates and assuming that $\overline{\psi} = \overline{\psi}(y)$, i.e., the stationary solution does not depend on zonal coordinate and that σ = ν = 0 for simplicity and using the periodic conditions for x and y, the eigenvalues problem of the operator S (where $2S = A + A^*$) has the form

$$\overline{u}\frac{\partial}{\partial x}\Delta\tilde{\varphi} - \Delta\left(\overline{u}\frac{\partial\tilde{\varphi}}{\partial x}\right) = \lambda\tilde{\varphi}. \qquad (3)$$

We shall look for the solution of the form $\tilde{\varphi}(x,y) = \tilde{\varphi}(y)e^{ikx}$. With this transformation, we obtain the following eigenvalues equation

$$\frac{\partial\overline{\varpi}\tilde{\varphi}}{\partial y} + \overline{\varpi}\frac{\partial\tilde{\varphi}}{\partial y} = \frac{\lambda}{ik}\tilde{\varphi}, \quad \overline{\varpi} = -\frac{\partial\overline{u}}{\partial y}. \qquad (4)$$

In principle, one should solve this equation to obtain the spectrum of eigenvalues λ, which depend upon $\overline{\varpi}$. Here, we make use of the Dymnikov et al. (1992) conjecture which suggests a strong correlation between the sum of the positive Lyapunov exponents (eigenvalues of the linearization of barotropic flows) and the domain integrated enstrophy, that is

$$\sum_i \lambda_i^+ \approx \int_D |\overline{\varpi}|^2(y)\,dxdy. \qquad (5)$$

A numerical implementation of equation for A via Crank-Nicholson scheme in Dymnikov et al. (1992) using data for a three-year period after applying a 15-day filter (planetary-scale) on domains D over the North Atlantic and Western Europe establishes validity of Eq. (5). Summarizing, to the extent that the average time for ω-trajectories to diverge decreases as the sum of the positive Lyapunov exponents increases, we get a preliminary indication of how physical quantities, such as enstrophy, can be viewed as forecasting indicators.

In the context of atmospheric blocking, the result can be illustrated by using the following observation

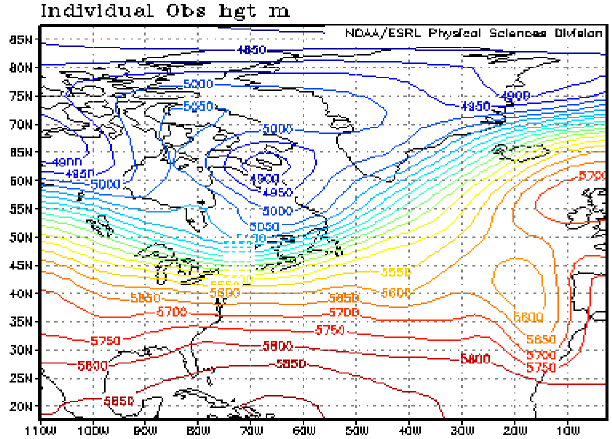


Fig 1a. The 500 hPa height for 0000 UTC 13 March 2003 of the blocking event over Atlantic ocean which lasted from 13 March 2003 through 24 March, 2003. The 500 hPa height is plotted every 50 m.

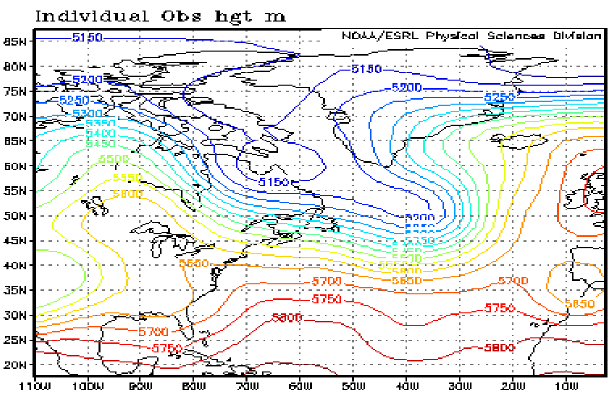


Fig 1b. Same as Fig. 1a, except for 0000 UTC 18 March, 2003 (peak activity period). The Rex shaped blocking pattern is clearly noticeable.

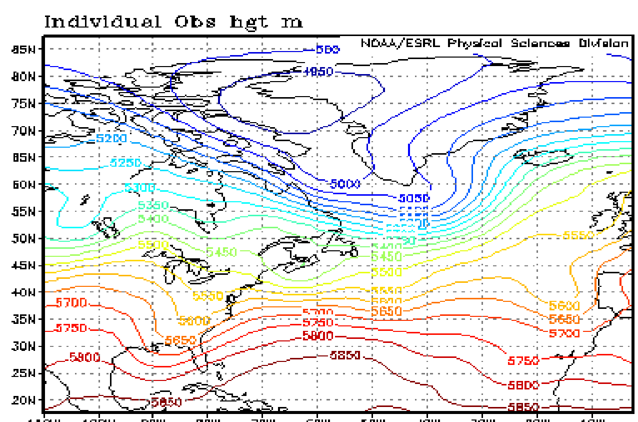


Fig 1c. Same as Fig. 1a, except for 0000 UTC 24 March, 2003 during the decaying stage.

from Lupo and Smith (1995a): Before blocking, there always exists upstream of the block, a ridge accompanied by an amplifying short wave within 0.5 ridge wavelength.

Given that the ridge and the short wave are associated with increasing gradients, the Lupo-Smith observation can be understood as the creation of sufficient instabilities to trigger the transition from zonal flow to blocking. Furthermore, initial data observations as in Lupo and Smith (1995a) as well as in Wiedenmann et al. (2002), indicate a strong correlation between the Block Intensity (BI) and the average (unintegrated) anti-cyclonic vorticity in the neighborhood of the block. For illustrative purposes, we next discuss in some detail the time variability of the mean geopotential height and of the Lyapunov exponents for a selected isolated blocking event occurring in midlatitude Northern Hemisphere only.

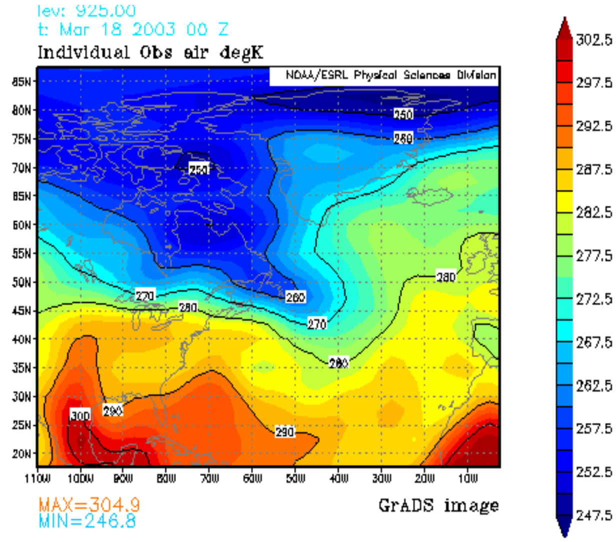


Fig. 2. The 925 hPa temperature field (^oK) of the unstable atmospheric flow region that contains the blocking event shown in Fig. 1 for 0000 UTC 18 March 2003. Below normal temperature is upstream (and downstream) the blocking ridge, whereas above normal temperature is noticed under the blocking high.

3. ANALYSIS

We shall first perform synoptic and then a dynamic analysis of the stability of the region of the atmospheric flow where the selected blocking event occurred.

3.1 Synoptic Analysis

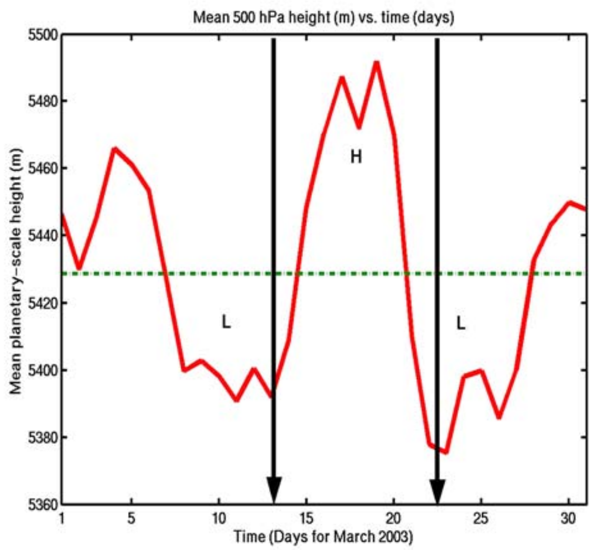
An isolated single blocking event occurring in NH at midlatitude during the winter of 2003 is studied in some detail as our illustrative case study example and is shown in Fig. 1 and Fig. 2. The blocking event occurred between 13 March through 24 March 2003. The blocking event lasted for approximately 11 days.

Mean blocking center latitude was located at 53.5^o N, whereas the mean blocking center longitude was located at 331.7^o E with a mean angular extension of 36.5^o. Mean Blocking Intensity (BI) was 4.3. Mean 500 hPa geopotential height computed at the blocking center was 5444 m. Maximum BI during the blocking life cycle was 7.7. Maximum geopotential height registered during the blocking life cycle 5779 m. Given the latitude and longitude boundaries of the event, this blocking event was considered to be in Atlantic sector.

Climatologically, overall, total number of blocking events which lasted for 5 or more days during the calendar year 2003 (Jan. through Dec.) was 30. About 37% (11/30) were in Europe sector, 27% (8/30) were in west Pacific sector, 20% (6/30) were in Atlantic sector, whereas remaining 16% (5/30) were in east Pacific sector. July was the most active period for the blocking events (6/30) with May, Aug and Dec (1/30) being the least. We note that in the specific year chosen for study here, blocking events are more frequent in summer season over the oceans. This finding is in line with climatological study of Barriopedro et al. (2006). The longest blocking event for the year 2003 lasted for 21 days and occurred in February in Europe sector. For details of case studies in SH, see Athar et al. (2007).

This blocking event impacted the upstream weather over continental US (as well as downstream over continental Europe). This is an instance where occurrence of midtropospheric level blocking effects the regional weather though the blocking is not occurring in exactly that region, as already pointed out in Section 1.

Surface data analysis indicates that a low pressure system was developed with center over north of the Great Lake area on 13 March 2003 with central pressure of 1012 hPa. This low pressure system persisted for 9 day and drifted slowly towards east. The low deepened to 1004 hPa and the stations under this area reported overcast and/or rainfall with similar temperature and dew point. Upper east cost reported overcast for those 9 days and then reported clear skies after the blocking ridge amplitude decayed over Atlantic ocean relaxing the stagnation of upstream cyclone over continental US (Fig. 1c) .



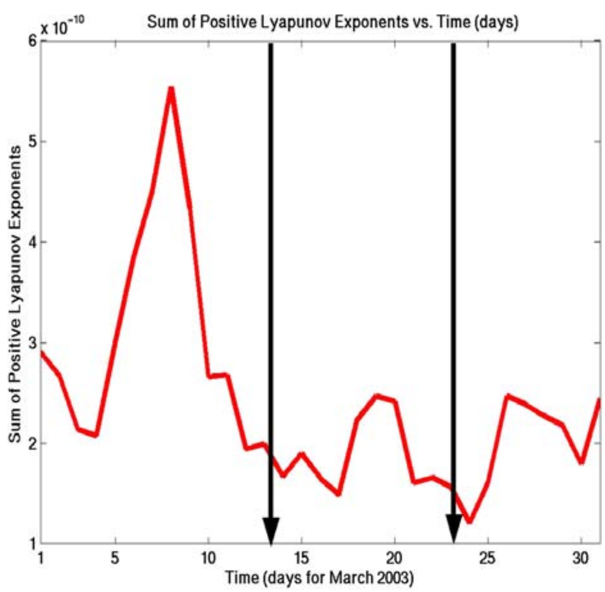


Figure 3. Upper panel: A diagram of mean 500 hPa geopotential height (m) along abscissa with respect to time (days) along ordinate for a stationary box (230^{o} E to 357.5^{o} E and 22.5^{o} N to 75^{o} N) in the mid-latitude Northern Hemisphere flow. Daily 6 hourly data is used here for 0000 UTC. The horizontal dashed line is the mean 500 hPa geopotential height for the displayed variations over the stationary box. *Lower Panel :* A calculation of area averaged enstrophy using Eq. (5) for the blocking event displayed in the upper panel which occurred during 13-24 March, 2003 (indicated by vertical lines with arrows).

3.2 Dynamic Analysis

The plot shown in Fig. 3 (upper panel) displays the mean planetary-scale height fields within the box described in Section 2.2 for the entire month of March 2003. Note a positive height anomaly during the blocking period. During March, the average heights within the box fall until just before block onset (day number 13). The heights rise until the block reaches a maximum and then fall again during and after decay (day number 22). They suggest changes in the behavior of the large-scale flow regime. The presence of two peak structure in the 500 hPa mean geopotential height is indicative of a rapidly falling height during blocking that lasted for less than 2 days.

Calculations of the Lyapunov exponents following Eq. (5) for our entire data set under study demonstrate a relationship between these value and upper panel of Fig. 3. From Fig. 3 (lower panel), we note that the area averaged enstrophy reaches a minimum shortly after block onset and is at a relative minimum during the lifecycle of this blocking event.

The enstrophy is calculated in Eq. (5) using the planetary-scale heights used to construct Fig. 3 (upper panel). This is also consistent with the view that, in a quasi-barotropic flow, the planetary-scale flow should be strongly barotropic, and that the blocking state represents a minimum state of enstrophy [and entropy see e.g., Dymnikov and Filatov (1995)]. Since these correlate to the positive Lyapunov exponents and are relatively small here indicate that negative values of fluid trapping (again implying more predictability, or a more stable condition) grow in concert with the intensity of the blocking event.

During blocking events, this implies that once the blocking event established itself, the large-scale flow is relatively more predictable. We note that the Lyapunov exponents give a relative change and are thus alone not sufficient to identify the blocking event unambiguously. The fluctuating behavior of the sum of positive Lyapunov exponents depicted in Fig. 3b is commonly observed in unstable dynamic systems (Lorenz 1963).

Another interesting observation is that a comparison of positive height anomaly and the sum of the positive Lyapunov exponents for the entire data set under discussion indicates a strong positive correlation between the two as a function of ln*p*. A comparison of lower panel with upper panel in Fig. 3 reveals this correlation at 500 hPa.

This correlation holds for all mandatory levels between 300 hPa and 925 hPa for the entire duration of the chosen blocking event further justifying the usefulness of Lyapunov exponents as a simple diagnostic tool. Deep blocking anticyclones extending from upper to lower troposphere may thus be diagnosed for their stability characteristics in terms of Lyapunov exponents.

3.3 Discussion

Changes in the nature of the planetary-scale flow can be correlated with block decay supporting the general implications of the work of Tsou and Smith (1990), Haines and Holland (1998), Colucci and Baumhefner (1998), and that the planetary-scale provides an important contribution to blocking lifecycles by providing a favorable environment for the blocking event to occur, in spite of the large contributions by the synoptic-scale flow and interaction components of the forcing.

Additionally, supporting evidence for the change in planetary-scale flow regimes comes from examining the Lyapunov exponent (flow stability) calculations. Fluid trapping values and the area integrated enstrophy values (Fig. 3, lower panel) fall to a minimum during the lifetime of the block in the blocked region in agreement with what would be expected for each blocking event implying that the planetary-scale flow became unstable around the time of block onset and decay for all cases studied here.

It is possible that the planetary-scale flow at these two times moved from one (geostrophically) stable state to another, and the corresponding behavior of the other metrics shown in Fig. 3 corroborate this interpretation. Thus, the methodologies shown here, which are relatively easy to generate, have at least some value as a diagnostic tool for atmospheric phenomena. They may even have value as a metric for predictability, however, more study is needed in order to adequately demonstrate such value.

Earlier studies have used statistics to make a similar point (Lejenas and Oakland 1983). However, decay can occur when there is no longer active synoptic-scale support for the events, when the synoptic-scale impacts negatively on the blocking events, or when the planetary-scale flow regime changes character.

4. SUMMARY AND CONCLUSIONS

Suitability of Lyapunov exponents as a diagnostic tool for atmospheric blocking is examined using Dymnikov et al. conjecture which correlates the sum of positive Lyapunov exponents with the enstrophy. Two diagnostic tools/variables (namely, the mean geopotential-scale height and the Lyapunov exponents) providing valuable information about the change in the flow pattern during the blocking event are estimated. A total of 30 blocking events are analyzed in terms of the above two diagnostic tools for NH blocking events for the year 2003.

It is noted that the planetary-scale height variation in the atmospheric flow follow the changes in the flow during the blocking event which are characteristically different than the unblocked flow pattern upstream as well as downstream. The typical 500 hPa geopotential height variation for all the blocking events that occurred during the year 2003 range approximately between 100 m to 200 m.

The above observation is true irrespective of whether the blocking event occurs entirely over land, over sea or partially over land and partially over sea. This may indicate more dominating role played by the different scales and their interactions of atmospheric flow (as noted in earlier studies too) once the blocking sets in instead of orographic forcings.

Lyapunov exponents can quantify the average predictability and stability properties of a dynamical system without the need to explicitly solve for the flow streamfunction. Thus, Lyapunov exponents (i.e., the eigenvalues of the linearlized decomposition of the steady state flow) may thus be used as a diagnostic tool especially in blocking studies. The role of the large-scale flow in formation, maintenance as well as in decay of a selected NH blocking event is discussed in terms of Lyapunov exponent time variability.

At least for the data set studied, the regional Lypunov exponents seems to characterize the stability of the planetary-scale flow in barotropic circulation. The stability theory (Lyapunov exponents) is thus investigated in order to evaluate its use as a simple diagnostic tool. A simultaneous knowledge of the two diagnostic tools seems to provide a more reliable scenario for the existence/occurrence, sustenance as well as decay of a blocking event. Geopotential height variations alone however do not provide any underlying insight into the stability of the atmospheric flow during the blocking period.

Combining with the results presented in Athar et al. (2007), where Lyapunov exponents for SH blocking events during the 3 year period (2002-2004) were calculated, we may arrive at a tentative conclusion that Lyapunov exponents seems to mimic the (in)stability of the planetary-scale atmospheric flow during blocking event and may thus qualify as a climatologically reliable diagnostic tool for atmospheric blocking over both hemispheres.

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