CHARACTERIZATION OF TURBULENT KINETIC ENERGY BUDGET IN THE ATMOSPHERIC SURFACE LAYER

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Urban activities and pollutant dispersions take place in the atmospheric surface layer, the bottom of the atmosphere, whose depth is on the order of several tens or hundreds meters. Characterizing turbulence in the surface layer is of great significance in urban air quality research, and has a major role in policy and regulatory decision, urban design, and air pollution control. The turbulence kinetic energy (TKE) budget, which associates the change of turbulence per unit time to mechanical production, buoyancy production or destruction, turbulent energy dissipation, and transport by mean flow, turbulent fluctuations, and pressure fluctuations, finds its applications in both empirical and computational approaches. This study utilizes experimental data collected in an open filed in Hanford, WA in 2002 to investigate the TKE budget in the surface layer. The TKE budget was normalized and all terms were parameterized as functions of a stability parameter *z/L*, where *z* is a distance from the ground and *L* is the Monin-Obukhov length. The mechanical, buoyancy and dissipation terms, are found to be imbalanced due to a large net transport term. This imbalance suggests imperfection of the Monin-Obukhov Similarity Theory (MOST), which is widely used as the major parameterization in the surface layer. Modifications on the TKE parameterization derived from MOST were attempted and generated good agreement with the experimental data.

1. INTRODUCTION

Since 1960s, Monin-Obukhov similarity theory (MOST) [Monin and Obukhov 1954] has been widely accepted as the major parameterization in the atmospheric surface layer. Based on MOST the turbulent kinetic energy (TKE) budget in the atmospheric environment has been studied extensively for several decades. Although deep understandings have been gained from many field campaigns since 1968 Kansas Experiment [Businger et al. 1971], many aspects of the TKE parameterization in the surface layer have still remained uncertain, which will be discussed in details in Section 2. Descriptions of data analysis from Hanford 2002 Field Experiment can be found in Section 3. The estimate of von Kármán constant is described in Section 4. The parameterization of dimensionless TKE budget terms is presented in details and compared to previous research in Section 5, followed by discussion and conclusion in Section 6.

2. BACKGROUND

The Monin-Obukhov length is the primary length scale in MOST, and is defined as:

$$L \equiv -\frac{u_*^3 \Theta}{\kappa g \left(\overline{w'\theta'}\right)_s} \tag{2}$$

where u_* is the friction velocity taken from the surface kinematic Reynolds shear stress, Θ is the mean potential temperature, θ' is the potential temperature fluctuation, g is the gravity constant, and κ is the von Kármán constant.

The stability parameter *z/L*, where *z* is the vertical coordinate defined as zero at the ground, represents the stability under three conditions: unstable when *z/L* has a negative value, neutral when *z/L* equals zero, and stable when *z/L* is positive.

The TKE is defined as $e = 0.5(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$. After applying the assumption of zero-subsidence and choosing the coordinate system aligned to the wind direction, the TKE budget equation will reduced to the form of:

$$\frac{\partial e}{\partial t} = -U\frac{\partial e}{\partial x} + \frac{g}{\Theta}\overline{w'\theta'} - \overline{u'w'}\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\left(\overline{w'e}\right) - \frac{\partial}{\partial z}\left(\overline{w'p'}\right) - \varepsilon \tag{3}$$

(I) (II) (III) (IV) (V) (VI) (VII)

Where term I presents the local storage of TKE; term II is the advection by the mean flow; the buoyancy production or destruction, the shear production, and the dissipation are given by term III, IV and VII; term V and term VI are the transport terms by turbulent velocity fluctuations and pressure fluctuation, sometimes simply referred to as the turbulent transport and the pressure transport.

The dimensionless form is obtained by normalizing the TKE budget by $u_*^3/\kappa z$:

$$\phi_s = \phi_a + \phi_b + \phi_m + \phi_t + \phi_p - \phi_\varepsilon \tag{4}$$

All the dimensionless 'ϕ' functions are functions of stability parameter *z/L*. Stationary assumption gives dimensionless local storage ϕ_s a zero value. Under horizontal homogeneity assumption, dimensionless advection ϕ_a also goes zero. Dimensionless buoyancy production ϕ_b is nothing but *–z/L* by definition. There are many proposed forms of dimensionless shear production ϕ_m [Panofsky et al. 1960, and Swinbank 1964], out of which the now well-known Businger-Dyer relation [Businger et al. 1971, Dyer 1974] has been widely used:

$$\phi_m(z/L) = \begin{cases} 1 + \dfrac{\delta z}{L}, & z/L > 0 \\ 1, & z/L = 0 \\ \left(1 - \dfrac{\gamma z}{L}\right)^{-1/4}, & z/L < 0 \end{cases} \tag{5}$$

where γ and δ are constants whose values have been determined from many studies. The values of γ include 15 [Businger et al. 1971, and Oncley et al.1990], 22 [Wieringa 1980], 28 [Dyer and Bradley 1982], 19 [Hogstrom 1988], and 22.6 [Frenzen and Vogel 1992]. The values of δ include 5 [Webb 1970], 4.7 [Businger et al. 1971], 6.9 [Wieringa 1980], 8.1 [Oncley et al. 1990], and 5.3 [Hogstrom 1996].

Carl et al. [1973] proposed a modified form for unstable condition, changing the exponent from -1/4 to -1/3:

$$\phi_m(z/L) = \left(1 - \frac{\gamma z}{L}\right)^{-1/3}, \quad z/L < 0 \tag{6}$$

The value is γ for this -1/3 law include 15 [Carl et al. 1973], 8 [Gavrilov and Petrov 1981], and 16 [Frenzen and Vogel 2001].

Dimensionless turbulent dissipation $\phi_\varepsilon = \varepsilon\kappa z/u_*^3$. Wyngaard and Cote [1971] proposed a parameterization as:

$$\phi_\varepsilon(z/L) = \begin{cases} \left(1 + 2.5(z/L)^{3/5}\right)^{3/2}, & z/L > 0 \\ 1, & z/L = 0 \\ \left(1 + \beta|z/L|^{2/3}\right)^{3/2}, & z/L < 0 \end{cases} \tag{7}$$

The value of β was initially found to be 0.5, and later reported to be 0.75 by Caughey and Wyngaard [1979]. Other investigators have reported good agreement with 0.5 value over limited range of instability [Champagne et al. 1977, McBean and Elliot 1975, Frenzen 1983].

Less is known about the two transport terms. Under neutral conditions, MOST predicts the two transport terms to be zero, and the local TKE shear production will be balanced by dissipation. However, this has been strongly challenged by many atmospheric data [McBean and Elliot 1975, Deacon 1988, Hogstrom 1996, etc.].

Note that while the turbulent transport term can be calculated directly from the measurements, the measurements of pressure transport term present difficulty, which arises from the fact that the pressure fluctuations are very small and fast response measurements are needed. Direct measurements of pressure transport term were made by McBean and Elliot [1975], Maitani and Seo [1985], Schols and Wartena [1986], Oncley et al. [1995], and Wilczak et al. [1995], but there has been a lot of uncertainty.

3. EXPERIMENTAL DATA

Hanford 2002 Field Experiment took place in an open field with flat terrain and bush and grass surface in Hanford, WA in the spring of 2002. Five sonic anemometers were deployed at five different heights: 3.96, 7.62, 10.97, 15.24, and 22.86 m above the ground level. The sampling frequency was 10Hz.

There are a total 6 days in May 2002 which were examined and presented in this work. Raw data were filtered to remove large scale motion's influence, and then averaged into 5 minute and 1 hour blocks. 5 minute averaged data were used to find von Kármán constant under near neutral condition (refer to Section 4), while 1 hour averaged data were used to calculate the TKE budget (refer to Section 5). The vertical profiles of mean velocity and flux $\overline{w'e}$ were fitted toward second order polynomials when estimating the vertical gradient. The turbulent dissipation ε was estimated from the Kolmogorov inertial subrange in the spectra by assuming horizontal homogeneity. Bin averaging was applied when parameterizing the dimensionless TKE terms to reduce scatter.

4. VON KÁRMÁN CONSTANT

After 1960s, there have been extensive atmospheric data showing that von Kármán constant κ is actually neither universal nor constant but depends on different surface types, ranging from 0.32 to 0.43 in value [e.g. Hogstrom 1996 Table 1, Frenzen and Vogel 2001]. There were also many researchers [e.g. Purtell et al. 1981, and Hogstrom 1996] believed that von Kármán constant should be a constant about 0.40 regardless of surface, declaring the deviations from the classical value were due to inadequate measuring techniques.

To test Hanford data against the above contradictive conclusions, 5 minute averaged near neutral condition data were utilized to determine the von Kármán constant. Stability (z/L) between -0.002 and 0.002 was set as the cutoff for near neutral data. The mean wind speed U was normalized by friction velocity u_*, and κ was determined from linear fitting after the logarithmic profile:

$$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{8}$$

Figure 1 shows the linear fitting of von Kármán constant in Hanford Experiment. The roughness length z_0 was found to be 0.06 m, which falls in the reasonable range for grass and bush surface. We determined κ to be 0.385. However sensitivity in estimating this constant left us unable to verify Purtell's hypothesis [1998].

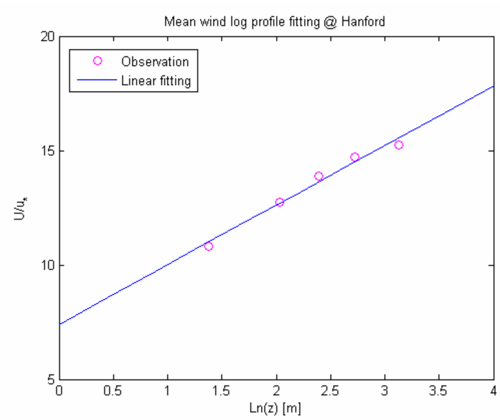

Figure 1- Logarithmic profile fitting for von Kármán constant and roughness height

5. DIMENSIONLESS TKE BUDGET

The TKE budget terms and their corresponding dimensionless forms were calculated from the 1 hour averaged data using equations (3) and (4). All terms in the dimensionless TKE budget were obtained except for the pressure transport and the advection. We combined these two unknown terms into one residual term, which can be determined under stationary assumption. Figure

2 presents the TKE budget terms from one sonic anemometer on May 24, 2002, showing the local storage of TKE is negligible compared with other terms. This actually happened consistently in all examined data, so we think the stationary assumption is valid. Figure 2 also indicates that contradictory to MOST, the three local TKE terms – the buoyancy production, the shear production, and the dissipation do not balance each other due to a non-zero net transport. The local imbalance of TKE will be presented from all examined data in Figure 5 showing the parameterized dimensionless TKE budget, which will be discussed in Subsection 5.4. Parameterization of the shear production, the turbulent dissipation, and the turbulent transport will be discussed in Subsections 5.1 through 5.3.

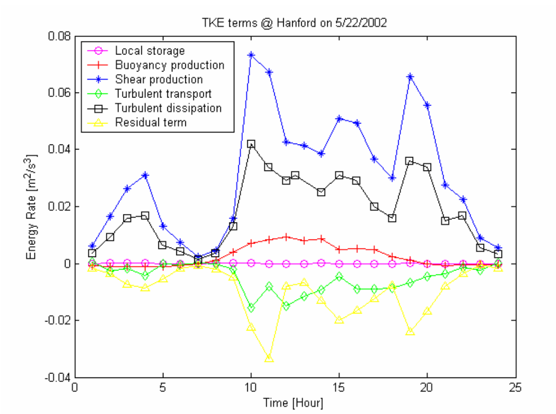

Figure 2 - The TKE budget on May 22, 2002

5.1 Dimensionless shear production

Dimensionless shear production was examined against Businger-Dyer relation, i.e., equations (5) and (6) with original coefficients. The new coefficients were also obtained by least square fitting as: under stable condition δ = 5.0 and under unstable condition γ = 16 for the -1/4 law, and γ = 9 for the -1/3 law, which are comparable to existing literatures. Figure 3 shows the bin-averaged data of dimensionless shear production and the parameterizations with new coefficients. Under unstable condition, the -1/3 law generates very similar results as the -1/4 law with the new coefficients, and both agree with the experimental data very well. Unstable data near neutral condition presents most scatter, which suggests the difficulty in obtaining very accurate estimates for spatial gradient near neutral condition due to the fact that sonic anemometer data come from a few discrete heights. The errors in estimating spatial gradients with a few sensors have been so large that we think a measuring technique with higher accuracy and higher spatial resolution is needed.

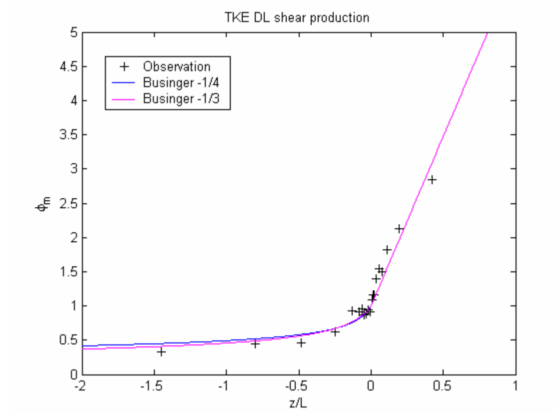

Figure 3 - Dimensionless shear production parameterization

5.2 Dimensionless turbulent dissipation

Dimensionless dissipation was tested against Wyngaard relation as in the equation (7). As expected, the formula failed to predict the data due to the imbalance. So we slightly modified the formula by introducing an imbalance coefficient ψ for all three stability conditions:

$$\phi_\varepsilon(z/L)=\begin{cases}\psi\left(1+\alpha(z/L)^{3/5}\right)^{3/2}, & z/L>0\\ \psi, & z/L=0\\ \psi\left(1+\beta|z/L|^{2/3}\right)^{3/2}, & z/L<0\end{cases} \tag{9}$$

From the near neutral data, we found the imbalance coefficient ψ to be 0.44. The best fitting generated α=2.1, and β=0.45, which are close to Wyngaard's original coefficients. We also tested dissipation term against Frenzen's [Frenzen and Vogel, 2001] formula as:

$$\phi_\varepsilon(z/L)=\begin{cases}\left(f_1+f_2(z/L)\right)\left(\phi_m-z/L\right), & z/L>0\\ f_1, & z/L=0\\ f_1\left(\phi_m-z/L\right), & z/L<0\end{cases} \tag{10}$$

After replacing their imbalance coefficient f_1 value 0.84 with ours 0.48, we found f_2=0.7, which is very close to their original value 0.6. These two parameterizations with new coefficients along with the experimental data can be found in Figure 4. Note that Frenzen's formula gives very different prediction from Wyngaard relation especially for very stable case. We believed this is because he proposed this formula based on a much narrower range of stability than ours. It is also noted that two formulas give very different transition near the neutral condition.

The result is interesting and enlightening: the previous formulas work pretty effectively if the right imbalance coefficient is determined. However the most intriguing question that why the imbalance differs in different field measurements still lacks a satisfactory answer.

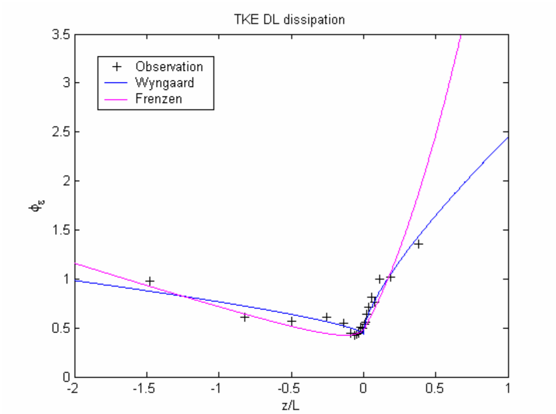

Figure 4 - Dimensionless turbulent dissipation parameterization

5.3 Dimensionless turbulent transport

Wyngaard and Cote [1971] proposed that dimensionless turbulent transport balances the buoyancy production under unstable condition, i.e., $\phi_t = z/L$ for $z/L < 0$. Our data were tested against this *z/L* law, as well as the modified linear form:

$$\phi_t = a + b\left(\frac{z}{L}\right), \qquad z/L < 0 \tag{11}$$

We also extended this formula to stable condition. Least square fitting generates the coefficients to be a = 0.36, b = -0.15. Figure 5 shows the observed dimensionless turbulent transport and how they compare with the two parameterizations. The *z/L* law fails apparently, which indicates the inability of turbulent transport to balance the buoyancy production in our case.

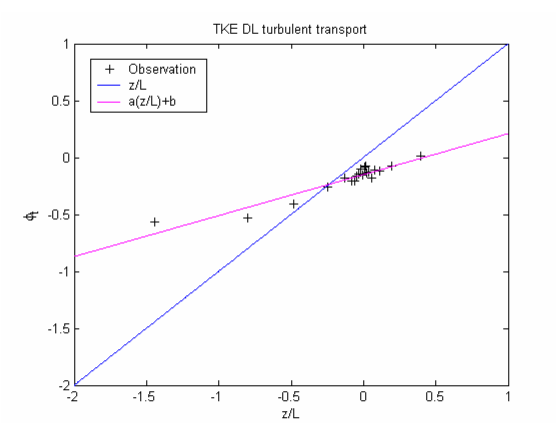

Figure 5 - Dimensionless turbulent transport parameterization

5.4 Parameterized dimensionless TKE budget

Finally all parameterizations with our new found coefficients were presented in Figure 5. The local imbalance of TKE is clearly shown in the range of stability -2 < *z/L* <1. We can also see the residual term, which is basically the pressure transport assuming stationary and horizontal homogeneity, can not be neglected in a very wide range.

Although the estimated pressure transport term provides us a possibility of interpreting all TKE terms, we are not very confident about this method. Different sources of error may influence the result. Stationary assumption has been validated by the data. Horizontal homogeneity was expected since the Hanford experiment has a large enough field, but this can not be verified without having any horizontal spatial measurements. Note that horizontal homogeneity introduces uncertainties in both the turbulent dissipation and the advection term, and thus the estimated pressure transport as well. Even if the assumption is correct, the uncertainty in the estimated pressure transport could still be large due to the uncertainty from turbulent transport term. So we believe in order to understand this mysterious term better, an accurate and inexpensive technique for measuring pressure fluctuations is needed.

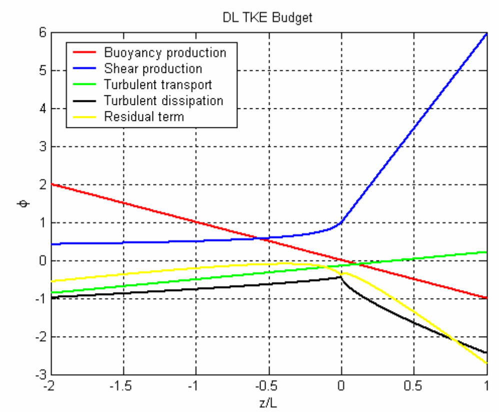

Figure 6 - Parameterized dimensionless TKE budget terms

6. DISCUSSION AND CONCLUSION

Atmospheric data from sonic anemometers in Hanford 2002 Field Experiment has been utilized to study parameterization of the TKE budget in the surface layer. The analysis included:

1) Parameterization of the dimensionless TKE budget for a wide range of stability (-2 < *z/L* <1).
2) Von Kármán constant was found to be 0.385.
3) Dimensionless shear production parameterization generated similar coefficients as previous literatures for Businger-Dyer relation.
4) An imbalance coefficient was introduced to improve Wyngaard relation and Frenzen's formula. The mechanism of imbalance still remained unclear.
5) Dimensionless turbulent transport parameterization showed a strong deviation from the traditional z/L law.

From the field data, a higher spatial resolution of velocity and temperature measurements will greatly improve the TKE parameterization. A technique of measuring pressure fluctuations is also needed.

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