DO GRAVITY WAVES TRANSPORT ANGULAR MOMENTUM AWAY FROM HURRICANES?

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1. INTRODUCTION

The importance of understanding the effect of convection on a tropical cyclone has been examined extensively in the past but with different approaches to the representation of convection around the storm. In many previous studies employing two-dimensional or balanced models (e.g. Montgomery and Kallenbach 1997; Nolan and Farrell 1999) balanced potential vorticity perturbations were used to represent convection because they were considered to be the end product of a rapid adjustment process to strong localized convective heating. However, more recent studies (Nolan and Montgomery 2002; Nolan and Grasso 2003; Nolan et al. 2007) have shown the importance of capturing the rapid three-dimensional nonhydrostatic adjustment processes such as gravity wave radiation to accurately compute the effect of convection on a tropical cyclone.

In earlier studies (*e.g.* Abdullah 1966; Kurihara 1976; Willoughby 1978), gravity waves were speculated to trigger the formation of spiral bands in tropical cyclone. However, inspired by the work of MacDonald (1968) that qualitatively proposed the existence of Rossby-like waves in tropical cyclones, spiral bands have been identified recently as more slowly moving potential vorticity bands (Guinn and Schubert 1993) and their propagation properties relative to the mean flow have been described in the theory of vortex-Rossby waves (Montgomery and Kallenbach 1997; Möller and Montgomery 2000). In addition, vortex-Rossby waves were shown to be dynamically more active than gravity waves in terms of wave activities in the analysis of a numerically simulated tropical cyclone (Chen et al. 2003).

However, this does not necessarily mean that gravity waves can be neglected when examining tropical cyclones. Gravity waves may be capable of transporting angular momentum out of the vortex core (Chow et al. 2002, hereafter CCL02; Chow and Chan 2003, hereafter CC03). CCL02 showed that a rotating elliptical vortex in two-dimensional shallow water equations can generate large-scale moving outer spiral bands that are often simulated in a full-physics numerical model. CC03 derived an analytical expression based on the shallow water equations and found that gravity waves in a tropical cyclone can transport a significant amount of angular momentum out of the vortex (~13% per hour). But CCL02 and CC03 were based on the two-dimensional hydrostatic framework, missing the importance of capturing the three-dimensional adjustment processes. The goal of this study is to revisit and clarify how effective gravity waves might be in transporting angular momentum away from a tropical cyclone by using a three-dimensional, nonhydrostatic, linear model.

2. MODEL AND METHOD

Angular momentum transport by gravity waves in a tropical cyclone is investigated here by using a three-dimensional, nonhydrostatic but linear model of vortex dynamics, now known as Three-Dimensional Vortex Perturbation Analysis and Simulation (3DVPAS: see Nolan and Montgomery 2002; Nolan and Grasso 2003; Hodyss and Nolan 2007; Nolan et al. 2007). 3DVPAS is based on the dry vortex-anelastic equations and allows for the simulation and analysis of unbalanced asymmetric perturbations on balanced, axisymmetric basic state vortices. It can simulate both asymmetric and symmetric motions, with some coupling between them by using eddy flux divergence tendencies arising from asymmetric motions as forcing for symmetric motions.

The basic state vortex is in hydrostatic and gradient wind balance and is modeled after tropical cyclones. Four different basic state vortices are constructed to have the maximum tangential wind of 35.0 ms^{-1} at a radius of 31.5 km, but with different radial structures: 1) a “Rankine-with-skirt” vortex (hereafter the RWS vortex), 2) a modified Rankine vortex (hereafter the MR vortex), 3) a sectionally continuous single-exponential vortex (Willoughby et al. 2006; hereafter the SC vortex), and 4) a Gaussian vortex (hereafter the GS vortex). Figs. 1a and 1b show their radial profiles of tangential velocity and relative vorticity. These radial velocity profiles are extended into the vertical direction through the use of some analytical functions to construct realistic tropical cyclone wind fields, as shown in Fig. 1c for the RWS vortex. Secondary circulation is absent in all of the basic state vortices considered here.

The model domain is 250 km in the radial direction and 20 km in the vertical direction. The grids are stretched in the radial direction so more points are used in the inner core region, but unstretched in the vertical direction. Free-slip, solid-wall boundary conditions are enforced on all sides and Rayleigh damping regions are placed at the upper and outer boundaries. Temperatures and pressure fields are calculated to satisfy hydrostatic and gradient wind balance.

To represent rotating inner-core asymmetries in the eyewall of the basic state, purely asymmetric wavenumber-two heat sources equivalent to heating of 5.0 K hr^{-1} are introduced at r = 30 km, z = 6 km (the eyewall region) and allowed to rotate around the core for 24 hours at the half of the maximum azimuthal velocity. The rotating heat sources have a structure

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similar to a Gaussian bubble. The choice of rotating asymmetries on a circular vortex is equivalent to the effect of a rotating elliptical vortex, as used in CCL02 and CC03.

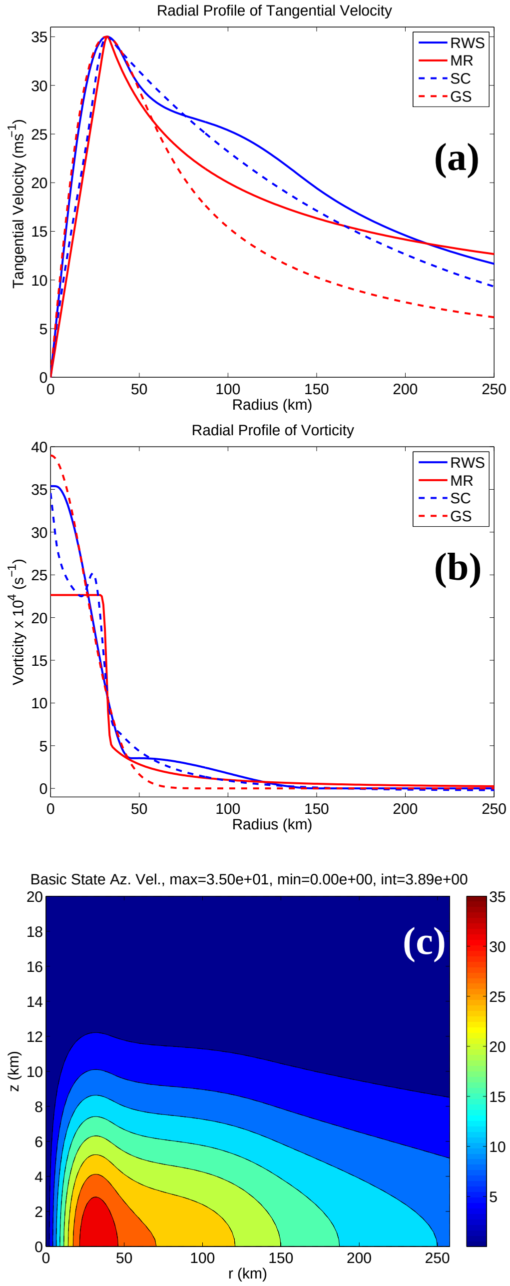

Fig. 1: Radial profiles of (a) tangential velocity and (b) relative vorticity of the basic state vortices; (c) radius-height cross-section of the tangential wind field of the RWS vortex.

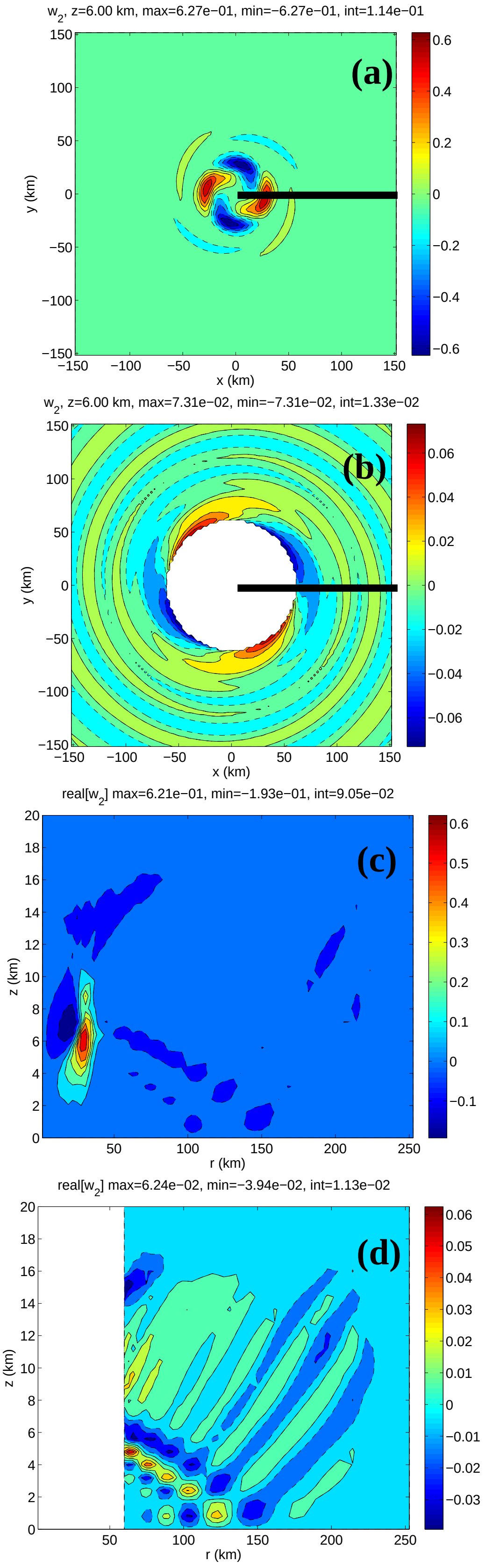

Fig. 2: (a) Horizontal cross-section of asymmetric vertical velocity field at z = 6 km, t = 24 h; (b) same as in (a) but focusing on the outer region; (c) radial-height cross-section along the line in (a); (d) same as (c) but along the line in (b).

3. SOME PRELIMINARY RESULTS

a. A control case

The result of a control case with the RWS vortex as the basic state is presented here first. After approximately 15 hours, the asymmetric response settles into a nearly steady state of radiating asymmetric gravity waves. Examining the vertical velocity field at *t* = 24 h (Figs. 2a and 2c) reveals that radiating gravity waves are not clearly evident at first because the asymmetric response is strongly localized near the eyewall region. However, focusing on the outer region (Figs. 2b and 2d) clearly shows the existence of gravity waves propagating radially outward from the inner core the eyewall in a cyclonic spiral fashion, despite their significantly smaller magnitude.

When unbalanced asymmetric temperature perturbations are placed at a location with nonzero vorticity gradient, both dry gravity waves and vortex-Rossby waves are excited, but only vortex-Rossby waves carry a potential vorticity signature. The asymmetric potential vorticity field at z = 6 km, *t* = 24 h (Fig. 3a) shows that potential vorticity is strongly localized near the eyewall region but is negligible in the outer storm environment. The fact that vortex-Rossby waves which are excited in the inner core region do not propagate far from the eyewall region indicates the existence of a stagnation radius for vortex-Rossby waves.

Fig. 3b shows that the eddy angular momentum flux divergence at *t* = 24 h is heavily localized near the eyewall region and is much smaller outside the inner core region, which indicates no significant interaction of radiating gravity waves in the outer storm environment. However, the eddy angular momentum flux vectors (not shown) are pointed radially outward, suggesting that angular momentum is transported radially outward by radiating gravity waves.

Since the stagnation radius for vortex-Rossby waves is located not too far from the eyewall region of the basic state vortex and gravity waves do not interact with the outer storm environment during propagation, the angular momentum transported away from the storm by the gravity waves may be approximated by the change of angular momentum in the *core* of the vortex, where the *core* is defined between the vortex center and the stagnation radius of vortex-Rossby waves.

The symmetric response of the RWS vortex to the rotating asymmetries is computed by using the asymmetric eddy flux divergences as the symmetric forcing, and Fig. 4 shows the 24-hour change in angular momentum field which is again heavily localized near the eyewall. The stagnation radius for n=2 vortex-Rossby waves is computed by using the analytic expression derived by Montgomery and Kallenbach (1997), which is 42 km. The 24-hour loss of angular momentum in the core of the RWS vortex is 6.0×10^{17} $\mathrm{kgm^2s^{-1}}$, compared to the angular momentum of the RWS vortex of 3.2×10^{19} $\mathrm{kgm^2s^{-1}}$. The 24-hour fractional loss of angular momentum is 1.9%, which is significantly lower than the CC03's estimation of 13% per hour.

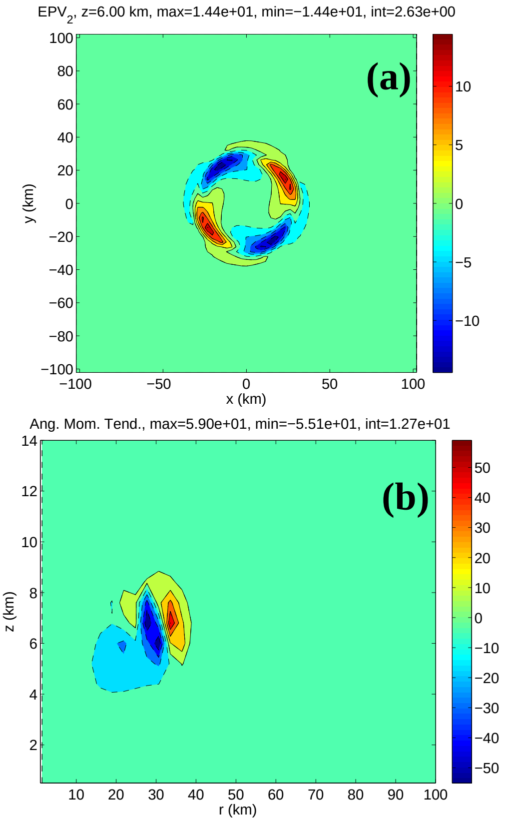

Fig. 3: (a) Horizontal cross-section of potential vorticity field at *z* = 6 km, *t* = 24 h. (b) Radial-height cross-section of the divergence of angular momentum flux at *t* = 24 h.

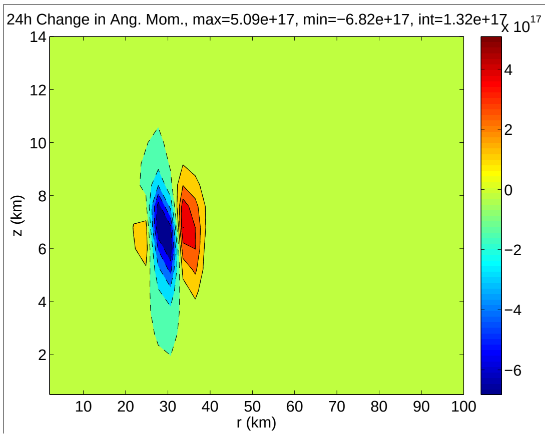

Fig. 4: The 24-hour change in angular momentum induced by the symmetric response to the rotating asymmetries.

b. Radial vorticity profiles of the basic state vortices

Simulations as in the control case are repeated but with different basic state vortices having different radial vorticity profiles. The 24-hour fractional losses of angular momentum for the MR, SC, and GS vortices are 0.1 %, 0.3 %, and 1.8 % respectively, as compared to 1.9 % for the control case with the RWS vortex. The difference may be due to the different radial vorticity gradient profiles, since both the stagnation radius and the damping rate of vortex-Rossby waves at that stagnation radius are closely related to the radial vorticity gradient profiles.

c. Rotation speed and wavenumber of the asymmetries

Simulations as in the control are repeated but with different rotation speed and wavenumber of the asymmetries. Five different rotation speeds (0%, 25%, 50%, 75%, and 100%) relative to the local surface tangential velocity are chosen. Increasing the rotation speed of the asymmetries from 25% to 100% leads to less angular momentum transport out of the vortex by gravity waves. However, for the case with the 0% rotation speed (the nonrotating asymmetries), the 24-hour fractional loss of angular momentum was negative; angular momentum is instead *deposited* into the core of the vortex. Because the higher rotation speed of the asymmetries helps the asymmetries maintain more coherent structure against shear, the asymmetries may become more "balanced", leading to weaker gravity wave radiation. Increasing the wavenumber (from $n = 1$ to $n = 4$) of the rotating asymmetries leads to a smaller angular momentum removal by radiating gravity waves.

4. SUMMARY AND FUTURE WORK

The efficiency of angular momentum transport by gravity waves is reexamined with a three-dimensional, nonhydrostatic but linear model. Numerical simulations show that gravity waves do transport angular momentum away from hurricanes in most cases. However, in contrast to the previous calculation by CC03 which concluded that gravity waves are very effective at removing angular momentum from tropical cyclones, this study reaches the conclusion that gravity waves do not transport a significant amount of angular momentum out of tropical cyclones. The disparity between CC03 and this study hints at the importance of capturing the three-dimensional adjustment process. Further analysis is underway.

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