1. PURPOSE

There is considerable interest in the question of how best to assess the economic value of weather forecasts (Stern and Dawkins, 2004; Stern, 2005a). The primary purpose of this paper is to report on how this might be achieved utilising an application of financial market mathematics (Stern, 2006a).

Specifically, the focus of the work presented here is upon a set of Day1 to Day7 maximum temperature forecasts, which have been generated by a system that mechanically integrates (that is, combines) judgmental (human) and automated predictions (Stern, 2006b; 2007a&b). The system has now been extended to provide forecasts out to ten days (Stern, 1999; Stern, 2005b&c).

2. INTRODUCTION

A "real-time" trial of the system has been ongoing since 20 August 2005. After 589 days, to 31 March 2007, the trial revealed that, overall, the various components (rainfall amount, sensible weather, minimum temperature, and maximum temperature) of Melbourne forecasts so generated explained 42.0% variance of the weather, 6.1% more variance than the 35.9% variance explained by the human (official) forecasts alone (Table 1).

3. FORECASTS OUT TO TEN DAYS

Since 20 August 2006, forecasts have also been generated for beyond Day7 (Day8 to Day10). After 224 days, to 31 March 2007, Day8 forecasts explained 11.7% of the variance, Day9 forecasts explained 6.3% of the variance, and Day10 forecasts explained 3.5% of the variance (Figure 1). For these longer range forecasts, the variance explained was mainly for the temperature components.

4. HIGH-IMPACT EVENTS

Shapiro and Thorpe (2004) note: "THORPEX addresses the influence of sub-seasonal time-scales on high-impact forecasts out to two weeks, and thereby aspires to bridge the 'middle ground' between medium range weather forecasting and climate prediction".

It was expected that mechanically integrating judgmental (human) and automated predictions via some kind of an averaging procedure would result in an inferior set of forecasts for extreme (high-impact) events.

Figure 2 shows that this proved to be the case for the hottest 10% of days (Decile 10), the average absolute error of the combined forecasts leading up to the hottest 10% of days being 0.27°C higher than the average absolute error of the corresponding official forecasts. However, this proved not to be the case for the coolest 10% of days (Decile 1), the average absolute error of the combined forecasts leading up to the coolest 10% of days being 0.11°C lower than the average absolute error of the corresponding official forecasts.

5. FORECAST CONSISTENCY

What is particularly interesting about the verification data is that the combined forecasts are more consistent than the official forecasts.

For example, the consistency, that is, the RMS inter-diurnal change in the sequences of combined forecasts of maximum temperature (7 days in advance, 6 days in advance, 5 days in advance, 4 days in advance, 3 days in advance, 2 days in advance, 1 day in advance) is 1.38°C (this RMS inter-diurnal change being well below the 1.82°C associated with the official forecasts).

6. OPTIONS PRICING THEORY

In a 1992 paper presented to the 5th International Meeting on Statistical Climatology, the author introduced a methodology for calculating the cost of protecting against the onset of global warming (Stern, 1992).

The paper, "The likelihood of climate change: A methodology to assess the risk and the appropriate defence", was presented to the meeting held in

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Toronto, Canada, under the auspices of the American Meteorology Society (AMS). In this first application of what later was to become known as 'weather derivatives' (Stern, 2001a,b,c,d; 2002a&amp;b; Dawkins and Stern, 2003, 2004; Stern and Dawkins, 2003, 2004) the methodology used options pricing theory from the financial markets to evaluate hedging and speculative instruments that may be applied to climate fluctuations.

What now follows is an application of options pricing theory where one uses the theory in the context of assessing the economic value of weather forecasts.

The theory shows that the more consistent forecasts are from one day to the next, between Day7 (when they are first issued) and Day1 (the final issue), the cheaper are the prices of option contracts (weather derivatives) that one may wish to purchase to protect against the eventuality that the forecasts might be incorrect (refer to the next section).

The implication from this is that, the more consistent forecasts are from one day to the next, the more valuable are the forecasts.

7. THE ECONOMIC VALUE OF FORECASTS

A challenge in pricing options on commodities is non-randomness in the evolution of many commodity prices. For example, the spot price of an agricultural product will generally rise prior to a harvest and fall following the harvest. Natural gas tends to be more expensive during winter months than summer months. Because of such non-randomness, many spot commodity prices cannot be modelled with a geometric Brownian motion, and the Black-Scholes (1973) or Merton (1973) models for options on stocks do not apply.

In 1976, Fischer Black published a paper (Black, 1976) addressing this problem and the Risk Glossary (2006) summarises the result of his work thus:

His (Black’s) solution was to model forward prices as opposed to spot prices. Forward prices do not exhibit the same non-randomness of spot prices. Consider a forward price for delivery shortly after a harvest of an agricultural product. Prior to the harvest, the spot price may be high, reflecting depleted supplies of the product, but the forward price will not be high. Because it is for delivery after the harvest, it will be low in anticipation of a drop in prices following the harvest. While it is not reasonable to model the spot price with a Brownian motion, it may be reasonable to model the forward price with one. The assumption that the spot price follows a log-normal process is replaced by the assumption that the forward price follows such a process. From there the derivation is identical to the Black-Scholes formula for evaluating stock options and so the final formula is the same except that the spot price is replaced by the forward - the forward price represents the expected future value discounted at the risk free rate. Black’s (1976) option pricing formula reflects this solution, modelling a forward price as an underlier in place of a spot price.

Pricing options on forecast weather elements, which may be employed in a weather risk management context, also requires one to address non-randomness in the evolution of many forecasts of these weather elements. For example, the predicted maximum temperature, for say, 4 days hence, will generally rise as a ridge of high-pressure approaches (anticipating warmer winds from lower latitudes once the ridge passes) even though the current temperature is relatively low. Because of such non-randomness, forecasts of weather elements cannot be modelled with a geometric Brownian motion, and Black’s (1976) option pricing formula also now can be applied to forecasts of a weather element.

From the foregoing, one may note that, in evaluating option contracts used in the context of applying “weather derivatives” to day to day forecasts in a risk management context, it may be demonstrated that the cost of the “weather derivative” option on a forecast increases as the volatility (σ) of the underlying forecast increases in precisely the same manner that the cost of an option on a forward contract increases as the volatility (σ) of the underlying forward increases.

The Black (1976) formula for a call option on an underlying struck at K, expiring T years in the future is

\[ c = e^{-rT}(FN(d_1) - KN(d_2)) \]

and the put price is

\[ p = e^{-rT}(KN(-d_2) - FN(-d_1)) \]

where

- \( r \) is the risk-free interest rate
- \( F \) is the current forward price of the underlying for the option maturity
- \( \sigma \) is the volatility of the forward price, and,
- \( N(.) \) is the standard cumulative Normal distribution function.

From the formula, the issue of what values to use is not a trivial one. To illustrate, let us suppose that one wishes to value a European call option (using a dividend yield of 0%) on the Day1 forecast maximum temperature being above 35°C when:

- The forecast at Day7 is for a temperature of 32°C.
The RMS Inter-Diurnal Change is 2°C,

- The interest rate is 5%, and
- The pay-off is $1.00 per each degree Celsius above 35°C.

The methodology is now illustrated:

**Step 1.** To neutralise the impact of the choice of units used, add a large number, say, 1000, to both F and K, which results in F=1035 and K=1032.

**Step 2.** To neutralise the impact of the units used for the volatility, divide the RMS inter-diurnal change (2°C) by the new value for K (1032), which approximates to the 1-day volatility that one would obtain under the assumption that the new forecast follows a lognormal process. This is because the new forecast series is a set of large numbers, and, as a consequence, the RMS inter-diurnal change ~

\[ \sqrt{\frac{1}{6} \sum \left( \ln \left( \frac{|Day6value|}{|Day7value|} \right)^2 + \ln \left( \frac{|Day5value|}{|Day6value|} \right)^2 + \ln \left( \frac{|Day4value|}{|Day5value|} \right)^2 + \ln \left( \frac{|Day3value|}{|Day4value|} \right)^2 + \ln \left( \frac{|Day2value|}{|Day3value|} \right)^2 + \ln \left( \frac{|Day1value|}{|Day2value|} \right)^2 \)\]

**Step 3.** To obtain the annualised volatility, multiply the 1-day volatility obtained at Step 2 by \( \sqrt{365} \) that, in the current case, yields 3.70%.

**Step 4.** Go to one of the many option calculators on the WEB (for example, Numa Financial Systems, 2006) to obtain a theoretical European call option value based on a maturity date of 6 days hence (Day-7 to Day-1) to yield $1.06 as the value of the call option.

The material presented demonstrates the proposition that, when undertaking a defensive strategy of purchasing weather derivatives, the cost of protecting against the possibility of weather forecasts being in error ($1.06 for an RMS Inter-Diurnal Change of 2°C) reduces as the forecast consistency increases, that is, as the RMS Inter-Diurnal Change decreases, is confirmed.

To illustrate:

- For an RMS Inter-Diurnal Change of 1°C, the value of the call option reduces to $0.25, but,
- For an RMS Inter-Diurnal Change of 3°C, the value of the call option increases to $1.97, and,

For an RMS Inter-Diurnal Change of 4°C, the value of the call option increases further to $2.93.

8. A COMPETITIVE ADVANTAGE

The American Marketing Association (2006) notes: “a ‘competitive advantage’ exists when there is a match between the distinctive competences of a firm and the factors critical for success within the industry that permits the firm to outperform its competitors. Advantages can be gained by having the lowest delivered costs and/or differentiation in terms of providing superior or unique performance on attributes that are important to customers.”

From the foregoing, it may be said that the value of a series of weather forecasts with a low volatility, that is, a series of forecasts that display a high level of consistency from one day to the next, is greater than the value of a series of forecasts with a high volatility.

This is because the cost of protecting against the possibility of such weather forecasts being incorrect by adopting a strategy of purchasing weather derivatives is lower.

This means that sellers of weather derivatives, who utilise low volatility forecasts to price their call and put options, are provided with a competitive advantage over sellers of weather derivatives who utilise high volatility forecasts. This arises because sellers of weather derivatives who utilise low volatility forecasts being able to charge lower, and, therefore, more competitive, prices to purchasers of weather derivatives who wish to use those weather derivatives to protect against the possibility of the weather forecasts being incorrect.

9. CONCLUSION

The verification data, in showing that the combined forecasts are more consistent than the official forecasts, are also showing that the combined forecasts are more valuable than the official forecasts.

Furthermore, that the combined forecasts are also more accurate than individual currently available predictions taken separately, also provides the small to medium sized companies involved in weather risk management and weather broadcasting with a potential competitive advantage (O’Donnell et al., 2002) over their peers should they choose to adopt a strategy of mechanically combining existing predictions.

And, there is a multiplicity of existing predictions to choose from (Australian Weather News, 2006; Bureau of Meteorology, 2006)).
10. REFERENCES


Numa Financial Systems, 2006: The Numa Option Calculator is available via the website http://www.numa.com, which was accessed on 15 July 2006.


Stern, H., 2006a: The application of financial market mathematics to translating climate forecasts into decision making, 3rd International Conference on Climate Impacts Assessments (TICCIA), Cairns, Australia, 24-27 Jul., 2006.


<table>
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<th>Weather Element</th>
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Table 1 Nearly two years of verification statistics show that the process of mechanically integrating (combining) the forecasts substantially improves the officially issued product.

Figure 1 Percentage variance of observed weather explained by forecasts between 1 and 10 days ahead.
Figure 2 Average change in absolute error through mechanically integrating forecasts for each maximum temperature decile range.