P2.21 PROBABILISTIC QUANTITATIVE PRECIPITATION FORECASTING

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1. INTRODUCTION

To provide enhanced customer service, the National Weather Service Weather Forecast Office in Tulsa, Oklahoma (WFO TSA), routinely makes probabilistic rainfall forecasts for arbitrarily selected rainfall amounts (0.10, 0.50, 1.00, 2.00 inches). These probabilistic quantitative precipitation forecasts (PQPFs) have been experimental since 2005 (Amburn, 2006), and provide the unconditional probability of exceedance (uPOE) for the selected rainfall amounts. The forecast method uses the fact that frequency distributions of rainfall amounts typically fit the exponential distribution. Even where they do not, the actual probability to exceed any given rainfall amount for a given event can be closely approximated using the probability density function (PDF) of the exponential distribution.

In the WFO's Gridded Forecast Editor (Global Systems Division, 2006) meteorologists at the WFO create the probability of precipitation (PoP) forecast and the quantitative precipitation forecast (QPF). At WFO TSA, the QPF is defined as the mean of the rainfall distribution expected for the period in question at each GFE 2.5 x 2.5 km grid box. Therefore, the QPF defines a unique probability density function for each grid box for the particular rain event. The exceedance probabilities are then computed for each grid box across the entire forecast area.

The development of this method to produce probabilistic QPF leads us to a specific definition for QPF. In calculating the unconditional probability of exceedance (uPOE), it is necessary to know and use the forecast mean of the expected distribution of rainfall. Specifically then, the desirable QPF should be defined as the ***expected mean of the rainfall distribution that a forecaster would expect at a point during the forecast time period, from a large number of similar rainfall events.*** This definition is also then consistent with the NWS PoP forecast which is also a point forecast (Hughes, 1980).

The benefit of producing probabilistic QPF as exceedance probabilities is to allow users who know their own cost/benefit ratios to use these exceedance probabilities in decision making. It also allows forecasters to convey much more information to the user community. A 50% probability of 0.10 inches of rain may not stop a farmer from cutting hay. However, a 10% chance for two inches of rain may be sufficient for an emergency manager or city storm water official to take action.

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2. CONCEPT AND THEORY

Theory is that typical rainfall events have a distribution of rainfall amounts that closely resembles a special case of the gamma distribution. Wilks (1995) states, "the versatility in shape of the gamma distribution makes it an attractive candidate for representing precipitation data, and it is often used for this purpose." The exponential distribution (a special case of the gamma distribution where the alpha parameter is 1) provides an excellent representation of precipitation climatologies. When applied, a unique probability density function (PDF) for any event can be defined using the QPF and PoP for that event. From the PDF, POEs can be computed for any selected rainfall amounts.

Ultimately, the Tulsa method produces POEs using four terms: 1) the PDF of the exponential distribution; 2) WFO generated QPFs that are used for the mean of the PDF; 3) WFO generated PoP; and 3) software in the Gridded Forecast Editor (GFE, Global Systems Division, 2005).

Examples of both text and graphic products are shown later in this paper. Although point POEs can be easily calculated, county averages are currently computed and distributed to conserve Internet bandwidth and data server resources.

2. MATHEMATICAL BACKGROUND

This formula-based method of producing POEs is based on the assumption that the distribution of given precipitation amounts can be approximated by the gamma distribution. The gamma function is shown in Equation (1). A brief mathematical explanation is provided here.

$\Gamma(\alpha) = \int x^{(\alpha-1)} e^{-x} \, dx$ **(1)**

,for $\alpha > 0$, integrated from 0 to ∞.

The gamma distribution takes on several different shapes, as shown in Figure 1, depending on the values of the shape parameter alpha (α). However, based on climatology, an appropriate distribution for most precipitation events is where alpha is equal to 1.0. In this case, the frequency of small rainfall amounts is highest, with a rapid decrease in frequencies of higher amounts. Where $\alpha = 1$, the gamma distribution simplifies to a special distribution called the exponential distribution which can be used in producing the POEs. The density function of the exponential distribution is defined by Equation (2), where the mean value of the distribution is given by μ (Mu).

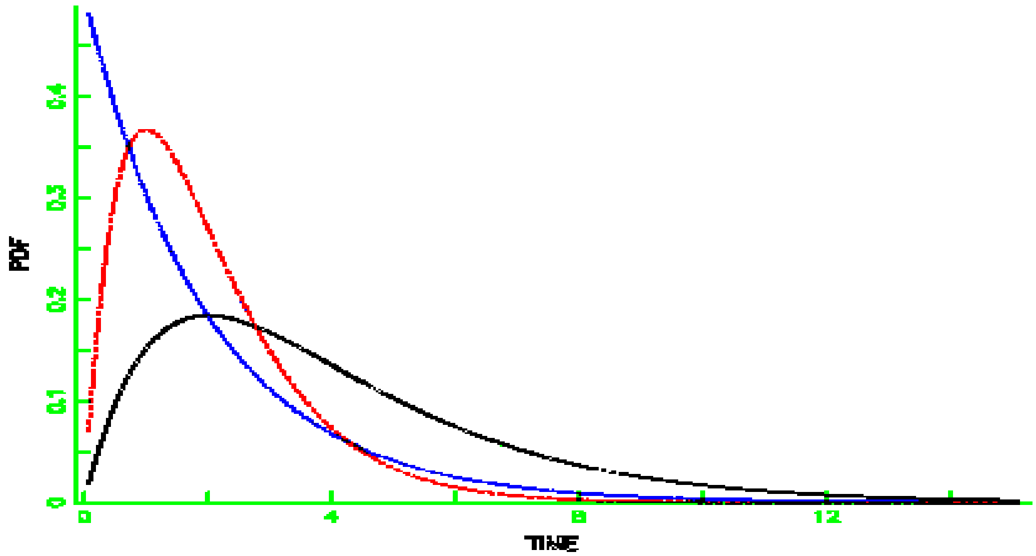


Figure 1. Examples of gamma PDF, where alpha = 1 (blue line), 2 (red line), 3 (black line), from Engineering Statistics Handbook (2005)

$$f(x) = (1/\mu) * e^{-x/\mu} \qquad (2)$$

, where μ is the mean

Integrating (2) yields the cumulative density function (Equation 3), where the POE can be computed for any selected rainfall amount, x. (A more rigorous explanation can be found in a number of statistics books, such as Wilks (1995)).

$$POE(x) = e^{-x/\mu} \qquad (3)$$

Examples of the exponential PDF are shown in Figure 2 for a variety of means. Note, as the mean increases, the PDF becomes “flatter” with a larger area under the right tail of the PDF. This indicates that events with higher average rainfalls will have a higher frequency of larger individual rainfall amounts, and therefore higher POEs for large rainfall amounts. Table 1 contains examples of POEs, using Equation (3), for certain threshold QPFs with different mean values. POEs in the table are conditional upon the occurrence of rain. The reader will notice that the probability for attaining the mean value is less than 50%. This is a characteristic of the exponential distribution. The third line in Table 1 provides a good example. The mean on that line is 0.50 inches, yet the POE for 0.50 inches is only 36.8%. Since the exponential distribution is skewed strongly toward lower values, the mean and median are not the same. The mean value will be higher than the median. More robust explanations can be found in statistics textbooks, such as Walpole and Myer (1978).

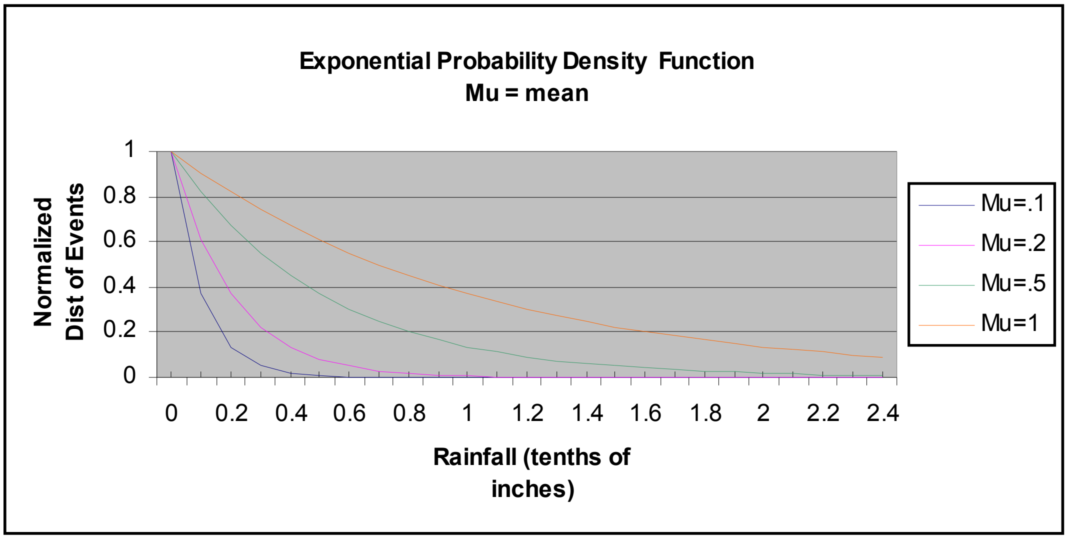


Figure 2. Exponential density functions for varying means (Mu). For larger means, the decline in the number of rainfall amounts is less dramatic, indicating a greater frequency of heavier amounts.

Table 1. The table shows examples of POEs for different mean values, given a PoP of 100%.

Mean QPF(in.)	POE(.10)	POE(0.25)	POE(0.50)	POE(1.00)	POE(2.00)
0.10	0.368	0.082	0.007	0.000	0.000
0.20	0.607	0.287	0.082	0.007	0.000
0.50	0.819	0.607	0.368	0.135	0.018
0.75	0.875	0.717	0.513	0.264	0.069
1.00	0.905	0.779	0.607	0.368	0.135
1.50	0.936	0.846	0.717	0.513	0.264
2.00	0.951	0.882	0.779	0.607	0.368
2.50	0.961	0.905	0.819	0.670	0.449

3. EXAMPLES OF RAINFALL FREQUENCIES AND DISTRIBUTIONS

Plots of precipitation data for a ten-year period from 1995 through early 2005 for sites in the TSA CWFA match the shape of the exponential distribution rather well. Figure 3 shows 12-hour rainfall data plots for two sites in the TSA forecast area. It can be seen that the distribution of rainfall amounts in each 0.05-inch category bin decreases rapidly as the amounts increase.

While the exponential distribution is valid as a composite of events, is it also valid for individual rainfall events? Figures 4 through 7 include 376 individual 12-hour rainfall events in the Tulsa WFO forecast area from mid summer 2005 through late winter 2007. (A 12-hour rainfall is defined here as any 12-hour period where measurable rain occurred anywhere in the WFO TSA forecast area.) The consistency in the shape of the plots would indicate that the exponential distribution also applies to individual events as well as single stations over long periods of time as shown in Figure 3.

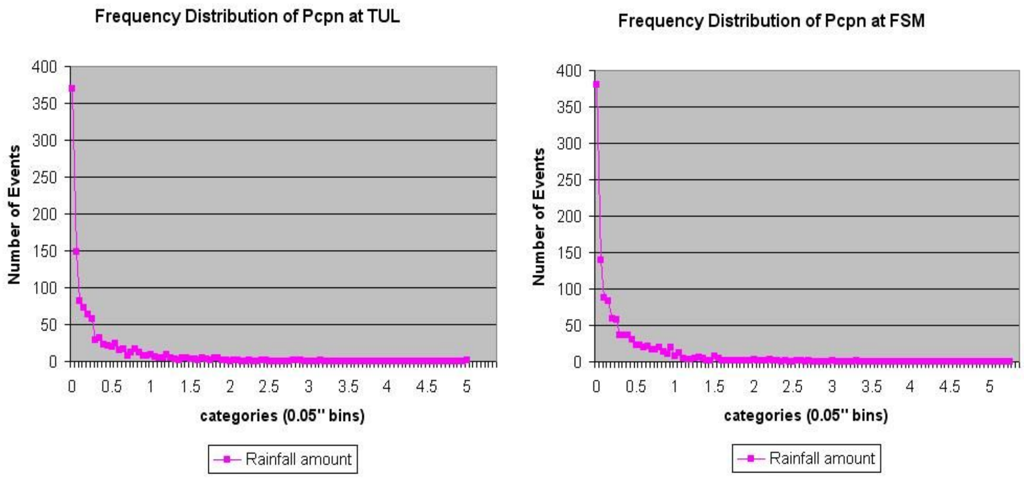


Figure 3. Rainfall frequency distributions from 1995 through early 2005 for Tulsa, OK (TUL), Fort Smith, AR (FSM). Frequencies are for 0.05 inch categories.

The data for Figures 4 through 7 are human quality controlled quantitative precipitation estimates from the NWS Arkansas Red Basin River Forecast Center in Tulsa, Oklahoma. These data analyses are performed hourly on a 4x4km grid that covers the Tulsa WFO forecast area. Each 4x4km grid is effectively considered a rain gage. The hourly analyses are summed over 12-hour periods and grouped into 0.05 inch categories to create the frequency distributions in the figure. A rainfall event was selected if it had an areal coverage of 10% or more of the forecast area. The events were then stratified by season, winter (DJF), spring (MAM), summer (JJA), autumn SON). Note that nearly all the distributions are exponential. There are a few exceptions, which generally fall in the winter period, probably related to more stratiform rain events with near 100% areal coverage. An example of that kind of event is shown in Figure 8, along with the actual POE and exponentially computed POE. However, even in this event, the computed POE provides more useful information than the current NWS PoP and QPF. Further analysis and study of a wider variety of events, particularly winter events, would be prudent. More appropriate distributions may be required for some events, depending on the synoptic scale influences.

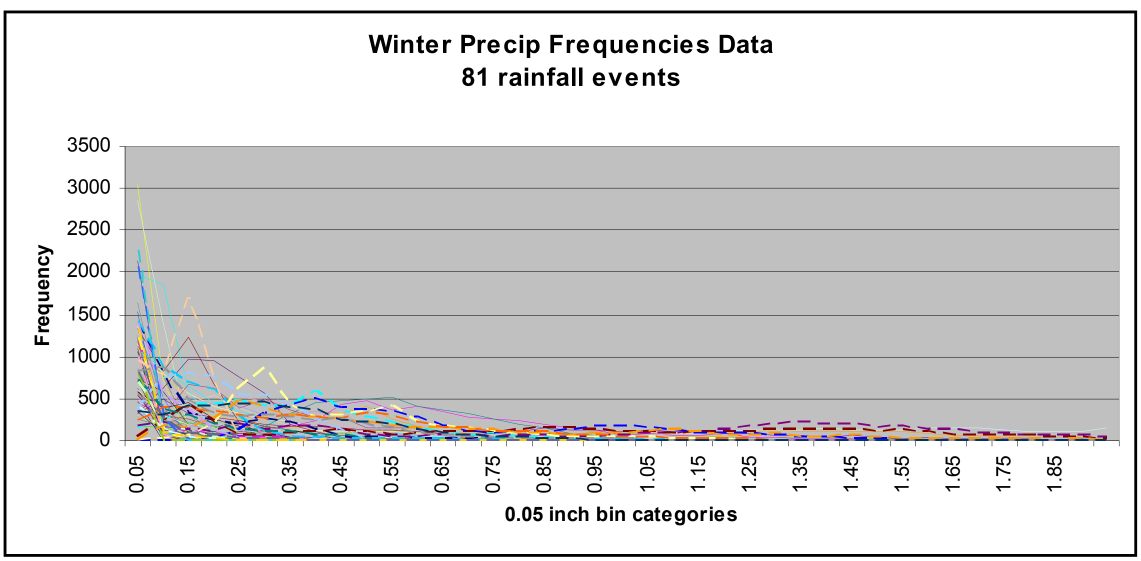


Figure 4. Winter rainfall distributions for 81 12-hour rainfall events in the TSA forecast area.

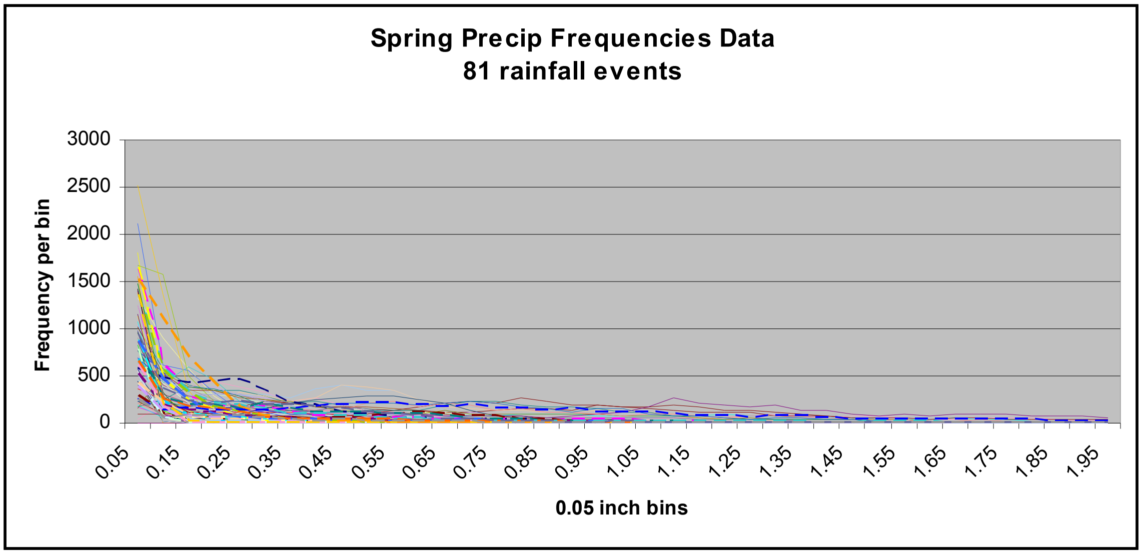


Figure 5 Spring rainfall distributions for 81 12-hour rainfall events in the TSA forecast area.

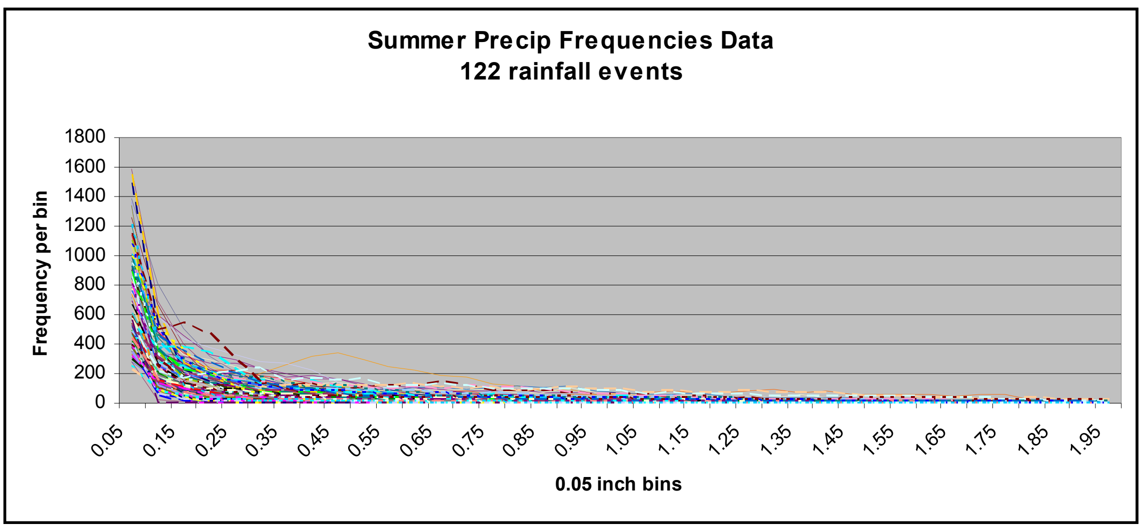


Figure 6. Summer rainfall distributions for 122 12-hour rainfall events in the TSA forecast area.

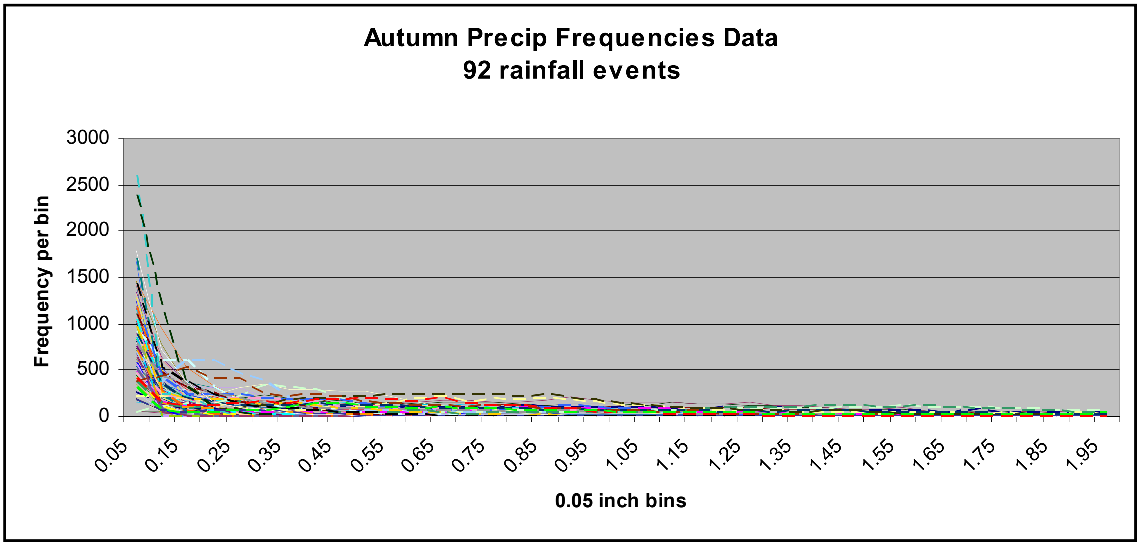
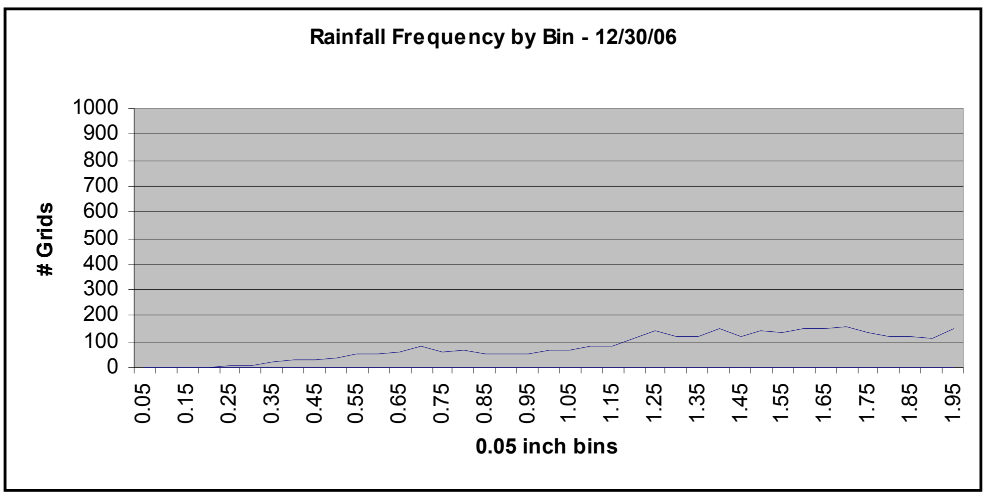


Figure 7. Autumn rainfall distributions for 92 12-hour rainfall events in the TSA forecast area.



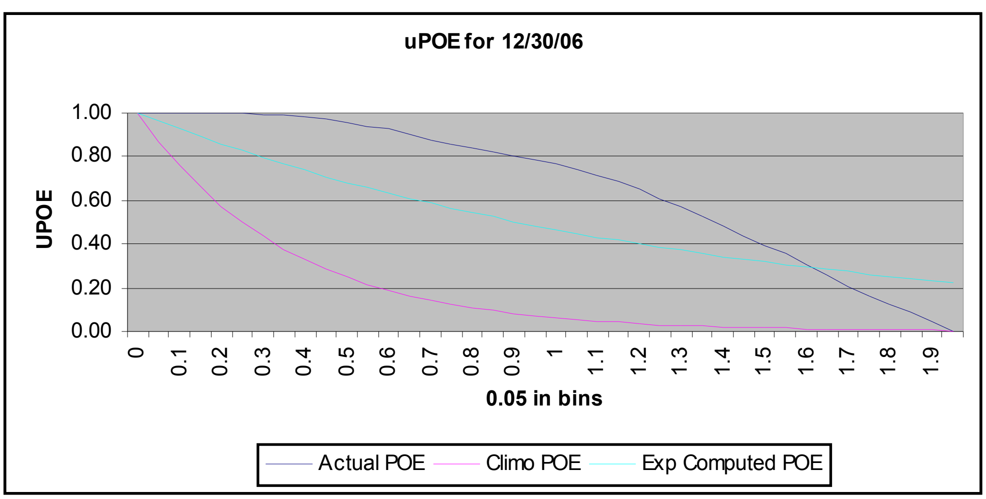


Figure 8. Non-typical rainfall distribution of rainfall for 12/30/06 in eastern Oklahoma (top) and corresponding actual POE compiled from the event (dark blue), and POEs computed using the exponential

PDF with climatological mean QPF (pink) and actual observed mean QPF (cyan).

4. THE FORECAST PROCESS

WFO meteorologists already forecast all the necessary parameters for the production of POEs. No additional workload is required. In fact, the process of producing the grid files, products and graphics is automated within the GFE. Forecasters create their PoP and QPF forecasts, as they currently do. Then, GFE calculates the POE grid fields for threshold rainfall amounts of 0.10, 0.50, 1.00 and 2.00 inches.

During the development phase of the Tulsa method, the type of POE had to be determined. Would it be conditional (based on the occurrence of rain), or unconditional (independent of the occurrence of rain). Following the lead of JK&R69, it was decided to produce the unconditional POE (uPOE). Equation (3) is then used to calculate the POE based on the mean (μ) rainfall amount based on the condition that rain occurs. That results in a conditional POE. The uPOE is then simply the product of the conditional POE and the NWS PoP as shown in Equation (4).

$$uPOE(x) = (e^{-x/\mu}) * PoP \qquad (4)$$

This solved an interesting problem. When calculated, conditional POEs frequently exceeded the standard PoP for that same period, since the value of μ is based on the condition that rain occurs. As an example, given that rain occurs, the conditional POE for 0.10 inches of rain may be 80%. However, the NWS PoP to measure 0.01 inches of rain may only be 30%. This might be confusing to the less sophisticated user.

One last obstacle had to be overcome. NWS QPF forecasts are unconditional, i.e., they are areal average amounts a forecaster expects when <u>all</u> gages are considered, including the ones that recorded no rain. Therefore, the NWS QPF needs to be converted to a conditional QPF, which is the value of μ used in Equation (4). Within the GFE, Equation (5) accomplishes this. This is also consistent with the work of JK&R69.

$$\mu = \text{Conditional QPF}$$
$$\mu = (\text{unconditional QPF}) / PoP \qquad (5)$$

The critical element to the entire process is the QPF supplied by the forecasters. That QPF is substituted for μ and changes the shape or "steepness" of the exponential PDF, thereby changing the resultant POEs. This step in the process takes advantage of a forecaster's expertise to identify events that may not match the "average" for that season. This should provide for much more accurate POEs than can be computed by simply using the seasonal mean as offered by JK&R69. Events not typical for the season will likely depend on the nature of the event (convective or non-convective).

After GFE performs the calculations using Equations (4) and (5), all output products are generated. The forecasters may then choose to alter the POEs, although that is not expected to happen very often. Verification and feedback to the forecaster should help determine if and when these adjustments will add value.

5. COMPARISONS TO PREVIOUS WORK

JK&R69 derived Equation (6), which defines the unconditional probability to exceed a certain amount of rainfall (r), for a given event. $P_t(r/0.01)$ is the conditional probability that an amount greater than "r" will occur, and is provided in the tables they compiled for 108 stations across the conterminous United States. P(0.01) is the probability of measurable rain (0.01 inches), which is the standard NWS PoP. An excerpt is provided in Appendix A.

$$P_t(r, 0.01) = P_t(r/0.01) * P(0.01) \qquad (6)$$

JK&R69 provided the following example where they compute the unconditional probability to exceed 0.50 inches of rain, based on a PoP of 60%. "Consider, for example, the problem of determining the probability of .50 inches or more of rain in the 'tonight' period for Atlanta during the spring months. Assume that the public probability forecast has assigned a .60 probability to the event of measurable precipitation for 'tonight' (00Z-12Z for Atlanta), so that P(.01) is .60. The data in table 1 provide the conditional probability P(.50/.01) = .27. Substituting into equation ([6]):

$$P_t(0.50/0.01) = P_t(0.50/0.01) * P(0.01)$$
$$= 0.27 * 0.60 = 0.16 \qquad (7)$$

The desired probability is then 0.16."

The rainfall mean for a 12-hour, spring event in Atlanta, obtained from the JK&R69 table is 0.36 inches. By substituting 0.36 for μ and using a PoP of 60% in equation 4, the Tulsa method yields the following:

$$uPOE(x) = (e^{-x/\mu}) * PoP \qquad (8)$$
$$= \text{Exp}(-.50/.36) * 0.60 = 0.15$$

The results of equations (7) and (8) are remarkably close. Table 2 shows other examples, given a PoP of 100%. Not all values are as close as the above example, but the results are probably well within the forecast errors of both the QPF and the PoP.

Table 2. Sample POEs for 0.25 and 0.50 inches as taken from Jorgenson and Klein (J&K) and also calculated from uPOE equation (6) using 100% for the PoP. Average difference between methods was 3.38%. A maximum difference was 8% at Detroit and Fort Worth.

City	Mean	JK&R(.25)	uPOE(.25)	JK&R(.50)	uPOE(.50)	Avg Diff
Detroit (winter)	0.11	13%	10%	4%	1%	3.00%
Detroit (spring)	0.14	22%	16%	6%	3%	4.50%
Detroit (summer)	0.25	29%	37%	16%	14%	5.00%
Detroit (autumn)	0.20	26%	28%	11%	8%	2.50%
Fort Worth (winter)	0.19	24%	27%	11%	7%	3.50%
Fort Worth (spring)	0.39	47%	53%	27%	28%	3.50%
Fort Worth (summer)	0.32	38%	46%	21%	21%	4.00%
Fort Worth(autumn)	0.30	38%	43%	18%	19%	3.00%
Atlanta (winter)	0.30	37%	43%	19%	19%	3.00%
Atlanta (spring)	0.36	44%	50%	27%	25%	4.00%
Atlanta (summer)	0.34	43%	48%	24%	23%	3.00%
Atlanta (autumn)	0.25	34%	37%	19%	14%	4.00%
Sacramento (winter)	0.24	32%	35%	14%	12%	2.50%
Sacramento (spring)	0.19	28%	27%	8%	7%	1.00%
Sacramento (summer)	0.11	14%	10%	7%	1%	5.00%
Sacramento (autumn)	0.26	36%	38%	18%	15%	2.50%
					Avg Diff	3.38%

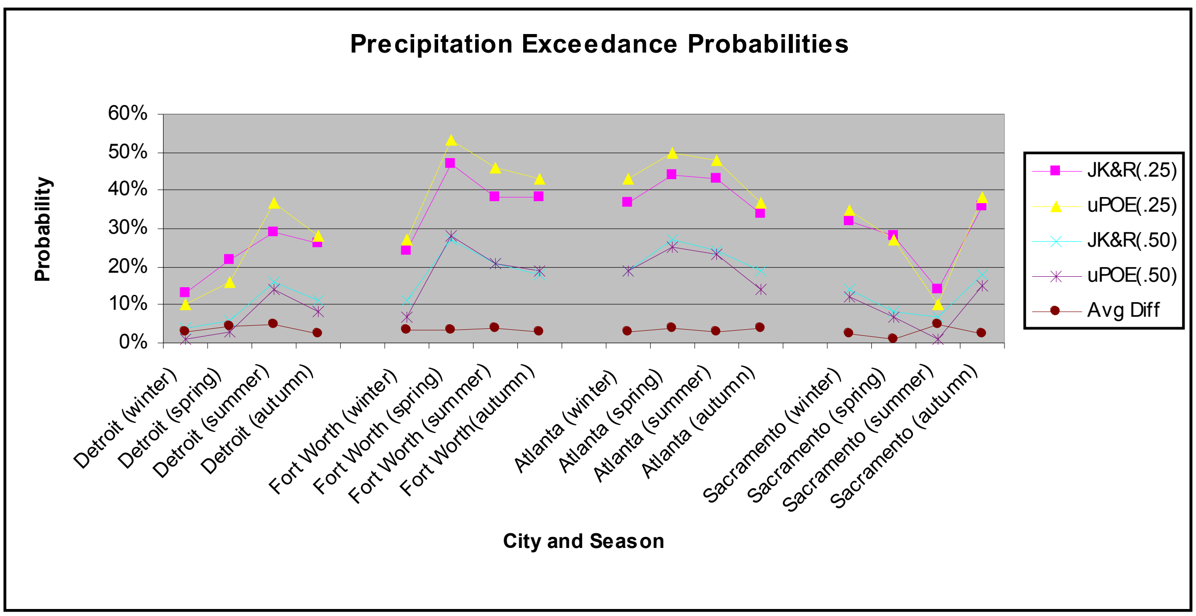


Figure 9. Plot of data in table 2.

6. DISTRIBUTIONS AND POES

Although data from this study indicates most precipitation events produce exponentially distributed rainfall amounts, those are obviously not always the case. Examples of non-exponentially distributed events are not uncommon. However, the POEs from the exponential PDFs are still reasonably close to the POEs computed from the actual rainfall distributions. Certainly, the POEs produced from the PDF equation provide more information than a standard QPF. Figures 7 through 15 show examples of several events which provide evidence supporting the use of the exponential PDF in producing POEs.

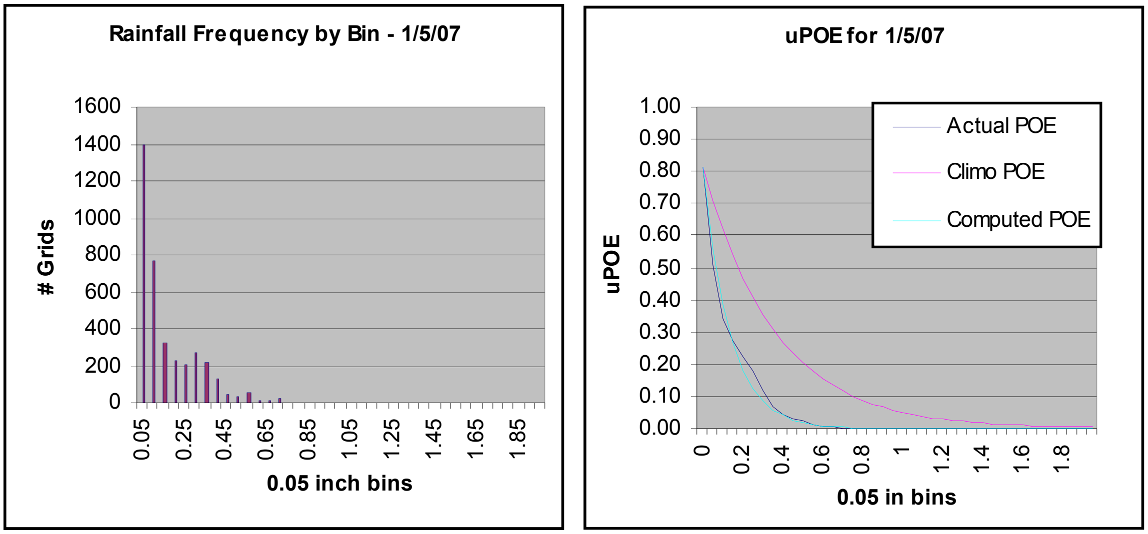


Figure 10. Rainfall frequency and uPOE for 1/5/07, TSA forecast area. Dark blue line is actual POE from data. Cyan line is POE computed from exponential PDF and observed rainfall mean and observed areal coverage (essentially a perfect forecast). Pink line is POE computed from exponential PDF, observed areal coverage, but climatological mean rainfall.

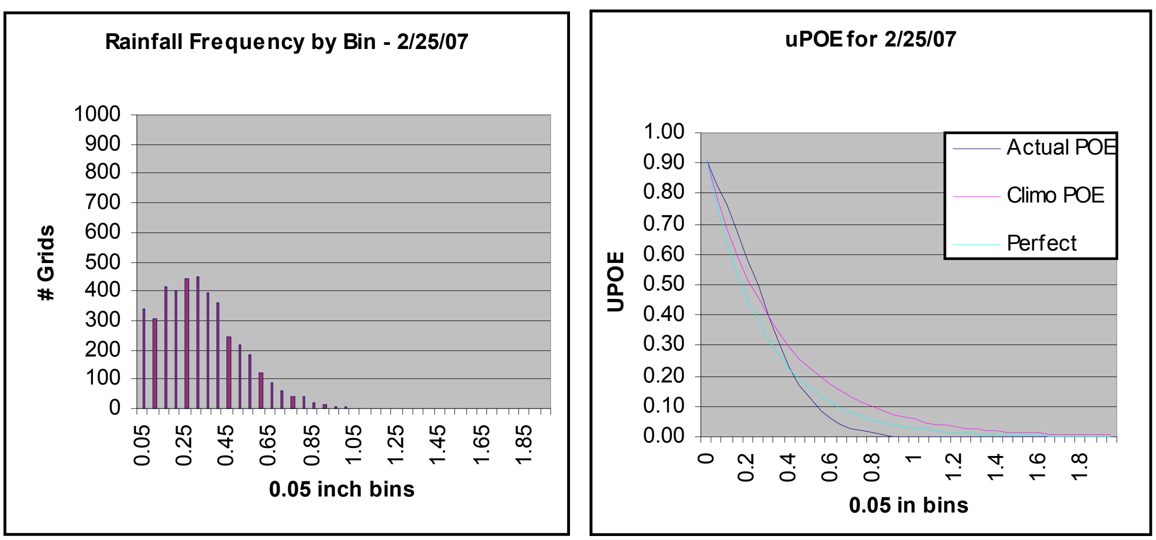


Figure 11. Rainfall frequency and uPOE for 2/25/07, TSA forecast area. Legends same as Figure 7.

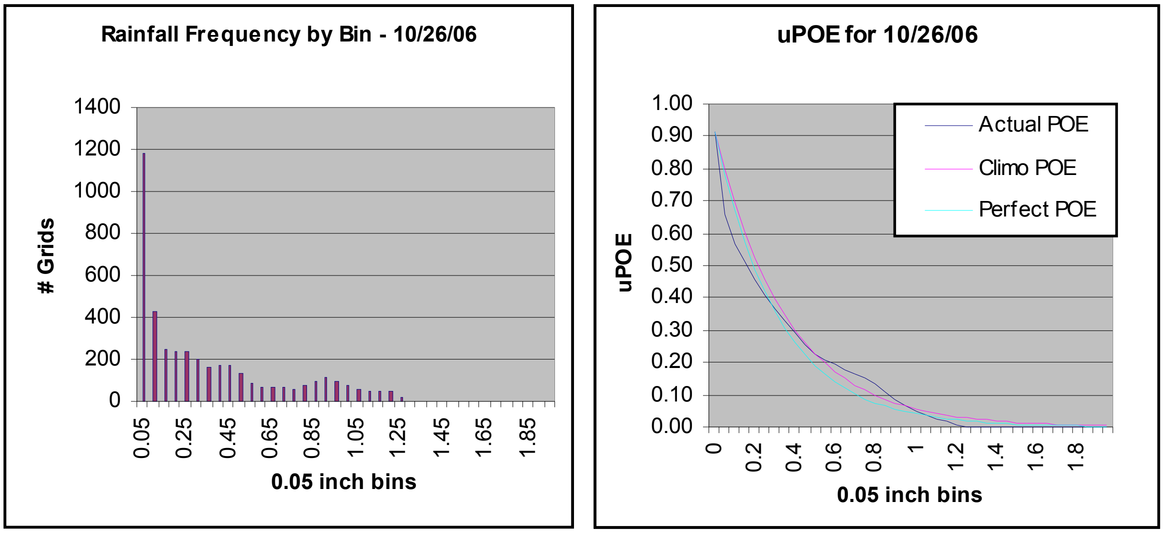


Figure 12. Rainfall frequency and uPOE for 10/26/06, TSA forecast area. Legends same as Figure 7.

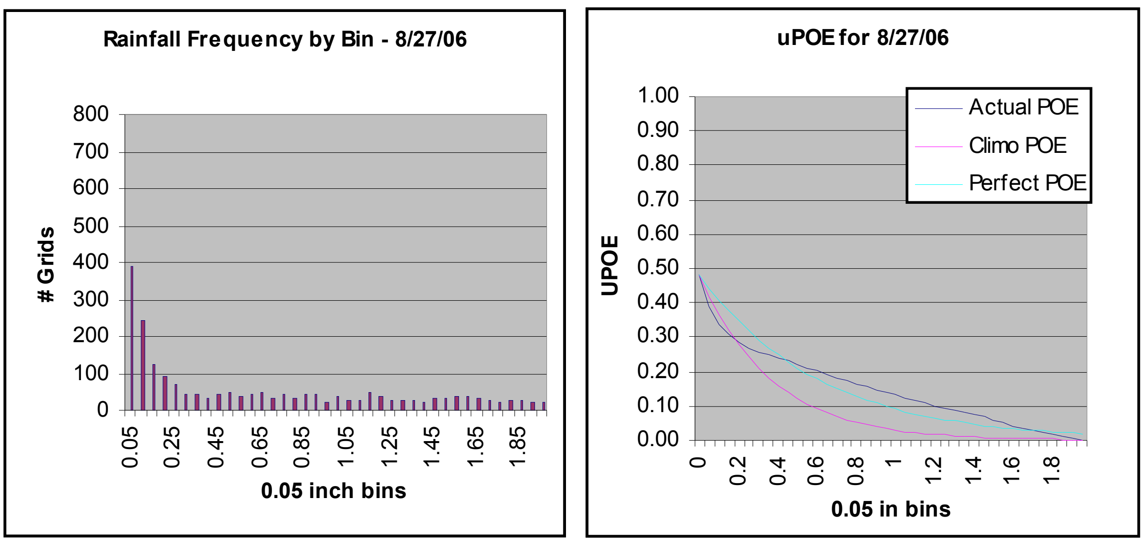
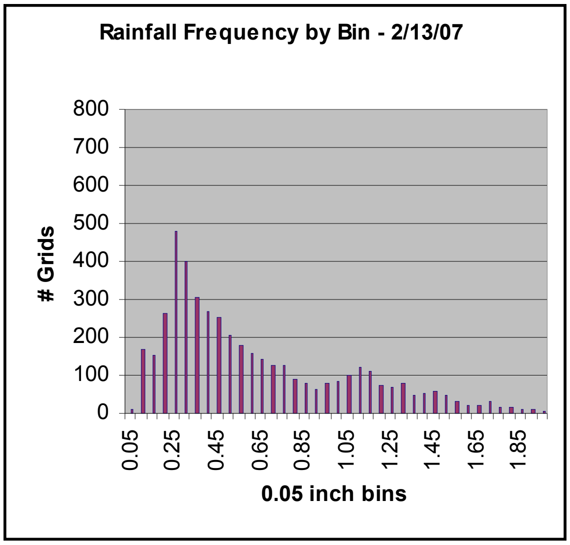


Figure 13. Rainfall frequency and uPOE for 8/27/06, TSA forecast area. Legends same as Figure 7.



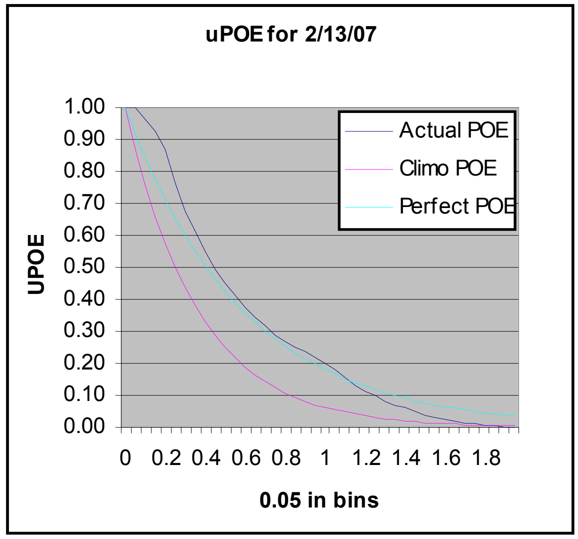


Figure 14. Rainfall frequency and uPOE for 2/13/07, TSA forecast area. Legends same as Figure 7.

7. JUSTIFICATION FOR THE METHOD

Simply put, accuracy is the justification for using formula-based POEs. Once the decision is made to provide probabilistic QPFs to the user community, it is incumbent upon the NWS to provide the best ones possible. Using a forecaster's mean QPF for μ in the exponential PDF rather than the climatological mean will result in much better POEs for those events. Also, using the single QPF for each grid to derive the various POEs will ensure forecast integrity.

Figure 15 shows the mean precipitation from 368 12-hour rainfall events from the end of July 2005 to the end of March 2007 (368 events where areal coverage was 10% or greater of the forecast area). The climatological mean for that same season, based on the 10-year WFO TSA data and also the 15-year JKR69 data, is approximately 0.36 inches. POEs calculated using 0.36 for the QPF (μ) would clearly not represent the variety of events shown in Figure 15. However, POEs based on a forecaster's best judgment should more closely match the actual means for each of those events and therefore should result in more accurate POEs. Near-real-time feedback through verification and also forecaster training is expected to make that true. Also, establishing a clear definition for QPF should also help.

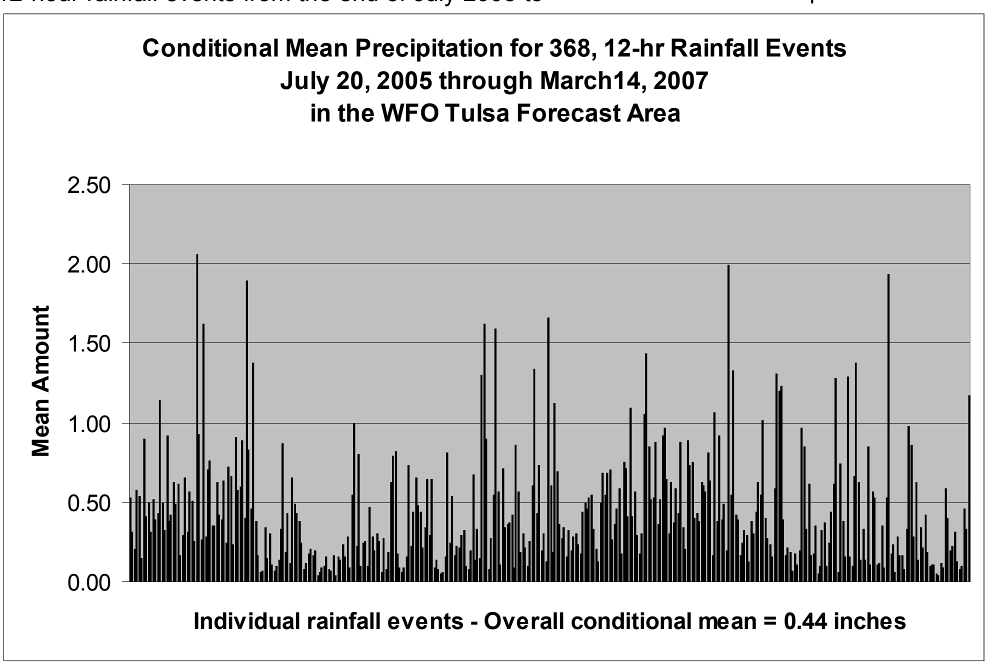


Figure 15. Late summer mean precipitation for 86 events in the WFO Tulsa forecast area. The line shows

the climatological mean for any event.

8. EXAMPLES OF POE OUTPUT

POE products are both graphic and text. Gridded POE within the GFE are available for 6-hour, 12-hour, or 24-hour periods. Figure 8 is a GFE depiction of the POE(0.10) for a specific 6-hour period. Those grids can be output directly to the TSA web page or used to generate other graphics, such as the bar graph shown in Figure 9. Finally, a text product is shown in Figure 10, depicting the average POEs for one of the 32 counties in the Tulsa forecast area.

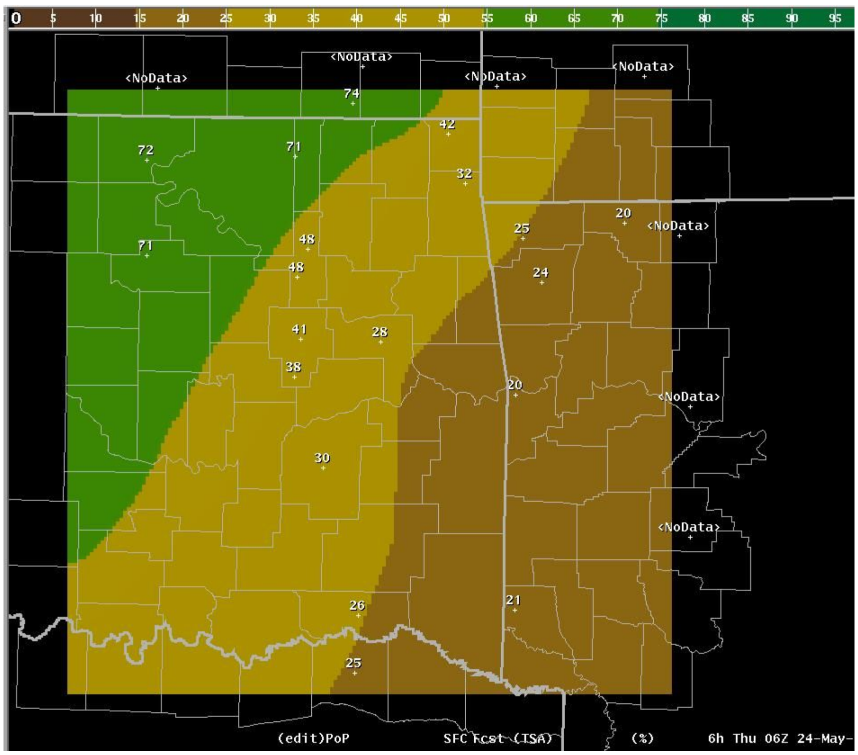


Figure 16. GFE depiction of WFO TSA PoP forecasts.

9. SUMMARY

Probabilistic QPFs, or probabilities of exceedance (POEs), are being produced at each forecast cycle at the WFO in Tulsa, Oklahoma. These POEs are generated in the Gridded Forecast Editor and are unique for each grid point across the TSA forecast area for threshold amounts of 0.10, 0.50, 1.00 and 2.00 inches. The meteorologist's unconditional QPF grid fields are used as input to the probability density function of the exponential distribution. Those QPFs effectively change the shape of the distribution so it will more closely match the expected distribution for the rainfall event. Conditional POEs are then generated for the specified threshold precipitation amounts. These conditional POEs are then multiplied by the PoPs at each grid point to arrive at the final unconditional POEs. This method is automated and requires no additional effort from the forecasters.

This Tulsa method of issuing a PQPF is experimental and still needs to be evaluated for accuracy and reliability. However, it does compare well with the previous results of Jorgenson, Klein and Roberts (1969). There is some concern that this PQPF method may not be entirely appropriate for some stratiform precipitation events. However, even single-event precipitation data that are not distributed exponentially will generally be well represented by POEs computed from a PDF of the exponential distribution. Examples of TSA PQPF product can be found at http://www.srh.noaa.gov/tsa/pqpf.htm, in both graphic and text modes.

Additional research should be conducted on rainfall climatologies across the United States to find if the POEs determined by the exponential distribution are appropriate for all climate zones. Random computations have indicated that POEs do not diminish as quickly as they should for some western sites when making comparisons to the JKR69 study. In similar instances, coefficients in the exponential PDF may provide better results.

Finally, this study and TSA efforts in producing probabilistic QPF in the form of POEs leads to a very specific definition of QPF. At this time, the NWS definition of QPF is "**The expected amount of liquid precipitation (in hundredths of inches) accumulated over a six hourly period**." For appropriate application of the WFO TSA method of producing POEs from QPF, the QPF should be defined more specifically as:

"**QPF: QPF is the forecast expected mean of the rainfall distribution that a forecaster would expect at a point during the forecast time period, from a large number of similar rainfall events**."

This definition is valid for point forecasts and is therefore consistent with NWS PoP forecasts.

10. ACKNOWLEDGEMENTS

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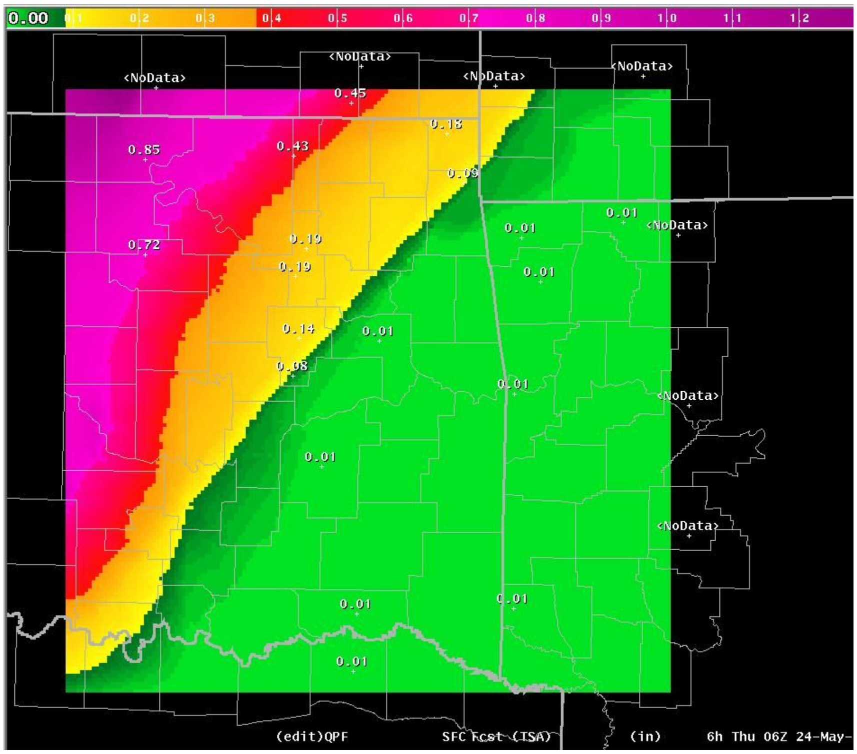

Figure 17. GFE depiction of WFO TSA QPF corresponding to PoP forecast in figure 16 above.

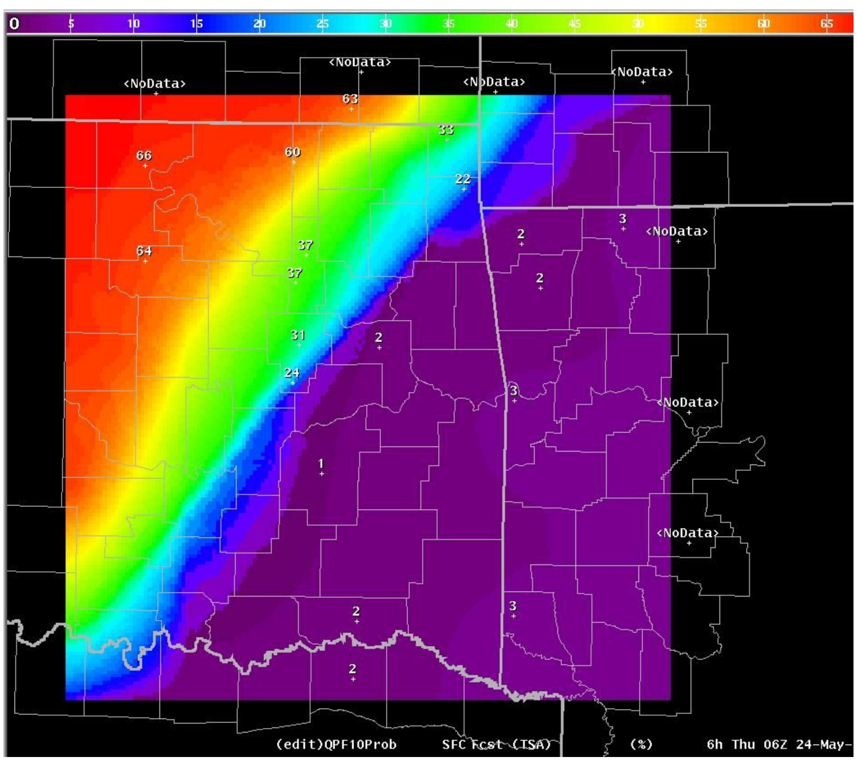

Figure 18. GFE depiction of WFO TSA probability to exceed 0.10 inches, corresponding to the PoP and QPF forecasts in figures 16 and 17 above.

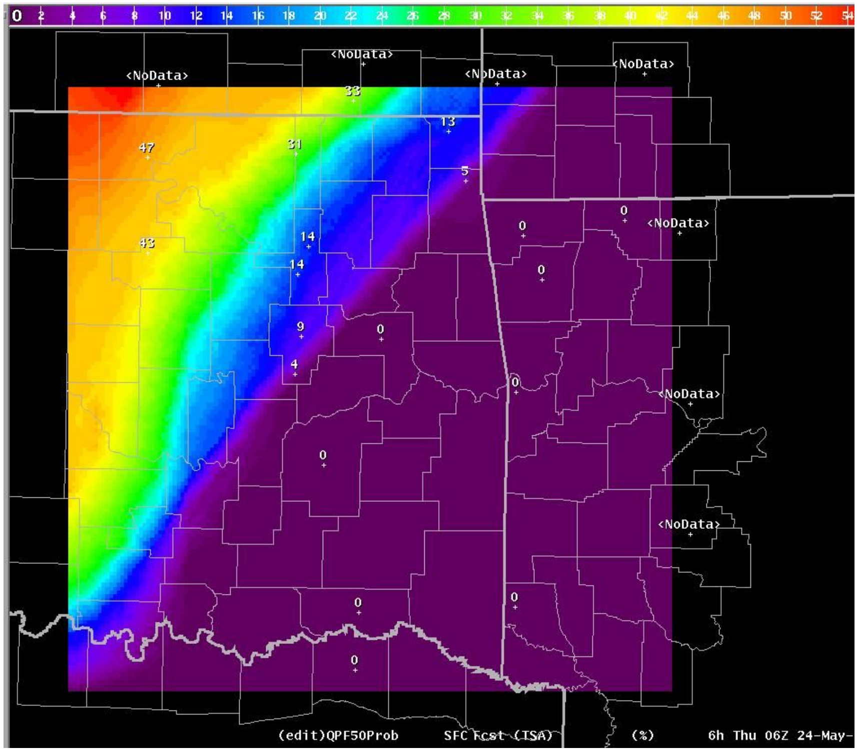

Figure 19. GFE depiction of WFO TSA probability to exceed 0.50 inches, corresponding to the PoP and QPF forecasts in figures 16 and 17 above.

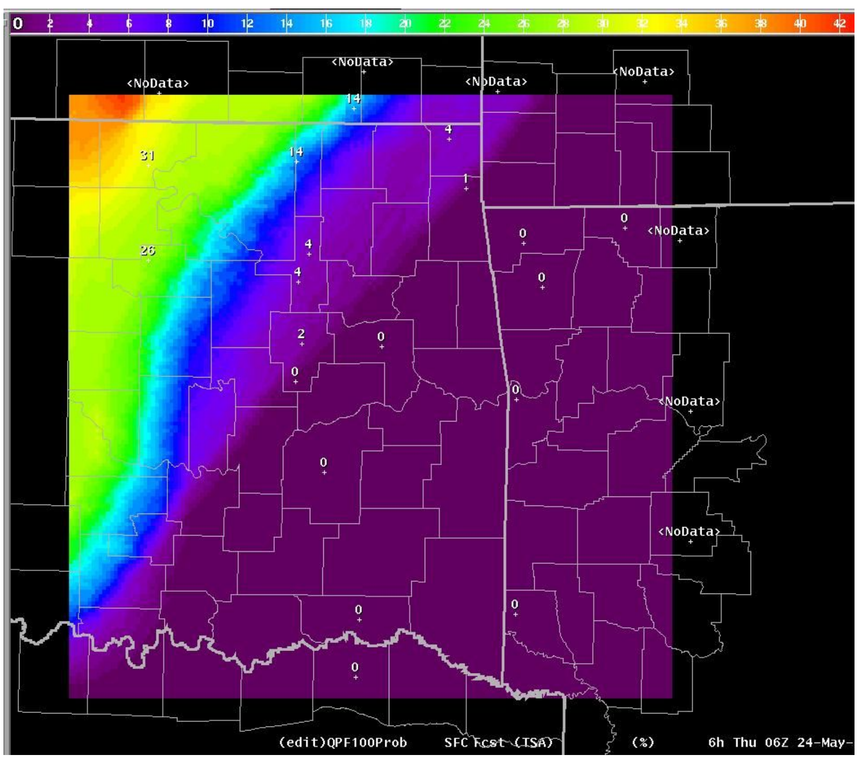

Figure 20. GFE depiction of WFO TSA probability to exceed 1.00 inches, corresponding to the PoP and QPF forecasts in figures16 and 17 above.

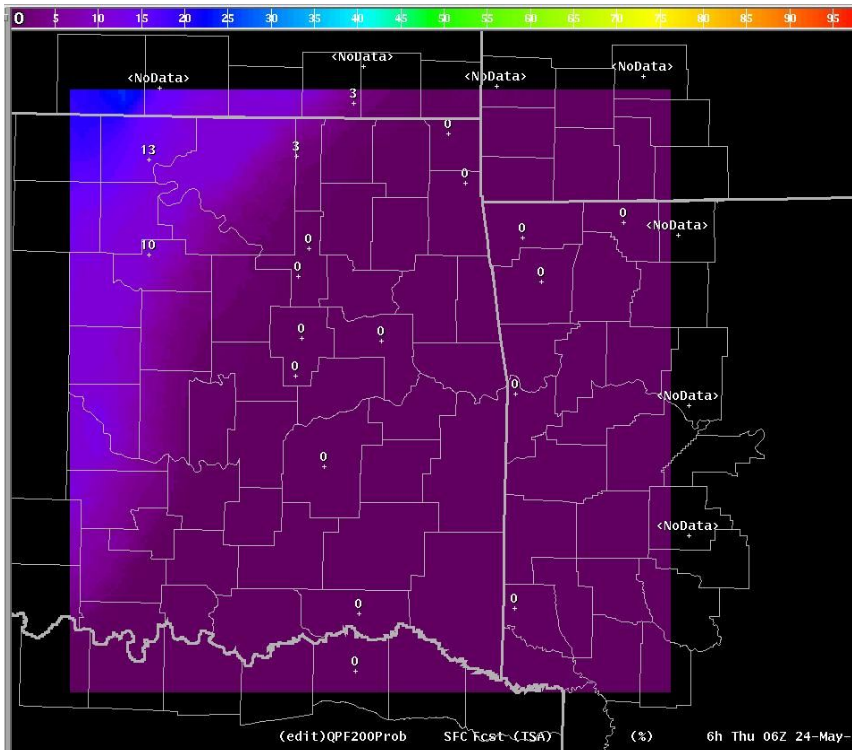

Figure 21. GFE depiction of WFO TSA probability to exceed 2.00 inches, corresponding to the PoP and QPF forecasts in figures 16 and 17 above.

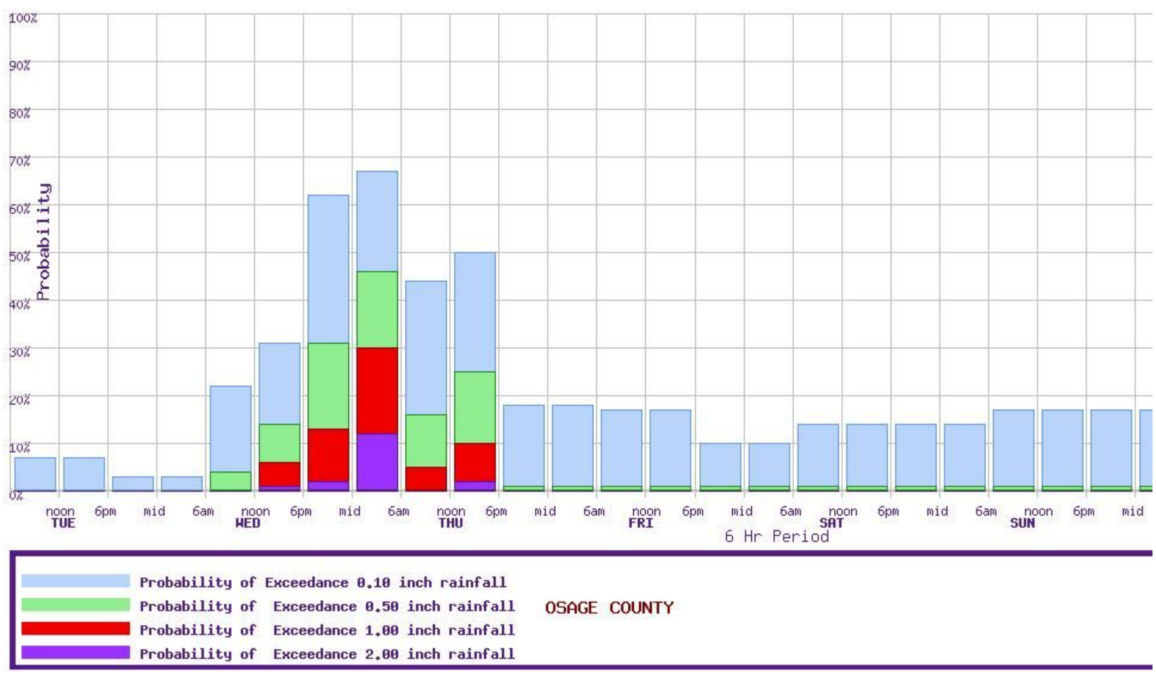

Figure 22. Bar graph output of the POE forecast for Osage County, OK, corresponding to the PoP and QPF forecasts in figures 16 and 17 above.

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OKZ054-230030-
OSAGE-
INCLUDING THE CITIES OF...PAWHUSKA
720 AM CDT TUE MAY 22 2007

DATE       TUE 05/22/07     \       WED 05/23/07     \       THU 05/24/07     \       FRI 05/25/07     \       SAT 05/26/07
CDT        06 09 12 15 18 21 00 03 06 09 12 15 18 21 00 03 06 09 12 15 18 21 00 03 06 09 12 15 18 21 00 03 06 09 12 15

POP 6HR       30    30    10    10    30    30    70    70    50    50    30    30    30    30    20    20    30    30
QPF 6HR     0.02  0.02     0     0  0.08  0.20  0.43  0.82  0.22  0.34  0.05  0.05  0.05  0.05  0.03  0.03  0.04  0.04
X 0.10         7     7     3     3    22    31    62    67    44    50    18    18    17    17    10    10    14    14
X 0.50         0     0     0     0     4    14    31    46    16    25     1     1     1     1     1     1     1     1
X 1.00         0     0     0     0     0     6    13    30     5    10     0     0     0     0     0     0     0     0
X 2.00         0     0     0     0     0     1     2    12     0     2     0     0     0     0     0     0     0     0
&&

POP 6HR.......6-HOUR PROBABILITY OF MEASURABLE PRECIPITATION (0.01 INCH OR MORE)
QPF 6HR ......6-HOUR UNCONDITIONAL LIQUID EQUIVALENT PRECIPITATION AMOUNT (INCHES)
X 0.10 .......PROBABILITY OF LIQUID EQUIVALENT PRECIPITATION EXCEEDING 0.10 INCH (PERCENT).
X 0.50 .......PROBABILITY OF LIQUID EQUIVALENT PRECIPITATION EXCEEDING 0.50 INCH (PERCENT).
X 1.00 .......PROBABILITY OF LIQUID EQUIVALENT PRECIPITATION EXCEEDING 1.00 INCH (PERCENT).
X 2.00 .......PROBABILITY OF LIQUID EQUIVALENT PRECIPITATION EXCEEDING 2.00 INCH (PERCENT).

$$
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Figure 23. Text output for Osage County, OK, showing the PoP and QPF for each 6-hour period, along with the POEs for 0.10, 0.50, 1.00, and 2.00 inches of rainfall, corresponding to the PoP and QPF forecasts in figures 16 and 17 above.

Appendix A
An Excerpt from Jorgenson, Klein and Roberts (1969)

This excerpt provides the statistical basis Jorgenson and Klein used for making probabilistic quantitative precipitation forecasts. Their Equation (4) defines the unconditional probability to exceed a selected rainfall amount, r. Table 1, to which they refer, is their tabulated data that gives the conditional probabilities of precipitation occurrence in seven quantitative ranges for 108 stations combined by seasons. The Tulsa Method to compute POEs uses Equation (4) below.

"To obtain the probability of a precipitation event consisting of any fixed amount of rain falling in a given time period, we can make use of the definition of conditional probability. The conditional probability of an event A given that event B will occur is

$$P(A/B) = P(A,B) / P(B) \qquad (2)$$

Where P(A/B) is the conditional probability of A, the condition being that B occurs, P(A,B) is the probability for the joint occurrence of A and B, and P(B) is the probability of B.

Applying this definition to a rain amount in excess of r in a period t, we write

$$Pt(r/0.01) = Pt(r,0.01) / P(0.01) \qquad (3)$$

Or

$$Pt(r,0.01) = Pt(r/0.01) * P(0.01) \qquad (4)$$

The conditional probability of an amount greater than r, Pt(r/0.01), is given in table 1 for time periods of 6, 12, and 24-hours. The probability of measurable rain, P(0.01), is obtained from the public probability forecast. The product of these two gives the desired probability."