## 11.3 SURFACE WAVE MODULATION OF ATMOSPHERIC REFRACTIVITY AND REMOTE SENSING OVER THE OCEAN

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### 1. INTRODUCTION

The propagation of electromagnetic and acoustic signals in the atmosphere is substantially affected by the atmospheric motion and the associated fluctuations of the refractivity and the speed of sound, respectively. Over the ocean, the atmospheric humidity decreases with height, thus leading to a vertically decreasing atmospheric refractivity. This vertical gradient leads to the formation of a waveguide within the first tens of meters above the ocean suface which confines the signals and allows them to propagate following the curvature of the Earth instead of being emitted into space (Figure 1). While the averaged refractivity determines the fraction of the signal's energy retained in the waveguide, the fluctuating component of refractivity causes the variation in intensity, phase and the angle of arrival. Turbulent fluctuations of refractivity have been studied extensively, starting from the 1960. A recent experiment has indicated that ocean surface waves also induce refractivity fluctuations.

While the wave-induced fluctuations in the atmospheric pressure are dominant and directly observable at low and moderate wind speeds (Hristov et al. (2004)), the wave-induced fluctuations in the wind velocity and passive scalar fields are comparatively weak, so that at moderate (above 6 m/s) and high winds they require a filtering to be extracted from the dominating turbulence (Hristov et al. (1998)). At low wind conditions, however, the turbulence intensity is sufficently low, so that the wave effects in the wind velocity can be recognized in the atmospheric flow without processing. Also, at low wind conditions the turbulent mixing is rather weak thus allowing vertical gradients of atmospheric humidity to be formed and maintained. Presence of vertical gradients is a necessary condition for the existence of wave-induced fluctuations in refractivity or in other passive scalar fields.

In this work we present observational evidence for the existence of wave-induced fluctuations and analyze the mechanism by which they occur. We also propose a model for the refractive structure function, an essential element in describing the propagtion pattern, which accounts for the surface waves influence.



Figure 1: Two patterns of signal propagation through the atmosphere. (a) Homogeneous refractive index case. (b) Refractive duct case. The radar beam is confined within the first tens of meters from the surface. The upper boundary is the height of the total internal reflection (figure reproduced from Barton and Leonov (1998)).

#### 2. EXPERIMENTAL SETTING

The data presented in this work were collected between June and November 2003 from the Air-Sea Interaction Tower observatory (Figure 2) located South of Martha's Vineyard, Massachusetts in a water depth of 15 meters. A mast in the tower was instrumented with 4 sonic anemometers measuring wind turbulence and air temperature at 20Hz, 3 hygrometers registering water vapor fluctuations at 20 Hz, and two pressure sensors sampling at 8Hz. Water surface elevation was measured directly beneath the instruments mast by a microwave sensor . Air's refractivity was recovered from the data of pressure, temperature and humidity (Bean and Dutton (1966)).

### 3. DATA INTERPRETATION

Figure 3 illustrates a low-wind regime, observed more than 20% of the time during CBLAST. In such regime the shear-generated turbulence has low intensity and the wave driving permeates the whole atmospheric boundary layer and is clearly observable. The time-series of wind velocity and pressure exhibit strong wave modulation. The wave-induced velocity fluctuations lead to a vertical displacement of the mean wind stream lines and an associated vertical displacement of the sheared humidity profiles. Consequently, a sensor at a fixed hight registers wave-induced fluctuations of humidity, demonstrated by the distinct peak at the frequency of the waves in the coherence between the atmospheric humidity and the surface waves. For a radio signal propagating over the

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Figure 2: ASI Tower with the instruments mast.

ocean, such fluctuations effectively create semi-periodic refractive medium bending the trajectory of the radar beam.

The propagation regimes through an idealized periodic refractive media can be illustrated by solving the ray propagation equation (Landau and Lifshitz (1984),  $\S$ 85)

$$d\mathbf{l}/d\mathbf{l} = \left[\nabla n - \mathbf{l}(\mathbf{l} \cdot \nabla n)\right]/n \equiv (\nabla_{\perp} n)/n \tag{1}$$

where l is an unit vector tangential to the ray and  $n \equiv n(\mathbf{r})$  is the refractive index. In such a medium a signal can be confined along the crests of the refractivity or accumulate displacement (i.e. form mirages) while propagating in an oblique direction. If propagating within the trough of the refractivity, the signal can be defocussed and switch to the other two regimes (Figure 4).

## 4. A STRUCTURE FUNCTION'S MODEL

The waves observed in the ocean have a nonmonochromatic spectrum (Figure 3). We need to describe the influence of the wave-induced refractivity fluctuations on the signal propagation statistically in a way that it is valid for ocean waves with finte-width spectrum. For Gaussian fluctuations of refractivity, key characteristics of the propagation pattern, such as variation of intensity, phase and the angle of arrival are determined from the refractivity's structure function (Tatarskii (1967))

$$D_n(\mathbf{r}_1, \mathbf{r}_2) = \langle [n'(\mathbf{r}_1) - n'(\mathbf{r}_2)]^2 \rangle.$$

Let us assume that the measured fluctuation of the refractivity consists of turbulent  $n_t$  and wave-induced  $\tilde{n}$ fluctuations, i.e.  $n' \equiv n - \langle n \rangle = n_t + \tilde{n}$  as well as that the turbulence and the wave effects are uncorrelated  $\langle n_t(\mathbf{r})\tilde{n}(\mathbf{r}) \rangle = 0$ . It is then easy to show that the structure function splits conveniently to a turbulent  $D_n^{\text{turb}}(\mathbf{r}_1, \mathbf{r}_2)$ and a wave-induced  $\tilde{D}_n(\mathbf{r}_1, \mathbf{r}_2)$  parts

$$D_n(\mathbf{r}_1, \mathbf{r}_2) = D_n^{\text{turb}}(\mathbf{r}_1, \mathbf{r}_2) + \widetilde{D}_n(\mathbf{r}_1, \mathbf{r}_2).$$

Since the light from astronomical sources is subjected to atmospheric distortion before it reaches terrestial telescopes (first explained by Newton (1704)), the refractive structure function due to turbulence  $D_n^{\text{turb}}(\mathbf{r}_1, \mathbf{r}_2)$  has been studied extensively starting from the 1960s (Chernov (1967); Tatarskii (1967)). Little is known about the wave-induced term  $\tilde{D}_n(\mathbf{r}_1, \mathbf{r}_2)$ .

To model the wave-induced refractivity we need to consider the mechanism responsible for it. Specifically, we will assume that the waves displace vertically the column of air with a stratified refractive index. Consequently, in ducting conditions a displacement upwards (i.e. over a wave crest) will bring to a fixed height z higher refractive index from below and a displacement downwards (over a wave trough) will bring a lower refractive index from above. Assuming small slope waves, the wave-induced variation of the refractive index  $\tilde{n}(\mathbf{r})$  will be proportional to the vertical displacement of the mean flow streamlines and the vertical refractivity gradient. To model the streamline displacement, below we will ignore any interaction between the turbulence in the air and the waves, and will rely on a linear wave-mean-flow interaction theory proposed by Miles (1957).

Consider a constant-stress turbulent atmospheric boundary layer over the ocean with mean horizontal velocity  $U(z) = (u_*/\kappa) \log(z/z_0)$ , where  $u_*$  is the friction velocity,  $\kappa$  is the von Karman constant, and  $z_0$  is the surface rougness. The stream function  $\phi$  describing the distortion of the mean air flow U(z) due to a monochromatic surface wave  $\eta = ae^{ik(ct-x)}$  satisfies the Rayleigh equation

$$U'' - k^2 \phi - U'' (U - c)^{-1} \phi = 0$$
 (2)

The horizontal velocity  $u(z; c/u_*)$  and the vertical velocity  $v(z; c/u_*)$  obtained from solving it are

$$u(z; c/u_*) = -\frac{u_*}{\kappa} \frac{\mathrm{d}\phi(z; c/u_*)}{\mathrm{d}z} \eta(x, t)$$
  
$$v(z; c/u_*) = \mathrm{i}k \left(u_*/\kappa\right) \phi(z; c/u_*) \eta(x, t)$$

and the corresponding streamlines are shown in Figure 5. The vertical displacement of a streamline is then

$$\delta z = \int v \mathrm{d}t = -c^{-1}(u_*/\kappa)\phi(y,c/u_*)\eta.$$



Figure 3: A low-wind regime frequently observed during CBLAST. The left-hand side plots show time-series over 100s of horizontal along-wind velocity, vertical velocity, pressure, and surface elevation (correspondingly). The colors in the velocity plots correspond to instruments heights, in the order blue (the lowest), green, red and cyan (the highest). The upper plot on the right-hand side shows the coherence function between the surface elevation and refractivity with colors indicating instrument height: blue (lowest), green, red (highest); the lower plot shows the surface waves spectrum.

Consider a vertical profile of refractive index N(z). The wave-induced fluctuation of the refractive index due to stream line displacement is  $\tilde{n} = -(dN/dz) \, \delta z$  or

$$\tilde{n} = -\left(\frac{\mathrm{d}N}{\mathrm{d}z}\right) \left(\frac{u_*}{\kappa} \frac{\mathbf{U} \cdot \mathbf{k}}{U(gk)^{1/2}}\right) \phi(z, \mathbf{k}) \eta(\mathbf{k})$$

Thus the transfer function relating the fluctuations of refractivity  $\tilde{n}$  with the surface elevation  $\eta$  (i.e. wave forcing) is found to be

$$\mathcal{T}(z;\mathbf{k}) = -\left(\frac{\mathrm{d}N}{\mathrm{d}z}\right) \left(\frac{u_*}{\kappa} \frac{\mathbf{U} \cdot \mathbf{k}}{U(gk)^{1/2}}\right) \phi(z,\mathbf{k})$$

and the wave-induced fluctuation of the refractive index is

$$\tilde{n}(\mathbf{r}, z) = \int \mathcal{T}(z; \mathbf{k}) \ \eta(\mathbf{r}, \mathbf{k}) \ \mathrm{d}\mathbf{k}$$

For the structure function we have

$$\widetilde{D}_n(\mathbf{r}_2 - \mathbf{r}_1; z_1, z_2) = \langle [\widetilde{n}(\mathbf{r}_1, z_1)]^2 \rangle + \langle [\widetilde{n}(\mathbf{r}_2, z_2)]^2 \rangle \quad (3)$$
$$-2 \langle \widetilde{n}(\mathbf{r}_1, z_1) \widetilde{n}(\mathbf{r}_2, z_2) \rangle,$$

where

$$\langle \tilde{n}(\mathbf{r}_{1}, z_{1}) \tilde{n}(\mathbf{r}_{2}, z_{2}) \rangle =$$

$$\int e^{i\mathbf{k} \cdot (\mathbf{r}_{2} - \mathbf{r}_{1})} \mathcal{T}(z_{1}, \mathbf{k}) \mathcal{T}^{*}(z_{2}, \mathbf{k}) S_{\eta \eta}(\mathbf{k}) d\mathbf{k}$$
(4)

and  $S_{\eta\eta}(\mathbf{k}) \equiv \langle \eta(\mathbf{k})\eta^*(\mathbf{k}) \rangle$  is the e surface waves spectrum. The local terms  $\langle [\tilde{n}(\mathbf{r}_1, z_1)]^2 \rangle$  and  $\langle [\tilde{n}(\mathbf{r}_2, z_2)]^2 \rangle$  also have the form (4) with  $\mathbf{r}_2 = \mathbf{r}_1$  and  $z_2 = z_1$ .

Simplifications for the transfer function are possible, considering that the wave modulation of refractivity should occur mostly in low-wind conditions. Then it can



Figure 4: An illustration of the propagation regimes through an idealized periodic refractive medium. The ray can be guided along the refractivity maxima or accumulate displacement and form mirages when it propagates in an oblique direction. Obtained from solving Eq. (1).

be reasonably assumed that the refractive duct is entirely below the critical height  $z_c$  (the height where the mean wind speed matches the phase speed of the surface waves, i.e.  $U(z_c) = c$ ). Away from the critical height  $z_c$  the term  $U''(U-c)^{-1}\phi$  in Eq. (2) can be ignored and the wave-induced fields decay vertically as  $\propto e^{-kz}$  with no change in phase.

#### 5. SUMMARY

Data collected during the Coupled Boundary Layers Air-Sea Transfer (*CBLAST*) experiment demonstrated detectable wave-driven variation of the atmospheric refractivity. Assuming turbulence and wave effects uncorrelated, we showed that the refractivity's structure function conveniently splits into turbulent and wave-induced components. The wave-induced structure function component was quantified from the critical layer theory of windwave interaction.

Although observed over the mid-latitude Atlantic, the wave modulation of the atmospheric refractivity may occur more often and be more pronounced in other locations. For instance, over the Indian Ocean dry air from the surrounding deserts can move over the water and create substantial vertical gradients of the atmospheric humidity. Low wind is commonly observed there along with sea swells, thus providing the necessary conditions.

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Figure 5: The wave-coherent flow streamlines as reconstructed according to the critical layer theory of Miles (1957). The solid line represents the air-water interface and the dashed line is the critical height. The whole eddy structure is propagating to the right following the wave. A Fourier sum of such flows over the wave spectrum forms the actual flow over the waves (Hristov et al. (2003)).

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