A Coordinate System Independent Surface Stress

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A small amount of instrument tilt can degrade the accuracy of eddy correlation flux measurements, yet in the field, it is difficult to mount sonic anemometers perfectly vertically. Although instrument tilt can be corrected with the planar fit tilt correction method (Wilczak, et al 2001), this method does not work well over complex surfaces such as those found in urban areas where mean streamlines may not be horizontal and the degree of tilt may differ for different wind directions (Klipp 2004), nor is it meant to be applied to data taken from within the roughness sublayer. In addition, the presence of the vertical surfaces of building walls complicates the definition of surface normal, which may no longer be parallel to the gravity vector.

A substitute for the surface stress for use inside the roughness sublayer has been developed. It is coordinate system independent, therefore sonic anemometer tilt correction is not necessary. Rotation of the coordinate system into streamwise coordinates is also optional.

Introduction

The Reynolds stress tensor can be diagonalized, resulting in three eigenvalues, λ_B, λ_M, λ_S independent of which coordinate system was chosen for the original data. Also independent of the coordinate system is the trace of the Reynolds stress tensor, $\lambda_B + \lambda_M + \lambda_S = 2 * \text{TKE}$. Since the Reynolds stress tensor is a real, symmetric matrix, the three eigenvectors associated with these eigenvalues form an orthogonal coordinate system since the original Reynolds stress tensor originates from data obtained from the *x, y, z* orthogonal coordinate system of the sonic anemometer.

The eigenvectors and magnitudes of the associated eigenvalues define an ellipsoid (fig 1) oriented at an angle to the streamwise, cross stream, vertical coordinate system used for most atmospheric boundary layer applications. This orientation in laboratory flows is a rotation about the cross stream axis in a manner which tilts the axis associated with the smallest eigenvalue about 17° away from vertical towards the mean wind direction (Liberzon et al. 2005, Hanjalic and Launder 1972). Our data from sonic anemometers in an open area agree with the laboratory results for cases where both $\overline{u'v'}$ and $\overline{v'w'}$ are much smaller than $\overline{u'w'}$.

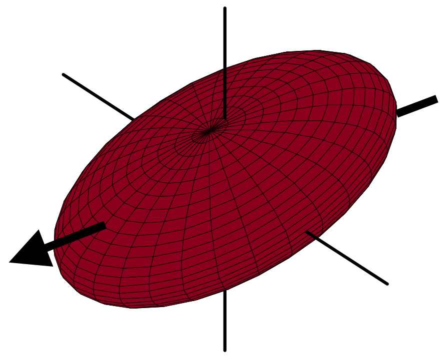

Fig. 1 Idealized TKE ellipsoid based on lab flows

Development of reverse transform for $\overline{u'w'}$

For these laboratory flows and the subset of field data, $\overline{u'w'}$ can be recreated by rotating the diagonalized Reynolds stress tensor 17° around the cross-stream direction to produce a covariance term in the upper right (and lower left) corner, $\overline{u'w'_R} = -(\lambda_B - \lambda_S)\cos(17°)\sin(17°)$. Under this definition, $\overline{u'w'_R}$ will equal zero for perfectly isotropic turbulence since all three eigenvalues are equal for the isotropic case.

In complex environments such as urban areas, the presence of non-horizontal surfaces changes the definition of wall normal. The surface stress over open terrain is $u_*^2 = \left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{1/2}$. Near a vertical surface this changes to $u_*^2 = \left(\overline{u'v'}^2 + \overline{w'v'}^2\right)^{1/2}$ with v as the wall normal

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direction and w as the cross stream direction (fig 2). In a location equidistant from both horizontal and vertical surfaces, the surface stress is not easily defined. Downwind from such a location, stress is even more difficult to define as the flow readjusts to new boundary conditions. But even in a complex environment, the Reynolds stress tensor can still be diagonalized and rotated by 17° as above to produce an idealized $\overline{u'w'_{\mathrm{R}}}$. The coordinate system independent surface stress is then taken as $u_{*\mathrm{R}} = \left(-\overline{u'w'_{\mathrm{R}}}\right)^{1/2}$.

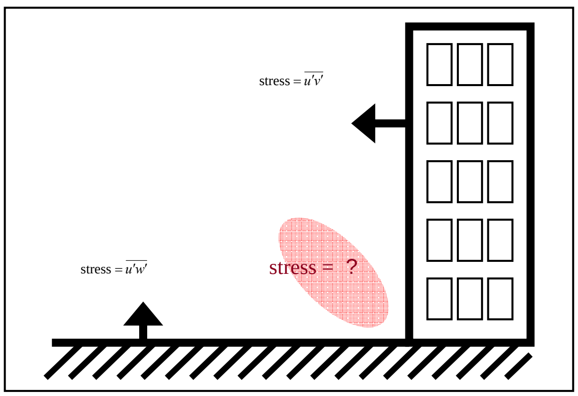

Fig. 2 Wall normal is no longer parallel to the gravity vector near buildings, making stress difficult to define

Data

This analysis uses sonic anemometer data from JU2003 taken in Oklahoma City, OK in July 2003. The open location was a city bus parking lot about 6 kilometers to the SW of the central business district. The urban location was in a light industrial area with 5-8m tall buildings and several trees in the area taller than the buildings, most notably a 12m tall tree about 15m SW of the tower. Day time is take to be 1400 – 2300 UTC and night is taken to be 0300 – 1100 UTC. (Local solar noon is about 1830 UTC.) All fluxes are calculated from deviations from 10 minute means.

Application of $u_{*\mathrm{R}}$ for open location

At night, at the open location, is when the winds are most likely to match the lab flow TKE ellipsoid orientations. As a result, the values for $u_{*\mathrm{R}}$ are very similar to the values for u_* from the standard definition (fig 3). TKE scaled by u_*^2 produces values fairly similar to TKE scaled by $u_{*\mathrm{R}}^2$, but the latter values show less scatter (fig 4).

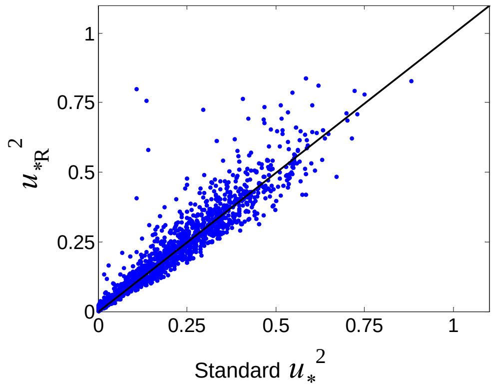

Fig. 3 Standard definition u_*^2 compared to idealized $u_{*\mathrm{R}}^2$, open location, night

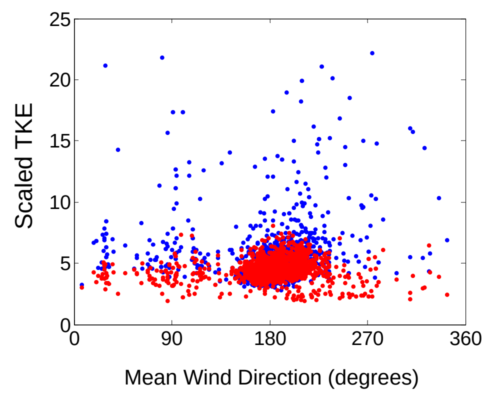

Fig. 4 Scaled TKE, open location, night, blue uses standard u_*, red uses $u_{*\mathrm{R}}$

In the day time, even at the open location, the TKE ellipsoid orientations are less likely to match the lab flow orientation. In the case of daytime convection, the eigenvector associated with the largest eigenvalue is at times more closely aligned with the cross stream direction than the streamwise direction. As a result, the values for $u_{*\mathrm{R}}$ are usually larger than the corresponding u_* value (fig 5). When TKE is scaled by $u_{*\mathrm{R}}^2$, the resulting values are less scattered than TKE scaled by the standard u_*^2 (fig 6). The TKE/ $u_{*\mathrm{R}}^2$ for daytime are similar to the TKE/ $u_{*\mathrm{R}}^2$ for night.

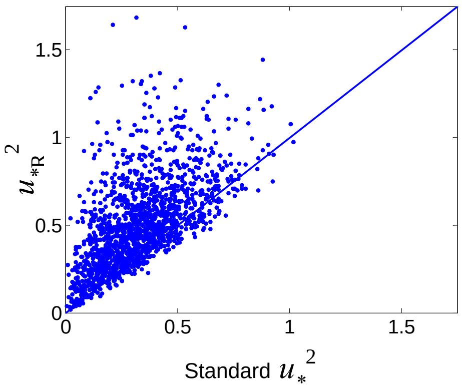

Fig. 5 Standard definition $u_*^{\ 2}$ compared to idealized $u_{*R}^{\ 2}$, open location, day

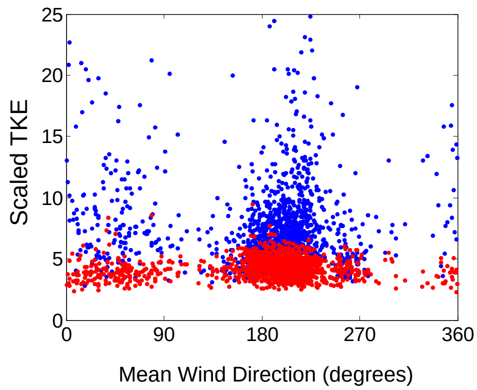

Fig. 6 Scaled TKE, open location, day, blue uses standard u_*, red uses u_{*R}

Application of u_{*R} for urban location

At the urban location, the presence of buildings nearby, and especially a tree about 15 m from the tower to the SW, affects the orientation of the wall normal direction in such a way that the standard definition of u_* can become very small in spite of significant turbulence and wind shear. The coordinate system independent u_{*R} values are only small when the turbulence level is small (fig 7) resulting in less extreme values for scaled TKE (fig 8). In addition to the tree at 220°, there is another large tree further away at 170°. This detail can be seen in the scaled TKE using u_{*R} but not in the scaled TKE using u_*.

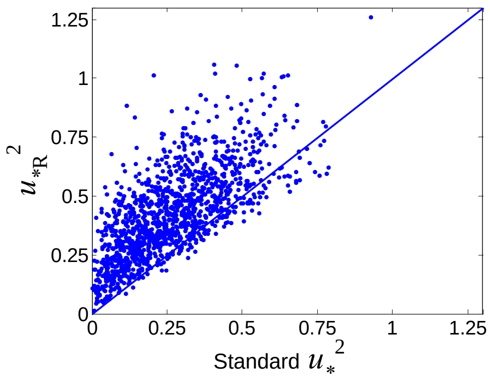

Fig. 7 Standard definition $u_*^{\ 2}$ compared to idealized $u_{*R}^{\ 2}$, urban location, day

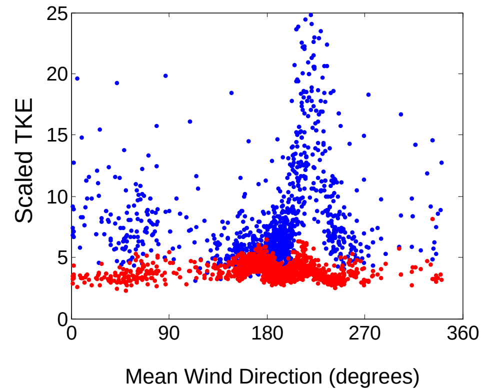

Fig. 8 Scaled TKE, urban location, day. Note the significantly smaller values for the coordinate system independent version downwind from a nearby tree at 220°. Blue uses standard u_*, red uses u_{*R}

Application of u_{*R} to the drag coefficient

Because the coordinate system independent u_{*R} values are sometimes significantly larger than the standard u_* values, the calculated values of C_D using u_{*R} are larger than the standard $C_D = u_*^2 / \overline{U}^2$ (fig 9). In the urban area, the standard calculation of C_D gives smaller values downwind of the nearby tree than for other fetch directions, implying that the surface is less rough in that direction (fig 10). The C_D values using u_{*R}, in addition to being larger than the standard values, shows significantly elevated values in the tree direction, more in line with the intuitive interpretation that C_D should be larger downstream from obstacles. On the other hand, the coefficient of drag may not be an appropriate parameter for use within roughness sublayers.

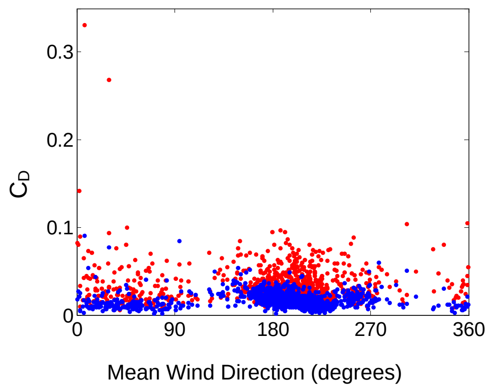

Fig. 9 Coefficient of drag, open location, day, blue uses standard u_*, red uses u_{*R}

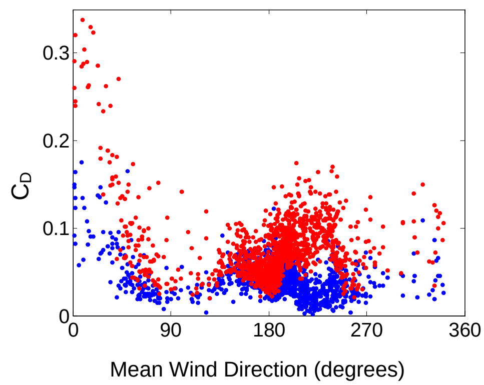

Fig. 10 Coefficient of drag, urban location, day, blue uses standard u_*, red uses u_{*R}

Discussion

The standard definition of u_* does not hold within roughness sublayers due to the proximity of surfaces oriented in various directions. By connecting the standard definition of u_* to invariants of the Reynolds stress tensor, the eigenvalues, a coordinate system independent method of calculating u_* is possible. The coordinate system independent version reduces scatter in the scaled TKE values, not just in the urban location, but also for the open location when convection is present.

The basis for a rotation of the eigenvalue matrix by 17° is purely empirical at this point in time. The author has no current theoretical explanation of this at the moment. It will be a subject of future research.

The performance of this new version of the surface stress needs to be investigated over a wider variety conditions and in a variety of other scaling schemes such as Monin-Obukhov similarity. Such an analysis is not possible with many data sets since evaluation of $\partial\overline{U}/\partial x$ and $\partial\overline{U}/\partial y$ become important in complex environments.

Acknowledgements

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