## 13.4 SPATIALLY AVERAGED PROPERTIES OF TURBULENT FLOW OVER STAGGERED ARRAYS OF CUBES WITH DIFFERENT PACKING DENSITIES: ANALYSIS OF THE SECTIONAL DRAG COEFICIENTS


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### 1. INTRODUCTION

The interaction between atmospheric flow and urban geometry (especially buildings) produces complex airflow patterns within the urban canopy. A detailed understanding of this interaction is very important from the point of view of urban canopy modeling. Recently, simplified urban canopy models have been developed using numerical approaches that do not resolve buildings explicitly, but that employ parameterizations of the drag and turbulence within the building canopy. Such approaches can be used in atmospheric models that do not run with a resolution high enough to represent individual buildings. Since the drag and turbulence clearly depend upon the building geometry, an important challenge is to develop parameterizations that are formulated explicitly in terms of the building geometry. CFD models that resolve every building can play an important role here. The detailed spatial data produced by CFD simulations of the turbulent flow around the buildings can be analyzed to compute spatial averages of flow variables over the grid cell volume of the urban canopy models.

An important input parameter for the simplified urban canopy models is the effective sectional drag coefficient ($C_D$(z)) of buildings in an array. Hence, the computation of $C_D$(z) for a range of building packing densities and geometries is very useful for these models. In this work, the turbulent flow over staggered arrays of cubes with different packing densities is simulated using a CFD RANS model and spatially averaged properties of the airflow within the urban canopy are deduced from the RANS results. Different sectional drag coefficients are computed and a first attempt to parameterize them for this type of configuration is made.


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### 2. THEORY

The sectional drag coefficient may be defined by the following expression (Macdonald, 2000; Coceal & Belcher, 2004):

$$\Delta p(z) = (1/2)\, r U^2(z) C_d(z) \qquad (1)$$

where $\Delta p(z)$ is the pressure deficit around an obstacle, $r$ is air density and $U(z)$ is the profile of horizontally-averaged mean velocity around the obstacle and $C_D$(z) is the sectional drag coefficient. $C_D$(z) takes very large values near the ground due to the small value of $U(z)$ there. In order to solve this problem, Martilli and Santiago (2007) have recently introduced a modified drag coefficient $C_{dmod}(z)$. This parameter includes two additional velocity scales that are relevant for the canopy drag, taking into account the spatially averaged velocity, time fluctuations ($v_{TKE}^2 = 2TKE$) and spatial fluctuations ($v_{DKE}^2 = 2DKE$) from it. *TKE* and *DKE* (definition in section 4) are turbulent kinetic energy and dispersive kinetic energy, respectively. The definition of $C_{dmod}$ is the following:

$$C_{d\,\mathrm{mod}} = \frac{2\Delta p}{r q_{tot}} \qquad (2)$$

with

$$q_{tot} = <\overline{U}>^2 + v_{TKE}^2 + v_{DKE}^2 \qquad (3)$$

Martilli and Santiago (2007) found that $C_{dmod}(z)$ does not vary substantially with height, making it an attractive alternative to $C_d$ $(z)$ in parameterising the drag, if TKE and DKE are known.

### 3. AIMS OF THE STUDY

The main objective of the study is to provide values of $C_D$(z) of buildings in staggered arrays with different packing densities (see next section). These values can be useful for urban canopy models. Recently, $C_d(z)$ for a particular array at a given packing density was computed by direct numerical simulations (DNS) (Coceal et al., 2006), but the dependence on packing density is largely unknown. The use of wind tunnel experiments and DNS simulations is impractical to do a parameter study due to the enormous time required. For this reason, to achieve this aim a RANS model with standard *k*-*e* turbulent closure is used in this study since it is less CPU time expensive and it gave reasonably good results for other configurations (Lien and Yee, 2004 and Santiago at al., 2007). The aims of the study are:

1) To compare one of our RANS simulations with a DNS simulation for a particular array at a given packing density (previously successfully validated against experimental data) to evaluate RANS performance.
2) To compute several spatially averaged properties and parameterize $C_D(z)$ for other configurations with different packing densities using RANS approach.

## 4. NUMERICAL METHODS AND SET UP

The flow is simulated over arrays of cubes in staggered configurations (see Figure 1). For RANS simulations, periodic conditions are imposed in streamwise direction and symmetric conditions in spanwise direction, representing an infinite array. Periodic conditions are imposed in horizontal directions in the DNS simulation. The flow is driven by a pressure gradient ( $ru_t^2/4h$ ) in the streamwise direction where $u_t$ is the total wall friction velocity. The domain height is $4h$, where $h$ is height of the cubes. A uniform Cartesian mesh with 16 points per cube was used. In addition, a grid sensitivity test indicated that this resolution is enough to represent the cube.

The packing densities of the array of cubes are characterized by the non-dimensional ratios $l_f$ and $l_p$, defined as follows:

$$l_f = \frac{A_f}{A_t} = \frac{h^2}{(h+S_y)(h+S_x)} \quad (4)$$

$$l_p = \frac{A_p}{A_t} = \frac{h^2}{(h+S_y)(h+S_x)} \quad (5)$$

where $h$ is the cube height, $A_f$ is the total frontal area of the cubes, $A_p$ is the total plan area of the cubes, $A_t$ is the total floor area, and the other parameters are described in Figure 1.

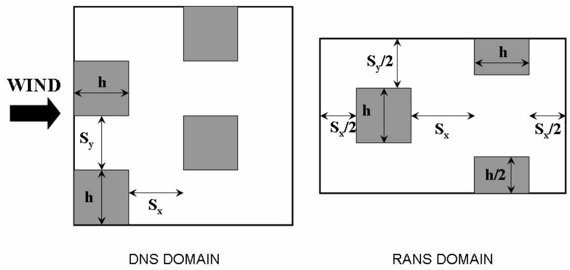


**Figure 1.** Plan view of the numerical domains corresponding to the DNS and RANS simulations.

The configuration used for the comparison against DNS results is $l_f$ = 0.25. In addition, other packing densities representative of values that typically occur in urban areas are studied using RANS simulations. These values are $l_f$ = 0.06225, 0.11, 0.16, 0.25, 0.33 and 0.44.

The objective of this work is to study the spatially average properties inside urban canopy. Therefore, horizontal spatial averages are made as follows:

$$<\overline{U}>_k = \frac{1}{N}\sum_i \sum_j \overline{U}_{i,j,k} \quad (6)$$

$$<\overline{u'w'}>_k = \frac{1}{N}\sum_i \sum_j \left(\overline{u'w'}\right)_{i,j,k} \quad (7)$$

$$<\tilde{u}\tilde{w}>_k = \frac{1}{N}\sum_i \sum_j \left(\overline{w}_{i,j,k} - <\overline{w}>_k\right)\left(\overline{u}_{i,j,k} - <\overline{u}>_k\right) \quad (8)$$

$$<\overline{TKE}>_k = \frac{1}{N}\sum_i \sum_j \overline{TKE}_{i,j,k} \quad (9)$$

$$DKE = 0.5(<\tilde{u}^2> + <\tilde{v}^2> + <\tilde{w}^2>) \quad (10)$$

where $N$ is the number of grid points within the averaging area at the vertical level $k$, excluding the space occupied by the cubes; overbars denote time averaging; brackets denote a horizontal spatial average performed over the entire domain; and tildes denote spatial fluctuations from the time and spatial average.

## 5. COMPARISON AGAINST DNS

The case selected for the comparison was $l_f$ = 0.25. The DNS simulation for this packing density was previously validated against the wind tunnel measurements performed by Cheng and Castro (2002) (Coceal et al., 2006).

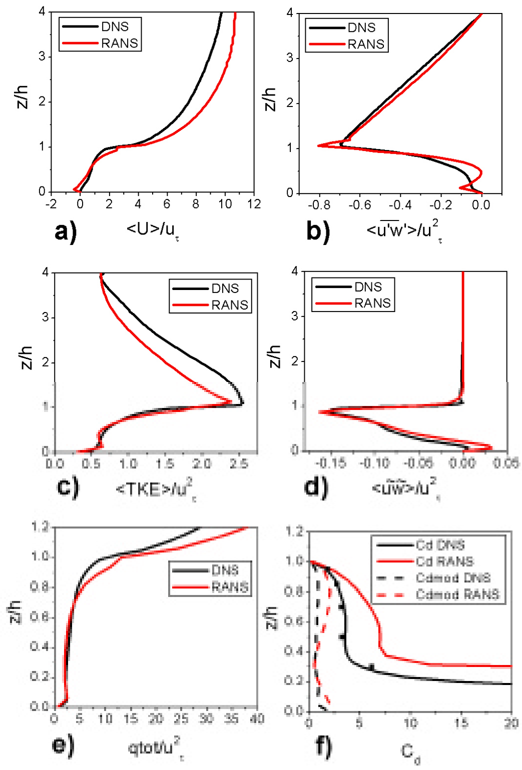


**Figure 2.** Comparison of vertical profiles of spatially-averaged statistics (a) mean streamwise wind speed, b) Reynolds shear stress, c) dispersive stress, d) turbulent kinetic energy, e) $q_{tot} = <\overline{U}>^2 + v_{TKE}^2 + v_{DKE}^2$ and f) $C_d(z)$ and $C_{dmod}$ from the DNS (solid line) and RANS simulations (dashed) in the case of $l_f$ = 0.25. Values of $C_d(z)$ computed from the experimental results of Cheng and Castro (2002) are also shown.

This work is focused on horizontally averaged properties of flow within the canopy. Some of the variables presented above are compared. The wind speed is normalised by $u_t$ and the fluxes and turbulent kinetic energy by $u_t^2$. Figure 2 shows the results obtained. The shapes of all profiles of spatial averaged properties (Figure 2a-e) are generally captured well by the RANS model. However, some differences in the values are found. Concerning the drag coefficients, the shape of the $C_d(z)$ curve is similar for the RANS, DNS and wind tunnel data, but the corresponding values of $C_d(z)$ calculated from the RANS results are a factor of 2 larger than those computed from the DNS and wind tunnel data of Cheng and Castro (2002). This is related to the underestimation of the $U(z)$ by the RANS simulation in the lower part of the canopy. $C_{dmod}(z)$ is also about a factor of 2 larger than the DNS for most of the canopy depth, but collapses well in the range 0.2 < z/h < 0.4. The profile of $C_{dmod}(z)$ from the DNS is remarkably constant, adding strong support to the results Martilli & Santiago (2007). Then $C_{dmod}(z)$ may be a better parameterisation of the drag force, if TKE and DKE are also known.

## 6. VARIATION WITH BUILDING DENSITY $l_f$ OF DRAG COEFFICIENTS

In the previous section, an overestimation of the drag coefficients by RANS simulation is observed. However, the shapes of the resulting profiles are generally captured well. Based on this fact, we shall now assume that the amount of over-prediction may be represented by a constant correction factor. This may be a first approach to compute drag coefficients. Using this assumption, the next step is to analyse how the drag coefficients vary with building packing density, taking into account this correction factor. In this way, RANS simulations are carried out for the same staggered configuration of cubes, but at different packing densities $l_f$ = 0.0625, 0.11, 0.16, 0.25, 0.33, 0.44.

The profiles of $C_d(z)$, shown in Figure 3a, present irregular shapes, especially for large packing densities and near the ground. This fact is due to the small values of mean velocity. These problems do not appear for $C_{dmod}$, which is relatively constant with height (Figure 3b). Indeed, the error made considering $C_{dmod}$ as constant in the calculation of the total drag force is small.

A simple parameterization of the evolution of $C_{dmod}$ averaged in z direction for each packing density (Figure 4a) is the following for $0.0625 \leq ?_f \leq 0.44$:

$$\overline{C}_{d\,mod}(l_f) = A + B \cdot l_f + C \cdot l_f^2 + D \cdot l_f^3 \quad (11)$$

where A = 1.0, B = 6.4, C = -29 and D = 29.

However, most of urban canopy models use $C_d(z)$ to parameterise the drag. In addition, only the mean velocity U is generally known. Figure 4a shows the variation of $C_d(z)$ with $l_f$ at different heights. The behaviour is different at each height. For z/h = 0.6 and 0.8, $C_d(z)$ increases with $l_f$. For z/h = 0.4, there is a maximum and after that decreases with $l_f$ and below z/h = 0.4 the variation is irregular (not shown in the Figure 4a). Therefore, a simple way to parameterise $C_d(z, l_f)$ for this range of values of packing density is:

At $z/h$ = 0.8 and 0.6:

$$C_d(z/h, l_f) = \mathrm{E} \cdot \exp(\mathrm{F} \cdot l_f) \qquad (10)$$

At $z/h$ = 0.4:

$$C_d(z/h = 0.4, l_f) = \begin{cases} \mathrm{E} \cdot \exp(\mathrm{F} \cdot l_f) & \text{if} \quad 0.0625 \le l_f \le 0.26 \\ \mathrm{G} - \mathrm{I} \cdot l_f & \text{if} \quad 0.26 < l_f \le 0.44 \end{cases}$$

(11)

where E = 1.5 and F = 4.8 for $z/h$ = 0.8, E = 1.7 and F = 5.4 for $z/h$ = 0.6 and E = 1.2, F = 7.2, G = 14 and I = 25 for $z/h$ = 0.4.

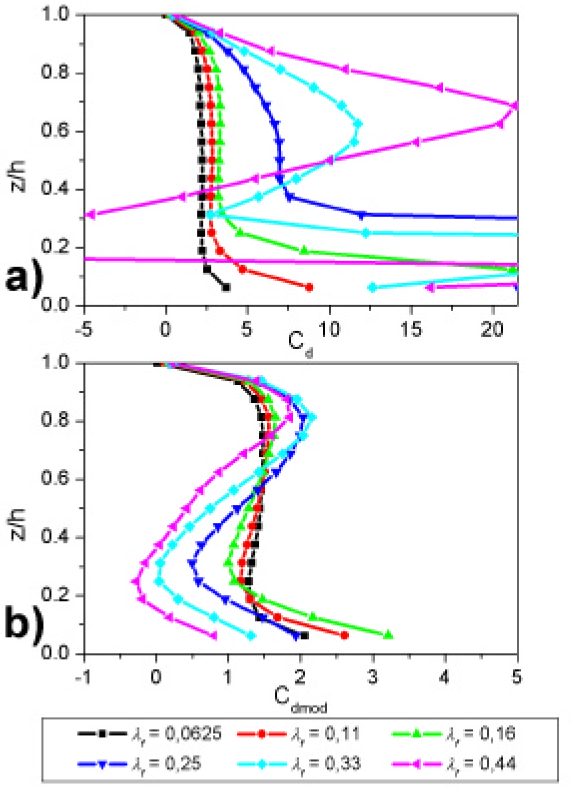


**Figure 3.** Vertical profiles of **a)** $C_d(z)$ and **b)** $C_{dmod}$ with packing density of the array of cubes calculated with RANS with *k-e*. Results are shown for $l_f$ = 0.0625, 0.11, 0.16, 0.25, 0.33, 0.44. Note that the horizontal scales for $C_d(z)$ and $C_{dmod}$ are not the same.

Based on the comparison with the DNS performed in section 5, a correction factor is necessary for the above parameterizations. A value of 0.5 is proposed for all $l_f$ but this assumption needs to be checked in future works with DNS simulations or wind tunnel experiments.

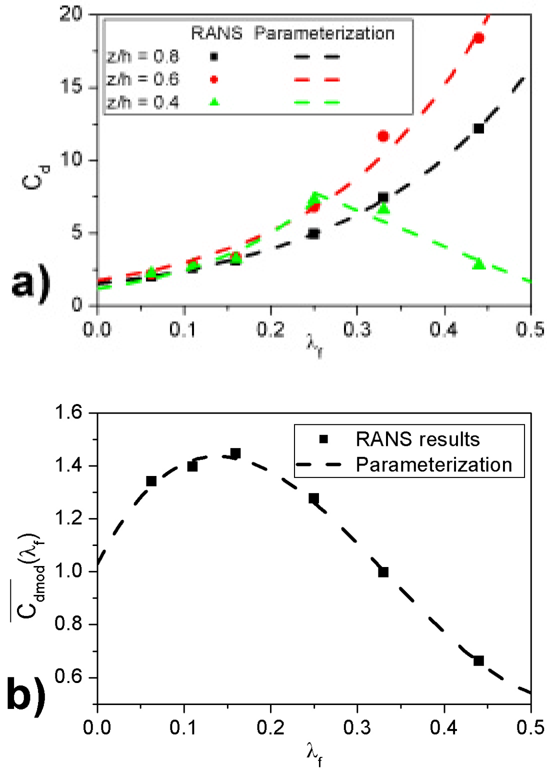


**Figure 4.** Variation of **a)** $C_d(z/h, l_f)$ with $l_f$ at $z/h$ = 0.8, 0.6 and 0.4 (dots are RANS model results and lines are the proposed parameterization) **b)** $\overline{C}_{d\,\mathrm{mod}}(l_f)$ with $l_f$ (squares are RANS model results and dash line is the proposed parameterization).

## 7. CONCLUSIONS

Urban canopy models needs information about drag coefficients of urban geometries. DNS simulations can provide accurate information concerning drag coefficient for a given packing density for a particular configuration but it is impractical to use them for a parameter study due to the prohibitive computational cost. For this reason, RANS simulations, that need much less CPU time, have been used. Firstly, a comparison against DNS for a given packing density has been performed where limitations of RANS have come to light. Taking into account the limitations of RANS, a study of the evolution of the drag coefficients with packing density has been made and simple parameterizations have been proposed. However, the limitations of RANS suggest making a "calibration" against DNS or wind tunnel data.

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