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ATMOSPHERIC TURBULENCE AND DIFFUSION ESTIMATES DERIVED FROM PLUME IMAGE ANALYSIS

Carmen J. Nappo
National Oceanic and Atmospheric Administration, Air Resources laboratory, Oak Ridge, TN

David R. Miller and April L. Hiscox
Natural Resources Management and Engineering, University of Connecticut, Storrs, CT

1. Introduction.

Of the many contributions made by Frank Gifford to the study of atmospheric turbulence and diffusion, his advancement and unique applications of the image analysis of plumes and puffs stand high among the significant. However, he did not consider this work seminal, and stated, "Nearly all students of atmospheric diffusion have attempted to make use of smoke puffs or plumes as a dispersion index…" (Gifford, 1957). Indeed, earlier studies of smoke puffs include, for example, Roberts (1923), Sutton (1932), Frenkiel and Katz (1956), Kellogg (1956). These studies focused on relative diffusion and the development of Lagrangian statistics. The studies by Frenkiel and Katz (1956) and Kellogg (1956) were concerned with the rate of spread of explosion clouds near the ground surface and in the stratosphere respectively. These analyses were made by (1) assuming the that visible edge of the cloud was a line of constant integrated line-of-site concentration of smoke particles, *i.e.* the opacity theorem of Roberts (1923); (2) the concentration distribution was within the cloud was Gaussian, and (3) the instantaneous cross-wind dispersion parameter was given by the Taylor diffusion law for short travel times and long sampling times, *i.e.* $\sigma_y^2 \propto t^2$, where t is the travel time. However, earlier studies by Brier (1950) and Batchelor (1950) showed that Taylor's law of diffusion law does not apply to the relative spreading of particles. Although the theory of relative diffusion is complicated, Gifford (1957) gives a clear description of the process with the result that

$$\overline{Y^2}(t) - \overline{Y^2}(0) \propto t^2 \qquad (1)$$

for small t, and

$$\overline{Y^2}(t) \propto t^3 \qquad (2)$$

for intermediate t, where $\overline{Y^2}(t)$ is the variance of the particle separations at travel time t. Gifford (1957, 1968) elegantly showed that it was not necessary to describe *a priori* the dispersion parameter of a spreading cloud, but that this could be determined by the geometry of the cloud.

Subsequent dispersion studies using smoke plumes were made by, for example, Bowne (1961), Högström (1964), and Byzova *et al.* (1970). Randerson *et al.* (1971) used satellite images of a long (~ 8 km) smoke plume to estimate atmospheric eddy diffusivities, Raynor *et al.* (1975) used a smoke plume to measure dispersion in an on-shore flow from open water to land. Gifford *et al.* (1978) used photo analysis of Martian streak lines observed with the Mariner 9 space craft to estimate eddy diffusivity in the Martial boundary layer. Nappo (1981) used high-altitude photographs of a long smoke plume to estimate single particle and relative diffusion as functions of travel time. Nappo (1984) compared wind tunnel turbulence and dispersion estimates obtained from a smoke plume photo analysis with those measured using a methane gas plume.

Photo analysis of tracer plumes is a valuable tool for remotely estimating turbulence and dispersion parameters; however, the method is limited to daylight conditions. Stable conditions are limited to those occurring shortly before sunrise. But recently, Hiscox *et al.* (2006 a,b) demonstrated that an elastic backscatter lidar images can be used to estimate dispersion in the night-time planetary boundary.

In section 2 of this note, we present a detailed development of the method for estimating dispersion parameters from a smoke plume image. In section 3, we show how these data can be used to estimate turbulence quantities that would otherwise be difficult to measure. In section 4, we describe the use of a lidar for estimating dispersion parameters in the stable boundary layer, and present some results from a field campaign.

2. Plume image analysis.

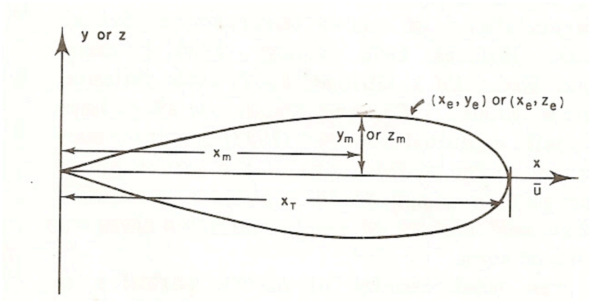

Figure 1. Meaning of various quantities used in smoke-plume analysis.

Figure 1 illustrates the plume parameters to be used in the smoke-plume analysis. If looking from above, then y is the horizontal distance; if looking from the side, then z is the vertical distance. The downwind coordinate is x_T, and $\overline{u}$ is the mean wind speed. We assume that the plume concentration is constant, and is given by :

$$C(x,y,z) = \frac{Q}{2\pi\sigma_y\sigma_z\overline{u}} e^{-y^2/2\sigma_y^2}\, e^{-z^2/2\sigma_z^2} \tag{3}$$

where Q is the constant source strength. We assume we are observing the plume from a high distance above. Integration of (3) gives:

$$C_e = \int_0^\infty C\,dz = \frac{Q}{2\pi\sigma_y\sigma_z\overline{u}} e^{-y_e^2/2\sigma_y^2} \int_0^\infty e^{-z^2/2\sigma_z^2}\,dz \tag{4}$$

Noting that

$$\int_0^\infty e^{-z^2/2\sigma_z^2}\,dz = \sqrt{\frac{\pi}{2}}\,\sigma_z \tag{5}$$

(4) becomes:

$$C_e = \frac{Q}{2^{1.5}\sqrt{\pi}\sigma_y\overline{u}} e^{-y_e^2/2\sigma_y^2} = \frac{k}{\sigma_y} e^{-y_e^2/2\sigma_y^2} \tag{6}$$

where k is a constant and y_e is the distance from the plume axis to the visible edge of the plume. We identify C_e with the constant integrated line-of-sight concentration along the visible edge of the plume (Roberts 1923).

Taking the logarithm of (6), and then differentiation with respect to x gives:

$$\frac{1}{x}\frac{\partial C_e}{\partial x} = -\frac{1}{\sigma_y}\frac{\partial \sigma_y}{\partial x} - \frac{z}{\sigma_y^2}\frac{\partial y_e}{\partial x} + \frac{y_e^2}{\sigma_y^3}\frac{\partial \sigma_y}{\partial x} = 0. \tag{7}$$

From Fig. 1, y_e is a maximum, y_m at x=x_m , and

$$\frac{\partial y_e}{\partial x} = 0. \tag{8}$$

Using (8) in (7) gives the simple result:

$$\sigma_{y,m}^2 = y_m^2 \tag{9}$$

where $\sigma_{y,m}$ is the value of σ_y at x_m. Thus, at x=x_m, (6) becomes:

$$C_e = \frac{k}{\sigma_{y,m}} e^{-1/2}. \tag{10}$$

Solving (10) for k and substituting this into (6) gives:

$$\sigma_y = y_m e^{1/2\left(1-y_e^2/\sigma_y^2\right)}. \tag{11}$$

Squaring both sides of (11) and taking the logarithm gives:

$$\ln\frac{\sigma_y^2}{y_m^2} = \ln e - \frac{y_e^2}{\sigma_y^2}. \tag{12}$$

After a rearrangement of the terms in (12), we obtain the final result:

$$\sigma_y^2 = y_e^2\left[\ln\left(\frac{e y_m^2}{\sigma_y^2}\right)\right]^{-1}. \tag{13}$$

Eq (13) is an implicit function which can be solved by any of several iteration techniques. A similar equation for σ_z is found in exactly the same way.

3. Applications

Dispersion

For long averaging times, Taylor diffusion theory gives:

$$\sigma_y^2 = 2\sigma_v^2 \tau_L t = 2k_y t \qquad (14)$$

where σ_v is the rms value of the horizontal cross-wind velocity, τ_L is the Lagrangian integral time scale, $t = x_t / \overline{u}$ is the travel time, and k_y is the eddy diffusivity in the *y* direction. Using (14) in (13), the eddy diffusivity can be evaluated using either:

$$k_y = \frac{1}{2} e\overline{u} \frac{y_m^2}{x_T} \qquad (15)$$

or:

$$k_y = \frac{1}{2} \overline{u} \frac{y_m^2}{x_m} \;, \qquad (16)$$

with similar equations for k_z.

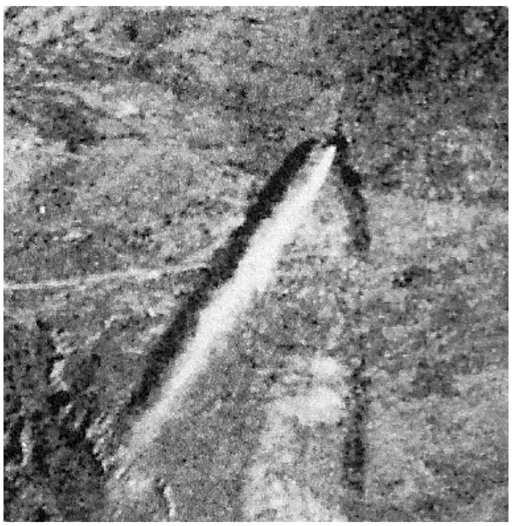

Figure 2. Smoke plume photographed from space and analyzed by Randerson *et al.* (1971).

Randerson *et al.* (1971) used the plume image shown in Figure 2. They estimated that x_T was about 4830 m. Assuming a wind speed of 2.2 ms^{-1} they determined that k_y equals 4.41 × 10^3 m^{2}s^{-1} from (15) and 4.46 × 10^3 m^{2}s^{-1} from (16).

Gifford *et al.* (1978) estimated eddy diffusivities on Mars by analyzing photographs of sand plumes on the ground surface. Figure 3 shows an example of these images. Values ranged from 0.2 - 8 × 10^6 m^{2}s^{-1}.

Nappo (1981) photo analyzed high-altitude photographs of a 5 km long oil fog plume. Photographs were taken at 15-min intervals from a U2 aircraft. Eq. 13 was applied to the individual snapshots. These results were considered to be estimates of relative diffusion or two-particles

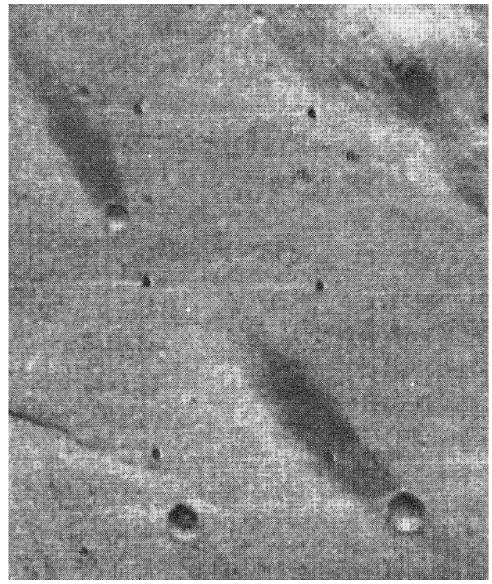

Figure 3. Sand plumes on the Martian surface

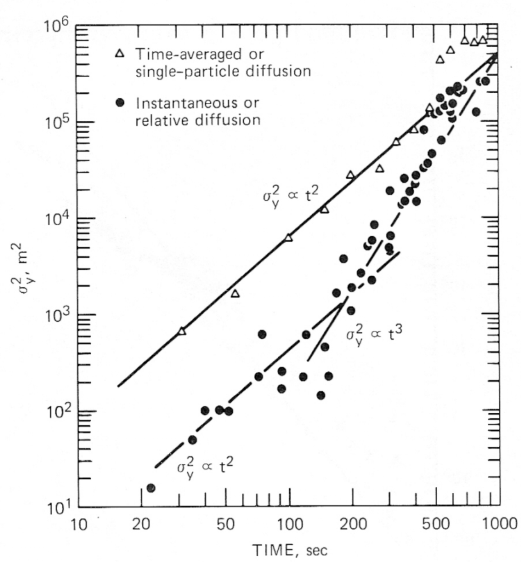

Figure 4. Single particle and relative diffusion estimates derived from a series of high-altitude smoke plume photographs (Nappo 1981)

diffusion, and are plotted in Figure 3. The negatives of the individual photographs were then superimposed forming in effect an ensemble average. This 'average' plume was analyzed using (13). These results were considered to be estimates of single-particle diffusion, and are plotted in Figure 3. From Figure 3, we see that the observed diffusion estimates agree fairly well with the theoretical relations.

Turbulence

Because dispersion is a function of turbulence, we can estimate a number of turbulence parameters using direct measures of dispersion. Thus, using either (15) or (16) to estimate k_y we can use (14) to calculate the Lagrangian time scale, τ_L, if we know σ_v. For small travel time, T, relative to the plume averaging time:

$$\sigma_y^2 = \sigma_v^2 T^2 , \qquad (17)$$

and we can estimate σ_v using σ_y values close to the source. If plume snapshots are manipulated so as to render an ensemble instantaneous plume, then analysis of this image using (13) gives the instantaneous or relative diffusion rate, σ_I. Batchlor (1952) expresses the relative diffusion in terms of the eddy dissipation rate, ε, *i.e.*

$$\sigma_{y,I}^2 = c_2 \varepsilon T^3 , \qquad (18)$$

where $c_2 = 0.4$. Alternatively, Byzova (1970) give

$$\varepsilon = \frac{\sigma_v^2}{2c_1\tau_L} , \qquad (19)$$

where $c_1 = 0.6$. Either (18) or (19) can be used to estimate ε. If an estimate of the mean wind, U, is available, then the turbulence intensity can be estimated using

$$i = \frac{\sigma_v}{U} . \qquad (20)$$

4. The JORNADA experiment.

A major limitation of the photo analysis technique is the need of light. This limits the method to al least twilight or else long exposures under full moon light. The use of a lidar allows plume visualization during nighttime conditions. The Joint Observational Research on Nocturnal Atmospheric Dispersion of Aerosols (JORNADA) program was designed to study the dispersion of elevated tracer plumes in the stable PBL. Direct measurements of cross-wind plume concentration were made using the University of Connecticut elastic backscatter lidar. The JORNADA field study was conducted in April 2005 at the New Mexico State University spray study site on the USDA Jornada Desert research ranch located north-east of Las Cruces, NM. The region is relatively flat with low, 1-2 m tall sparse desert vegetation with an unobstructed fetch in all directions for at least 10 km. The surface roughness length is estimated to be about 0.06 m.

On each of six nights, an oil fog plume released 11 m AGL was scanned in a cross-wind vertical plane by the UCONN elastic back scatter lidar (see Hiscox 2007a,b for details of the lidar). Figure 4 shows a typical scan. A scan takes about 3 seconds to complete. Dispersion parameters are estimated by assuming a Gaussian distribution of scattering particles, *i.e.*,

$$\chi_E = \chi_M e^{-\frac{z_E^2}{2\sigma_z^2}} \qquad (21)$$

where z_E is the vertical distance from the plume center to some line of constant concentration χ_E, and χ_M is the concentration at the plume center. An ensemble average of individual values of z_E is

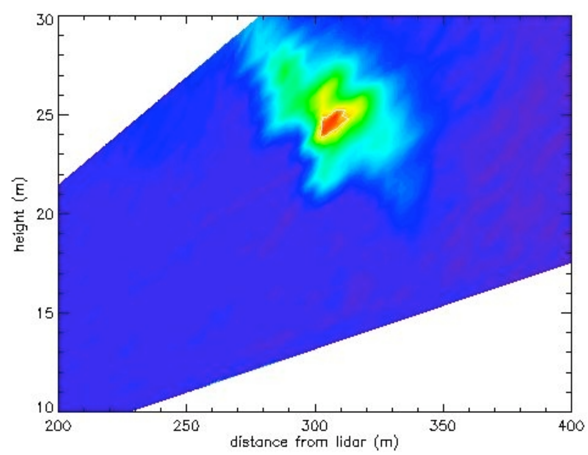

Figure 5. Contours of lidar backscatter from a cross -section of a smoke plume. Red is high value.

used in (21) to estimate the instantaneous dispersion parameter $\sigma_{y,I}$. The single value of z_E derived from the average of all the scans over some time period is used in (21) to calculate the total diffusion, $\sigma_{z,T}$. To illustrate the utility of the lidar scans, we show in Figure 5 the time series of $\sigma_{z,I}$ and vertical velocity, w, observed on 21 April 2005. At about 05:15 a large-amplitude gravity wave propagated through the field site and generated an abrupt and sustained increase in turbulence (Nappo, *et al.* 2008). The

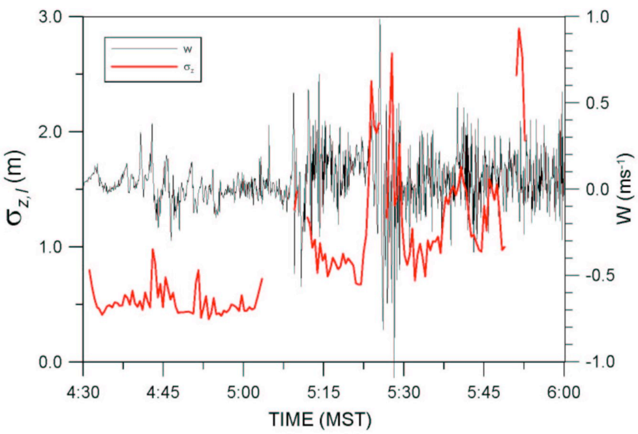

Figure 6. $\sigma_{z,I}$ and vertical velocity, *w*, on 21 April 2005 during the JORNADA experiment.

accompanying increase in $\sigma_{z,I}$ is clearly illustrated in Figure 6. The plume was scanned about 25 m from the plume source, and *w* was measured near the source point. Data such as shown in Figure 5 can not obtained using conventional plume sampling techniques.

5. Conclusions.

The analysis of plume images as proposed by Frank Gifford is an efficient and accurate way to obtain turbulence and dispersion data from regions or under conditions that are otherwise inaccessible to instrumentation. Of particular utility is the use of lidar scans of tracer plumes to analyze dispersion and turbulence throughout the depth of the nighttime PBL.

Acknowledgements

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