6.6

BALANCING THE EARTH'S RADIATION BUDGET

G. Louis Smith[1], Norman G. Loeb[2] and David Doelling[2]

1 National Institute for Aerospace, Hampton, Virginia
2. Science Directorate, Langley Research Center, NASA, Hampton, Virginia

ABSTRACT

The annual mean of the global-average net radiation budget should be quite small. Because of errors in the measurement and data analysis, the annual mean global-average net radiation as computed by the CERES program does not meet this requirement. A method is presented for computing the most likely errors in the measurement and data production which will bring the radiation into balance within the range of interannual variations. The need for a data set which is globally balanced is demonstrated by the computation of the annual-mean zonal distribution of heat flux by the atmosphere and oceans.

1. INTRODUCTION

The radiation budget of the Earth governs its climate. Our knowledge of the Earth's radiation budget is based on satellite measurements from which monthly mean maps of outgoing longwave radiation and reflected solar radiation are computed. When the annual average-mean global average net radiation is calculated for several years, the Earth should be in radiation balance within a W-m^{-2}. However, our data products give us a net imbalance of a few W-m^{-2} because of the accumulation of errors in the measurements and the methods by which the monthly-mean maps of radiation fluxes are produced. Even though we may improve our measurements and algorithms for generating data products, there will always be errors. In this paper we present a technique for computing the most likely corrections to the major errors so as to bring the Earth's annual-mean net radiation into balance.

The method for balancing the Earth's radiation budget is to select the major error sources and estimate their variances. Next, the sensitivity of the annual average global mean net radiation to each error is modeled. A set of corrections is then computed by maximizing the likelihood of this set, constrained by the requirement that these corrections bring the average net radiation into balance.

Corresponding author address: G. Louis Smith, Mail Stop 420, Langley Research Centre, NASA, Hampton, Virginia, 23681,
e-mail: george.l.smith@larc.nasa.gov

The method is applied to radiation budget data from the Clouds and Earth Radiant Energy System (CERES) project (Wielicki et al., 1995; 1998). From the CERES radiometer measurements, radiances are computed by use of the instrument gains. The spectral responses of the radiometer channels are taken into account to compute broadband longwave and shortwave radiances. Fluxes at the "top of the atmosphere" are computed by use of bidirectional reflectance functions for the shortwave fluxes and limb-darkening functions for the longwave fluxes. The instruments are on Sun-synchronous spacecraft, so that measurements are taken at two local times for each spacecraft; thus it is necessary to include the diurnal cycle of radiation in order to compute the average flux for the day. Each of these steps has an associated error. In order to apply the method presented here, the variance of each error must be estimated and the sensitivity of the average net radiation to each error must be computed.

One application of Earth radiation budget data is to compute the annual-mean zonal-average latitudinal distribution of meridianal heat flux by the atmosphere and oceans. This computation was done with the early radiation budget data set developed by Raschke et al. (1973) and similar studies have continued with successive data sets. The method requires a net balance of zero radiation, so the imbalance of the data products is quite bothersome. One starts at a pole, where the zero length of the meridian requires that the transport must equal zero, and integrates the net radiation to get meridianal transport of energy. This transport should become zero at the other pole. However, if the global net radiation is not zero, the flux of heat as one approaches the pole will become infinite due to the vanishing of the meridianal length at the pole. The method developed here removes this problem in an objective manner. Also, the model for the effects of errors permits computation of the zonal distribution of the corrections for net radiation.

Typically in remote sensing of the atmosphere, the measurements and algorithms for generating data products have errors which accumulate. The technique presented here may be adapted to many of these problems.

Wielicki et al. (1998) discussed the errors associated with the CERES instruments and data products and the imbalance of the global net radiation. Although efforts continue to reduce these errors, for the foreseeable future one must expect an imbalance of the Earth's energy budget as measured. A part of this imbalance is due to interannual variations of the Earth's radiation budget, but most of the imbalance is due to errors in the measurement and data production process. This paper presents a method for estimating the errors which will bring the radiation into global annual mean balance, within the limit expected due to interannual variability, based on their likelihood of occurrence. Section 2 demonstrates the formulation for the method. Section 3 discusses the error sources in the data products and gives the equations for the net radiation errors in terms of these sources. In Section 4 the analysis is used to compute the errors in the measurement and data generation processes which are most likely and the consequent changes in the data products are computed. Finally, Section 5 uses these radiation fluxes which have been forced into balance to compute the annua- mean meridional flux of energy.

2. FORMULATION OF METHOD

The CERES instruments aboard the Terra and Aqua spacecraft measure instantaneous reflected solar and Earth-emitted radiances. From these measurements, estimates of monthly-mean net ii. radiation fluxes RN' are computed for each region n covering the Earth for each month m. If the Earth were in balance over the year,

$$\sum_m \sum_n RN(m, n) = \overline{RN} = 0$$

In the measurement and data product generation, there are many parameters p_i which are used to compute RN. These p_i include the instrument gains, the bidirectional reflectance models and the quantities which are used to temporally interpolate the instantaneous regional fluxes to monthly-mean values. Because these parameters are not known perfectly, the computed fluxes do not balance, i.e.

$$\overline{RN} = \varepsilon_{RN}$$

where ε_{RN} is the imbalance. One way to balance the budget is to modify these parameters p_i by some amount x_i to revise RN so as to be in balance:

$$\widehat{RN} = \overline{RN} + \sum_i \frac{d\overline{RN}}{dp_i} x_i = 0$$

Combining these two equations gives

$$\sum_i \frac{d\overline{RN}}{dp_i} x_i = -\varepsilon_{RN}$$

Because of the importance of these derivatives, denote

$$a_i = \frac{d\overline{RN}}{dp_i}$$

This term is the partial derivative of annual mean global net radiation with parameter p_i. Vectors x and **a** are defined with components x_i and a_i, so that eq. (4) may be written as

$$a^t x = -\varepsilon_R \qquad (1)$$

We have one equation with k unknowns, where k is the number of parameters to adjust. The criterion for selecting these parameters will be to choose the most likely set of parameters x_i which will satisfy eq. (4), i.e. we use a maximum likelihood estimate for the x_i. The assumptions will be made that

i. x_i has mean 0 for all i.
II. All x_i are normally distributed.

The probability that x_i is between x_i and dx_i is then

$$P\{x_i\} = (2\pi)^{-n/2}(\det C)^{-n/2} \exp(-xC^{-1}x/2) \qquad (2)$$

The probability is maximum when $xC^{-1}x$ is minimum subject to the constraint of eq. (1), which is done by the method of Lagrangian constraints. The function

$$\Omega = \frac{1}{2}\mathbf{x^t C^{-1} x} + \lambda \mathbf{a^t x}$$

is minimized, in which λ is the Lagangian multiplier, whence

$$\left[\frac{d\Omega}{dx_j}\right] = \mathbf{C^{-1}x} + \lambda\mathbf{a} = \mathbf{0}$$

for all j ε [1,k].

The most likely value of x_i is then given by

$$\mathbf{x} = -\lambda\mathbf{C}\mathbf{a} \quad (3)$$

This result is used with eq. (1), whence

$$\lambda = (\mathbf{a^t C a})^{-1}\varepsilon_{RN}$$

Equation (3) is k equations, which together with λ, produce the x_i that are the most likely values of errors in the measurement and data product generation for which the annual mean global net radiation is zero.

Figurer 1 is a schematic of the problem. Equation (1) states that the values of x_i for which the global net radiation is zero are in a subspace of dimension k-1, normal to $\mathbf{a}$, with a distance from the origin defined by ε_R. Equation (2) defines a set of ellipsoids of constant probability, as $xC^{-1}x$ = const., and the axes of the ellipsoids are proportional to the eigenvectors of C. The solution is the point where the subspace given by eq. (1) is tangent to an ellipsoid defined by eq. (2). In eq. (3), **Ca** is the vector in the k dimensional space of parameters from the origin in the direction of the most likely set of errors to produce global net radiation balance. The Lagrange multiplier λ specifies the distance in this direction to this solution.

It is possible that ε_R contains some true interannual variation. The procedure just described is modified to allow for interannual variations in RN. The interannual variation of RN is denoted as x_N and is assumed to be normally distributed with standard deviation σ_N. The interannual variations are independent of the errors of the measurement and data product generation, so that the probability density function for the system including x_N is that given by eq. (2) times $(2\pi)^{-1/2}\sigma^{-1} \exp[-(x_N/\sigma_N)^{-2}/2]$. The formulation simply increases the order of x by one.

3. ERROR MODEL FOR MEASUREMENT AND DATA PRODUCT GENERATION

The error sources and the estimate of errors is now. The annual-mean global average net radiation balance of Earth is

RNbar = 1/4S – RSRbar – OLRbar

Where S is the solar flux at Earth, RSRbar is the global mean reflected solar radiation flux and OLRbar is the global mean outgoing longwave radiation flux. The factor of ¼ appears with the solar flux because the average radiation is computed for the area of the spherical Earth, but the solar flux is incident only over the circular disc of the cross section of the Earth.

RSRbar and OLRbar are computed from the measurements by a number of steps, each of which introduce errors. An error model is required which must include the standard deviations of the errors and their correlations. The next subsection

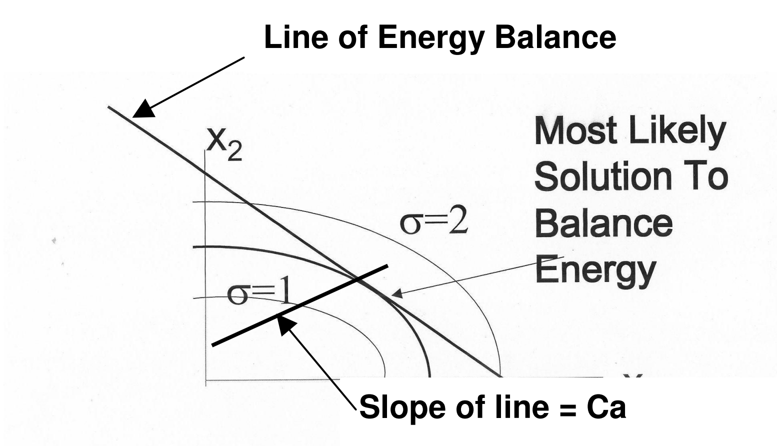

Figure 1: Schematic in 2 variables of most likely solution for balancing Earth radiation budget.

gives an overview of the data product generation process and the error sources, after which the computation of their effects on the monthly-mean net radiation for a region and the annual mean global-average net radiation is treated. Only the dominant terms are considered here, as the uncertainties of these major terms overwhelm the effects of the lesser terms. Each parameter which is considered as an error source is assigned an index, as required by the formulation of Section 2.

3.1 Measurement and Data Product Generation Errors

The individual measurements of the reflected solar radiation and Earth-emitted radiances are denoted as Level 0. The shortwave and longwave fluxes at the "top of the atmosphere" are computed from these radiances and are denoted as Level 1. The pixel values are used to compute instantaneous regional average fluxes, constituting Level 2. The regional average fluxes are averaged over a month to generate Level 3 data products.

3.1.1 Level 0 Errors: The desired measurements are the solar radiation which is reflected from Earth, or shortwave radiance, and the Earth-emitted radiance, or longwave radiance. The measured quantities are the radiances after they pass through the imperfect optics of the instrument. These radiances are called filtered radiances. The CERES instrument is calibrated frequently in orbit so as to minimize the meaaurement errors (Lee et al., 1996, 1998, 1999, 2000; Priestley et al., 2007). Errors in shortwave and longwave filtered radiances are given indices 1 and 2. The unfiltered radiances are computed from the filtered radiances by use of spectral factors, which take into account the spectral responses of the two channels, as well as the spectral overlap of the reflected solar and Earth emitted radiation. There are errors associated with this unfiltering process due to the imperfect description of the spectra of the filtered radiances and the variety of spectra of the broadband radiances. Also, the CERES instrument derives longwave radiance at night from the total channel and during day from the total channel minus the shortwave channel. Nighttime longwave has a lower error than daytime longwave, which reuires two channels to derive. Unfiltered shortwave radiance error is indexed as 3, nighttime unfiltered longwave as 4 and daytime longwave as 5.

3.1.2 Level 1 and level 2 Errors: The reflected solar radiation flux at the top of the atmosphere TOA is computed from the shortwave radiance by use of a a bidirectional reflectance function BRDF and the Earth-emitted flux is computed from the longwave radiance by use of a limb-darkening function (Loeb et al., 2003a, 2003b, 2005, 2007). These functions depend on the scene, so that a scene identification algorithm is required. The limb-darkening functions and BRDFs have errors, which cause errors in the TOA fluxes. Scene identification errors result in selection of the wrong BRDF, thus inducing errors in OLR and RSR fluxes. Because a scene identification error causes selection of the wrong functions, the resulting OLR and RSR errors are correlated. As with other terms, only the bias error in the limb-darkening functions and BRDFs are of concern in this work, as much of the differences between the directional models and the instantaneous true function largely cancel in the monthly-mean and even more for global averages. The errors incurred in the computation of TOA flux from the unfiltered radiances are numbered 6 and 7. The next step is to form instantaneous regional average fluxes, i.e. level 2 data products, from the instantaneous fluxes. This process has errors due to the overlapping of the measurement footprints with adjacent regions and non-uniformity of the measurement over the region. However, these errors in regional average fluxes average out in the computation of zonally and globally averaged fluxes and are negligible in the present study.

3.1.3 Level 3 Errors: The final step in data processing is to compute the monthly mean RSR and OLR for each region on the basis of instantaneous regional values. This computation requires that the RSR and OLR be interpolated between measurements and then integrated over the day and month. The temporal interpolation error for RSR is error 8 and for OLR is error 9.

When one computes the solar radiation incident on the Earth, the effective thickness of the atmsophere enters the calculation (Loeb et al., 2002). The radius of Earth plus the thicknss of the atmosphere is the radius of the sphere over whose surface energy is radiated. The nominal number used by CERES data processing is 20 km, but the effective atmospheric thickness varies with latitude, being less at the Poles than at the Equator. Also, deep convective clouds extend above 20 km to interact with sunlight. Consequently, there is an error in the use of the nominal thickness of the atmsophere. This error contribution is 10 for shortwave and 11 for longwave. The net radiation is the solar irradiance minus the RSR and OLR. The error of solar irradiance is number 12.

4. NUMERICAL APPLICATION

The sensitivity of the monthly-mean regional average to an error in parameter *i* is dR_n/dp_i and its annual mean global average value was denoted as a_i. The a_i are required in order to compute the set of errors which are most likely to have caused the imbalance of radiation budget, or converesly, the set of corrections to the radiation budget in order to balance the radiation budget. The distribution of dR_n/dp_i with month and zone is of interest, as it describes the times and zones when the error in R_n tends to be large. The evaluation of these terms requires an error model.

In this paper the errors are stated in terms of fractions of the annual-mean global-average RSR and OLR, and solar output S, so that the a_i. are simply the means of these quantities. Furthermore, the errors for these parameters are uncorrelated. For Edition 2 results from CERES the RSR is 97.8 W-m^{-2}, the OLR is 237.1 and the solar input is 340.0, so that the Earth has a net imbalance of 5.1 W-m^{-2}. Hansen et al.(2004) compute that the oceans stored an average of 0.75 W-m^{-2} during 1993-2003, and 0.85 W-m-2 during 2003 alone. For the present computation we use 0.85 W-m^{-2} as the storage term, so the imbalance is 4.25 W-m^{-2}. Table 1 lists the error sources, the sensitivity a_i and the standard deviation σ_i of the error.

Because the errors are considered to be uncorrelated, the covariance matrix C is diagonal and eq. (3) reduces to

$$x_i = \lambda a_i \sigma_i^2$$

Table 1 lists the $a_i\sigma_i^2$ for each term. The Lagrangian multiplier is computed as 0.411 W^{-1}-m^2 so that the corrections to be applied x_i are calculated and listed in Table 1. The x_i are then multiplied by the sensitivity to give the effect of each correction on the annual mean global-average net radiation flux. These effects are listed in the last olumn of Table 1 and are shown in fig. (3) as a bar chart. The corrections to the longwave and shortwave measurements account for nearly all of the error in the net radiation; the other corrections are quite small. This is because the probability of the total set of errors is the product of the probabilities of the individual errors. A small increase in the dominant term, longwave measurement, gives a larger effect with little change of probability than a proportional effect of a term with little effect, e.g. the reference level for longwave, which would have a low probability.

5. MERIDIONAL HEAT FLUX DISTRIBUTION

By conservation of energy, assuming no heat sources or sinks, the meridional heat flow carried by the atmosphere and ocean may be computed by integrating the net energy flux in the latitudinal direction, as was done by Vonder Haar and Oort (1973) using early satellite measurements of Earth radiation budget. The result for Edition 2 CERES data products is shown by fig. 3. When the computation starts at the South Pole, the energy flux at that point is zero and builds up (negatively) due to the southward flow of energy from the Tropics to about 40^{o}S, then increases until it becomes positive near the Equator, where the flow of energy is now northward. Between 40^{o}S and 40^{o}N there is a surplus of radiation which flows poleward. The northward flow increases until about 40^{o}N, after which the horizontal flow decreases due to the net loss due to radiation. However, near the North Pole the horizontal flow does not go to zero, but to 4.2 W-m^{-2}, due to the net imbalance of the data product. (The proper unit is Watts, as the iintegral is over the surface of the Earth.) This energy is flowing into a point, a physical impossibility. The same procedure is followed starting at the North Pole, giving the second blue line.

The corrections listed in Table 1 are applied to the annual-mean zonal-average net radiation as a proportion of the RSR and OLR and the resulting net radiation flux is integrated to give a revised horizontal heat flow, as indicated by the red line. This curve goes to zero at both poles as required.

The energy flow northward is divided by the circumference of the zone (in terms of The equatorial distance) and fig. (4) shows the resulting flux. For the uncorrected net flux, the meridional flux is shown by a blue line. As the net flux is integrated from one pole to get horizontal zonal flux, the result goes to infinity as the integration approaches the other pole. The red line shows the horizontal flux using the corrections. Due to numerical integration errors the result still becomes infinite very near the pole due to the division by zero. However, if one begins at each pole and integrates to the Equator, the two lines come vey close and provide a useful result.

6. CONCLUSIONS

The annual-mean globally-averaged net radiation flux as computed from satellite measurements is a few W-m^{-2} for CERES Edition 2 data products. This imbalance of radiation is due to errors in the measurements and the processing of the data to generate useful data products. A method is presented to compute the most likely errors which can be used to correct the relfected solar radiation and outgoing longwave radiation to balance the Earth's energy budget. This method is demonstrated by computing the corrections needed for the Edition 2 CERES data products and computing the poleward energy transport.

Acknowledgments: This work was supported by the Earth Observations Office of NASA through the CERES Program in the Sciences Directorate of Langley Research Centre. GLS is funded by contract to the National Institute for Aerospace.

SYMBOLS

a_i sensitivity of annual mean net global radiation flux for parameter p_i

a vector of a_i values

C Covariance matrix of error sources

p_i i-th parameter in measurement and product generation process.

x_i value of parameter pi for which probability is maximum subject to constraint of net radiation balance.

x vector of x_i values

λ Lagrangian multiplier

Ω exponent of probability of errors

Overbar annual-mean globally-averaged quantity

REFERENCES

Barkstrom, B. R. and G. L. Smith, 1986: The Earth Radiation Budget Experiment, Science and Implementation, *Rev. of Geophys., 24*, 379-390.

Lee, R. B. III, B. R. Barkstrom, G. L. Smith, J. E. Cooper, L. P. Kopia, R. S. Lawrence, S. Thomas, D. K. Pandey, and D. H. Crommelynck, 1996: "The Clouds and the Earth's Radiant Energy System (CERES) sensors and Preflight calibration Plans," *J. Atmos. and Oceanic Tech., 13*, pp. 300-313.

Lee, R. B. III, B. R. Barkstrom, H. C. Bitting, D. H. Crommelynck, J. Paden, D. K. Pandey, K. J. Priestley, G. L. Smith, S. Thomas, K. L. Thornhill, and R. S. Wilson, 1998: Prelaunch calibrations of the Clouds and the Earth's Radiant Energy System (CERES) Tropical Rainfall Measuring Mission and Earth Observing System Morning (EOS-AM1) Thermistor bolometer sensors, *IEEE Trans. Geosci. and Rem. Sens., 36*, 1173-1185.

Lee, R. B. III, S. Thomas, R. S. Wilson, K. J. Priestley, J. Paden, D. K. Pandey and A. Al-hajjah, 1999: Ground through on-orbit transfer of the International Temperature Scale of 1990 (ITS-90) radiometric scale using the CERES thermistor bolometers and built-in flight calibration systems, *SPIE, 3870*, 389-396.

Lee, R. B. III, K. J. Priestley, B. R. Barkstrom, S. Thomas, A. Al-Hajjah, J. Paden, D. K. Pandey, R. S. Wilson, and G. L. Smith, 2000: Terra Spacecraft CERES flight model 1 and 2 sensor measurement precisions: ground to flight determinations, *Proc. SPIE, 4135-01*.

Loeb, N. G., S. Kato and B. A. Wielicki, 2002: Defining top-of-atmosphere reference level for Earth radiation budget studies, *J. Clim., 15*, 3301-3309.

Loeb, N. G., N. Manalo-Smith, S. Kato, W. F. Miller, S. K. Gupta, P. Minnis and B. A. Wielicki, 2003: Angular Distribution models for top-of-atmosphere radiative flux estimation from the Clouds and the Earth's Radiant Energy System instrument on the Tropical Rainfall Measuring Mission satellite, Part I: Methodology, *J. Appl. Met., 42*, 240-265.

Loeb, N. G., K. Loukachine, N. Manalo-Smith, B. A. Wielicki and D. F. Young, 2003: Angular Distribution models for top-of-atmosphere radiative flux estimation from the Clouds and the Earth's Radiant Energy System instrument on the Tropical Rainfall Measuring Mission satellite satellite, Part II: Validation, *J. Appl. Met., 43*, 1748-1769.

Loeb, N. G., , S. Kato, K. Loukachine and N. Manalo-Smith, 2005: Angular Distribution models for top-of-atmosphere radiative flux estimation from the Clouds and the Earth's Radiant Energy System instrument on the Terra satellite, Part I: Methodology, *J. Atmos. & Ocean. Tech., 22,* 338-351.

Loeb, N. G., , S. Kato, K. Loukachine, N. Manalo-Smith and D. R. Doelling, 2007: Angular Distribution models for top-of-atmosphere radiative flux estimation from the Clouds and the Earth's Radiant Energy System instrument on the Terra satellite, Part II: Validation, *J. Atmos. & Ocean. Tech., 24,* 564-584.

Priestley, G. L. Smith, S. Thomas, D. Cooper, R. B. Lee, III , D. Walikainen, P. Hess, Z. P. Szewczyk and R. Wilson, 2007: Radiometric performance of the CERES Earth Radiation Budget climate record sensors on the EOS Aqua and Terra Spacecraft Calibration

Raschke, E., T. H. Vonder Haar, W. R. Bandeen, and M. Pasternak, 1973: The annual radiation balance of the Earth-atmosphere system during 1969 - 1970 from Nimbus 3 measurements, *J. Atmos. Sci., 30*, 341 - 364.

Vonder Haar and A. H. Oort, 1973: New estimate of annual poleward energy transport by Northern Hemisphere oceans, *J. Phys.*

Ocean., 3, 169-364.

Wielicki, B. A., B. R. Barkstrom, E. F. Harrison, R. B. Lee III, G. L. Smith, and J. E. Cooper, 1996: Cloud and the Earth's Radiant Energy System (CERES): an Earth Observing System Experiment, *Bull. Amer. Met. Soc., 77,* 853-868.

Wielicki, B. A., B. R. Barkstrom, B. A. Baum, T. P. Charlock, R. N. Green, D. P. Kratz, R. B. Lee III, P. Minnis, G. L. Smith, T. Wong, D. F. Young, R. D. Cess, J. A. Coakley Jr., D. H. Crommelynck, L. Donner, R. Kandel, M. D. King, J. Miller, V. Ramanathan, D. A. Randall, L. L. Stowe, and R. M. Welch, 1998: Clouds and the Earth's Radiant Energy System (CERES): algorithm overview, *IEEE Trans. Geosci. and Rem. Sens., 36*, pp. 1127-1141, 1998.

Table 1: Error sources, sensitivity, standard deviation of error, potential and most likely error.

	Error Term	Sensitivity, W-m^{-2}	σ_I x 100	$a_i\sigma_i^2$, W-m^{-2}	Correction to parameter	Effect on Net, W-m^{-2}
1	SW Measurement	97.82	2.	0.039	0.016	1.6
2	LW Measurement	237.15	1.	0.024	0.0010	2.4
3	Unfiltering SW	97.82	0.5	0	0.001	0.1
4	Unfiltering LW Night	118.58	0.2	0.24	.0002	0.0
5	Unfiltering LW Day	118.58	0.4	0.45	.0007	0.1
6	SW Radiance to Flux	97.82	0.2	0.20	.0002	0.0
7	LW Radiance to Flux	237.15	0.12	0.30	.0002	0.0
8	Time Averaging SW	97.82	0.3	0.29	.0004	0.0
9	Time averaging LW	237.15	0.12	0.30	.0002	0.0
10	Reference Level SW	97.82	0.1	0.10	.0	0.0
11	Reference Level LW	237.15	0.08	0.20	.0001	0.0
12	Incoming Solar	339.96	0.0006	0.20	0.	0.0
	Total					4.2

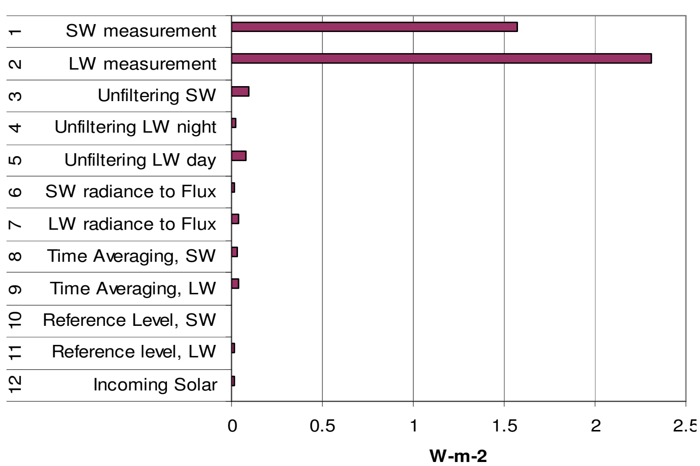

Figure 2: Effect of adjustment of each error source, W-m-2.

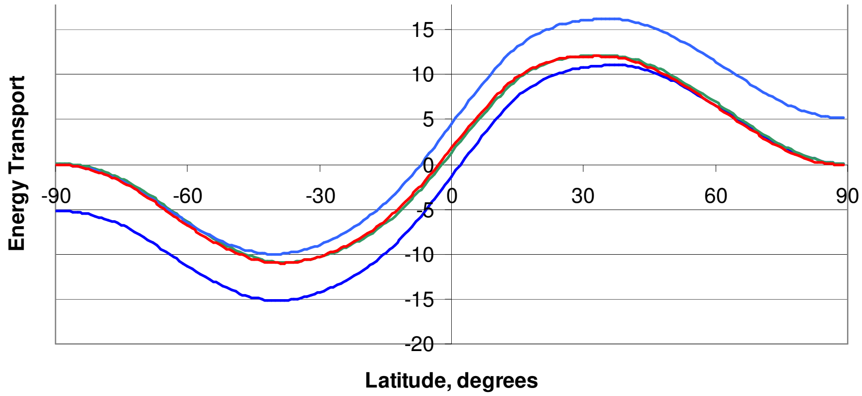

Figure 3: Annual Mean Meridianal Energy Flow.
Blue lines are based on uncorrected TOA fluxes,
starting computation from each pole.
Red line is based on corrected fluxes

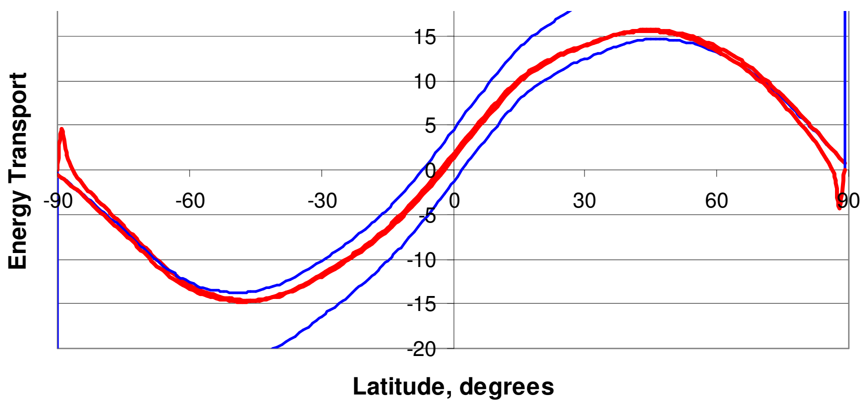

Figure 4: Annual Mean Meridianal Energy Flux Density.
Blue lines are based on uncorrected TOA fluxes,
starting computation from each pole.
Red line is based on corrected fluxes