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1. INTRODUCTION

Detecting and characterizing intermittent turbulence in stable boundary layer is important for both understanding the meteorological processes and for mitigation [1, 2]. However, these turbulence are often short in duration with high amplitudes and extremely random yet often contain highly non-stationary structures, providing a great challenge in signal processing. The traditional stationary signal processing often fails to catch the important physical phenomena.

Recently, the empirical mode decomposition (EMD) method has gained more and more popularity for analyzing nonlinear and non-stationary signals [3]. In particular, this method extracts concentrated structures without leaving the time domain, making physical phenomena identification possible. Using the so-called sifting process based on certain stopping criterion, the intrinsic mode function (IMF) of the time-series are extracted by the EMD method:

Definition: *Intrinsic Mode Function*

1. In the whole data set, the numbers of extrema and the number of zero crossing must be equal or differ by 1;
2. At any point, the mean value of the envelop defined by the local maximum and the envelope defined by the local minima must be zero.

A cubic spline is used during the sifting process and the resulting first IMF contains the short oscillation of the time-series. Our research focus on the sums of IMFs since individual components may not carry well-defined physical meaning.

In this work, we use the EMD method to characterize intermittent turbulence in stable boundary layer. A rigorous statistical approach, based on the randomized reference distribution is developed here to identify the intermittent turbulence embedded in the signals. The intermittency rate associated with various stability conditions based on the statistical

test is explored. This technique is applied to vertical velocity measurements under various stability conditions that are clearly non-stationary.

2. THRESHOLD TECHNIQUE AND INTERMITTENT TURBULENCE

The goal of our research is to develop a statistical technique for signal detection in the atmospheric boundary layer based on the EMD method. Thus, we must choose a threshold to extract intermittent turbulence from the incoherent noise. To do so, let us first consider the following definitions [5].

Definition: *Consider a discretely sampled real-valued signal*

$$x = (x_1, x_2, \dots, x_N).$$

Using the notation \hat{x} for the discrete Fourier transform of x , we can write, for each frequency component, $\hat{x}_i = A_i e^{j\theta_i}$, for $i = 1, 2, \dots, N/2$. Following standard practice, we call a signal incoherent if the θ_i are uniform $[0, 2\pi]$ and independent of each other and the A_i .

An exemplar is a signal which has the same spectrum as x but is known to be incoherent.

Definition: *The exemplar x^e of a real signal x with $\hat{x}_i = A_i e^{j\theta_i}$ has Fourier coefficients $\hat{x}_l^e = A_l e^{j\theta_l^e}$, where θ_l^e for $l = 1, 2, \dots, N/2$ are uniform $[0, 2\pi]$ and independent of A_l .*

Exemplars are easily made in the frequency domain by randomizing phase. The statistical test is based on building n exemplars x_i^e , $i = 1, 2, \dots, n$ of the original signal x .

With these definitions, we consider the test hypothesis:

$$H_0: x \text{ is incoherent.}$$

$$H_1: x \text{ contains intermittent turbulence.}$$

Based on the EMD method and the resulting IMF for our data, the test statistic $IMF4$ is obtained by

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the combinations of the first four IMF modes (see example 1). Our statistical decision is based on the following: we conclude H1 (detection at each time t), if the test statistic $IMF4$ is greater than the threshold th . We will choose th in such a way the probability of detection is maximized and the probability of the false alarm is less than or equal to a given parameter α . Hence, the probability of false alarm is given by

$$P_{FA} = P(|IMF4| > th | H_0) = \alpha.$$

We use n incoherent exemplars to determine the threshold. In this case, each exemplar based on the test statistic called $IMF4_i^e$ for $i = 1, 2, \dots, n$ is obtained by randomizing the phase of the spectrum of the test statistic used to build the randomized reference distribution. Thus, the resulting exemplars will have the same spectrum shape as the test statistic and known to be incoherent. The order statistic of the exemplars is then obtained and the threshold is defined as :

$$th = \left\{ \sup b : \left(\frac{1}{n} \sum_{i=1}^n I_{|IMF4_i^e| > b} > \alpha \right) \right\},$$

where α is the size of the test (or the probability of false alarm [4, 5]).

3. ILLUSTRATIONS

In our first example, the EMD method is applied to a vertical velocity measurement obtained from the stable boundary layer. This original signal is shown in Figure 1a, which is clearly non-stationary. In the next step, the EMD method is applied to the time-series to obtain the resulting IMF time-series. The first four IMF time-series of the original signal is shown in Figure 1b to Figure 1e. Since each of the four IMF time-series consists of interesting intermittent oscillations, the sum of the four IMF modes is used as our test statistic. Next, we obtain the threshold based on the randomized reference distribution using 100 exemplars $n = 100$, from which, the detection of the signal is obtained. Figure 2 shows the time-series of the test statistic as well as the detection based on the level of significance (or size of the test) $\alpha = 0.01$. (Note that some other test size may also be considered). Finally, based on this test size, the suggested detection to which intermittent turbulence may have occurred in the original time-series is shown in Figure 3.

We now address the issues of the stable boundary layer associated with the intermittent turbulence in our next example. In this example, 160 vertical

velocity time-series obtained from various stability conditions are considered. In this case, we first define the intermittent rate of the boundary layer as:

$$Intermittent\ Rate = \frac{Total\ Detection}{Length\ of\ the\ Signal}.$$

Based on our statistical test, the intermittent rate of all 160 time series are obtained.

Next, the time-series are divided into four class intervals based on the stability:

- Class 1 (Column 1): $Stability \leq 1$
- Class 2 (Column 2): $1 < Stability \leq 2$
- Class 3 (Column 3): $2 < Stability \leq 3$
- Class 4 (Column 4): $Stability > 3$

The Preliminary result based on the technique is shown in Figure 4. Clearly, the intermittent rate for each classes are different. More analysis is under way.

4. Acknowledgments:

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References

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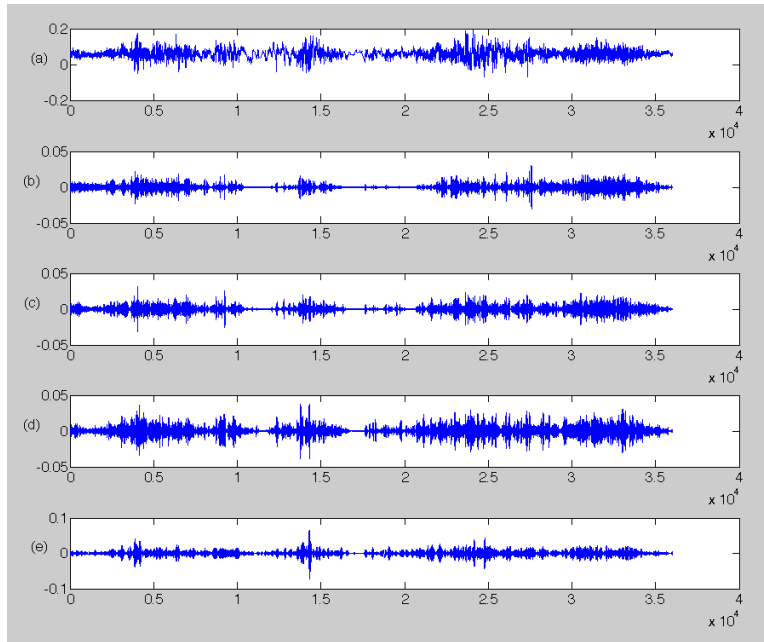


Figure 1: The original Signal b.to e. IMF 1 to IMF 4 time-series

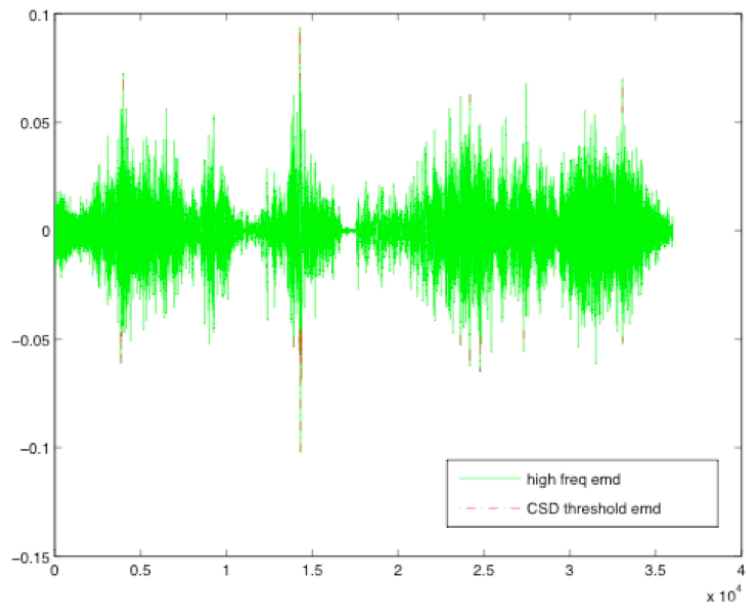


Figure 2: Test static time-series and detections

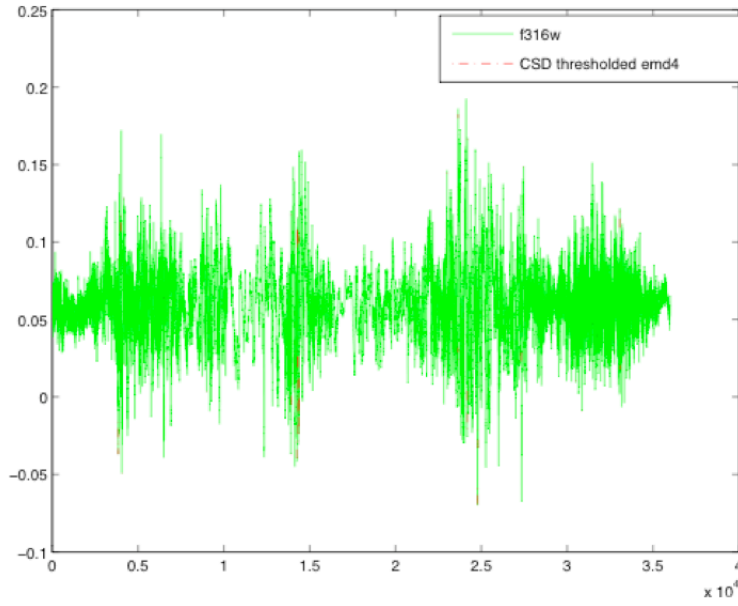


Figure 3: Original time-series and detections

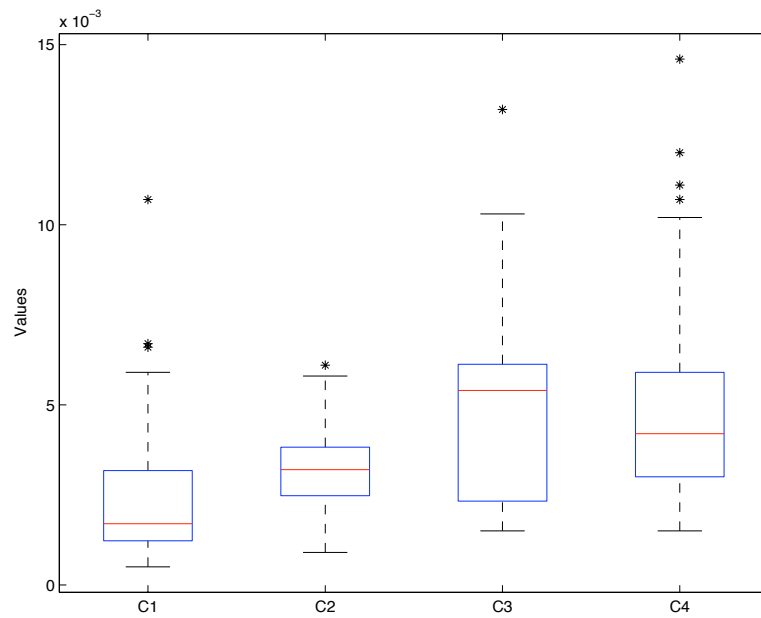


Figure 4: Stability VS. Intermittent Rate