Theoretical Threshold Reflectivity for Drizzling Clouds and Aerosol Indirect Effects

Yangang Liu[1], Bart Geerts[2], Mark Miller[1], Peter Daum[1], and Robert McGraw[1]
(1- Brookhaven National Laboratory; 2- University of Wyoming)

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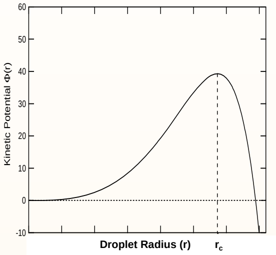
1. Introduction

Empirical studies have suggested a range of threshold radar reflectivity that separates nonprecipitating from precipitating clouds. Here we present a theory for threshold reflectivity by relating it to the threshold function associated with autoconversion process, and then seek applications.

2. General Formulation

2.1. Kinetic potential theory & critical radius



Kinetic potential theory considers rain initiation a statistical barrier-crossing process. Critical radius corresponds to the maximum kinetic potential, and depends on liquid water content and droplet concentration (McGraw & Liu 2003, 2004; Liu et al. 2004).

Figure 1 illustrates the idea of the kinetic potential theory and critical radius r_c.

2.2. Theoretical threshold function

For a bulk quantity $Y = \alpha \int r^{\delta} n(r) dr$,

Autoconversion rate $P_Y = P_{Y0} T_Y$ (1)

$P_{Y0} \equiv$ Rate Function; $T_Y \equiv$ Threshold Function.

For $n(r) = \frac{q}{r_0^q} N r^{q-1} \exp\left[-\left(\frac{r}{r_0} \right)^q \right]$, we have

$$T_Y = \frac{P_Y}{P_{Y0}} = \gamma\left(\frac{6+q}{q}, x_{cq} \right) \gamma\left(\frac{\delta+q}{q}, x_{cq} \right), \qquad (2a)$$

$$x_{cq} = \left(\frac{r_c}{r_0} \right)^q = \Gamma^{q/3} \left(\frac{3+q}{q} \right) x_c^{q/3}, \qquad (2b)$$

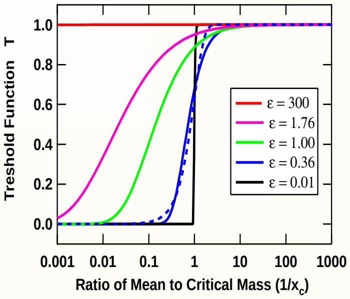
$$x_c = \left(\frac{r_c}{r_3} \right)^3 = 9.7\text{x}10^{-17} N^{3/2} L^{-2}, \qquad q \approx \varepsilon^{-1} \qquad (2c)$$

N = droplet concentration; L = liquid water content; r_3 = mean-volume radius; ε = relative dispersion of the cloud droplet size distribution; γ = incomplete Γ function

3. Number Threshold Function

Setting α =1 and δ =0 in the above general expressions, we obtain the expression for the number threshold function:

$$T_N = \gamma\left(\frac{6+q}{q}, x_{cq} \right) \gamma\left(1, x_{cq} \right) \qquad (3)$$



Threshold function is determined by two dimensionless quantities: x_c, and ε. Given x_c, the cloud-to rain transition becomes increasingly sharper when the cloud droplet size distribution narrows.

Figure 2 shows the dependence of the number threshold function on the ratio of the mean to critical mass ($1/x_c$) at different values of relative dispersion ε. The blue dashed line represents the approximate Sundqvist expression.

4. Radar Threshold Reflectivity

4.1. Theoretical expression

For a typical size distribution with q = 3 and in terms of radar reflectivity Z, Eq. (3) becomes

$$T_N = \gamma\left(3, x_c \right) \gamma\left(1, x_c \right), \qquad (4a)$$

$$x_c = 7.1\text{x}10^{-16} N^{1/2} Z^{-1}, \qquad (4b)$$

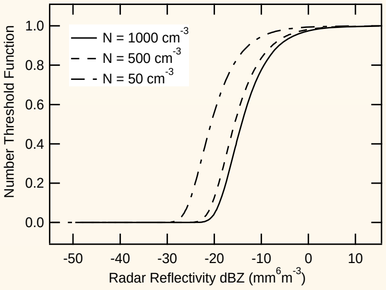
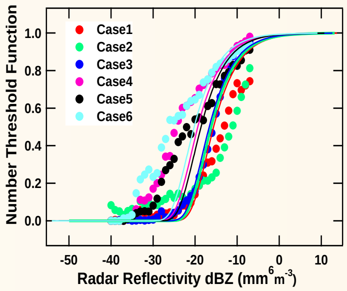


Figure 3 shows the dependence of the number threshold function on radar reflectivity at different values of droplet concentration N calculated from Eq. (4)

4.2. Comparison with observations



Measurements were made from stratiform clouds off the coast of the Northern California and Oregon using the PMS FSSP and 2D-C probes, and the University of Wyoming cloud radar onboard the same UW King Air aircraft.

Figure 4 shows the comparison of the theoretical threshold function to the observational results. The solid curves and dots are theoretical results and measurements, respectively. The colors of the theoretical curves correspond to those representing the observational results as given in the Figure Legend. Evidently, the theoretical expression compares favorably with observations, and discrepancies may be due to relative dispersion and measurement uncertainties.

4.3. Dependence of threshold reflectivity on droplet concentration and 2nd AIE

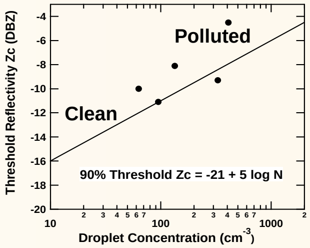
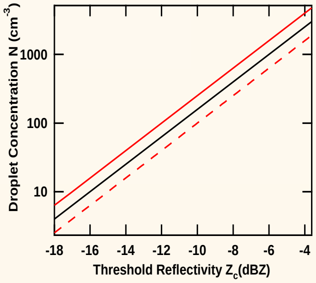


Figure 5 shows the dependence of the threshold reflectivity on droplet concentration. The black line and dots are the theoretical and observations, respectively. That threshold reflectivity is an increasing function of droplet concentration and hence aerosol loading is manifestation of the 2nd aerosol indirect effect (AIE) in radar reflectivity.

5. Retrieval of Droplet Concentration ?



$$N = 10^{\frac{Z_c + 21}{5}}$$

$$\Delta N = \frac{N}{5} \Delta Z_c$$

$$\Delta Z_c = 1 \text{dBZ}$$

$$\Delta N = 0.2 N$$

Figure 6 shows droplet concentration inferred from threshold reflectivity (solid black). The two red lines bracket the uncertainty in retrieved N given an uncertainty of Z_c = 1 dBZ.

References: Liu et al. 2007, Geophys. Res., Lett, 34, L16821; Liu et al., 2008, Geophys. Res., Lett, in press.