## A SIMPLFIED DYNAMICAL SYSTEM FOR TROPICAL CYCLONE INTENSITY EVOLUTION

Mark DeMaria NOAA/NESDIS/StAR, Fort Collins, CO

## **1. INTRODUCTION**

Tropical cyclone (TC) track forecast errors have decreased considerably over the past several decades. However, there have been only modest improvements in intensity forecasts (DeMaria et al 2007). Because of the complexity of the physical processes affecting intensity changes, statistical forecast models have remained competitive with much more complex prediction systems. For this reason, the National Hurricane Center (NHC) continues to run a hierarchy of operational intensity models that range from the simple Statistical Hurricane Intensity Prediction Scheme (SHIPS) (DeMaria et al 2005) to the fully coupled atmosphere-ocean Hurricane Weather Research and Forecast (HWRF) system (Surgi et al 2008). The HWRF model became operational in 2007, and is the follow on to the National Centers for Environmental Prediction (NCEP) version of the Geophysical Fluid Dynamics Laboratory (GFDL) coupled hurricane model (Bender et al 2007).

Over the past decade, SHIPS has generally been the most skillful of NHC's operational intensity forecast models (DeMaria et al 2007). Although gradual improvements have been made to SHIPS by including predictors from new data sources such as GOES imagery and satellite altimetry, further improvements may be limited by the underlying linear nature of the model. Also, a relatively large number of coefficients are needed to represent the intensity evolution. For example, the 2007 version of SHIPS included 21 predictors, and separate regression equations for each 6 hour forecast interval out to 120 h for a total of 420 coefficients.

In this study, a simple dynamical prediction system is introduced that can represent the basic evolution of TCs with a much smaller number of free parameters than SHIPS. The prediction system, which is based on a logistic growth equation (LGE), only requires six constants for the case when the storm center is over water. The LGE automatically bounds the solution to lie between zero and an upper bound intensity, and uses a time stepping procedure, rather than relating the intensity changes over relatively long forecast intervals to time averaged values of predictors, as in SHIPS. The details of LGE model and the method for estimating the model parameters are described in detail by DeMaria (2008), so only a brief summary is provided in section 2. Applications of this system are presented in section 3.

## 2. THE LOGISTIC GROWTH EQUATION MODEL

The basic equation for the intensity evolution is based on an analogy with a differential equation commonly used to model population growth (Thieme 2003), which can be written as

$$dV/dt = \kappa V - \beta V (V/V_{mpi})^n$$
(1)

where *V* is the maximum sustained surface wind and *t* is time. Equation (1) has four free parameters as follows:  $V_{mpi}$  is the time dependent Maximum Potential Intensity (MPI) in terms of a maximum surface wind;  $\kappa$  is the time dependent growth rate; and  $\beta$  and *n* are positive constants that determine how rapidly and how close the solution for *V* can come to  $V_{mpi}$ . Equation (1) has two families of solutions. For  $\kappa \leq 0$ , the solution decays towards zero and for  $\kappa > 0$  the solution approaches a steady state intensity  $V_s$  given by

$$V_{\rm s} = (V_{mpi})(\kappa/\beta)^{1/n} \tag{2}$$

When the LGE parameters are determined by a fit to observations,  $\beta$  is almost always greater than  $\kappa$  and  $n \approx 3$ , so the steady state solution is a fraction of  $V_{moi}$ .

The evolution of V when the storm is over land is determined from an empirical decay equation (DeMaria et al 2006) given by

$$dV/dt = -\alpha(V-V_b) \tag{3}$$

where the decay rate  $\alpha$  and the background maximum wind V<sub>b</sub> are known functions of time along the storm track. Equations (1) and (3) are referred to as the Logistic Growth Equation Model (LGEM).

The MPI along the storm track ( $V_{mpi}$ ) is determined from an empirical formula as a function of sea surface temperature (SST) plus a factor that accounts for the storm translational speed. The MPI formulas are described by DeMaria and Kaplan (1994) for the Atlantic and Whitney and Hobgood (1997) for the east Pacific, and are evaluated using the weekly SST analyses developed by Reynolds et al (2002).

Once  $V_{mpi}$  is specified, the remaining parameters in the LGE are the time dependent growth rate  $\kappa$  and the constants *n* and  $\beta$ . A preliminary version of LGEM has been run in real time at NHC since 2006, where these parameters were estimated from a multiple regression approach, using most of the same predictors as the SHIPS model (referred to as LGEM-MR). Results show that for the 2006-2007 forecasts, LGEM-MR had smaller average errors than SHIPS except at 12-24 h. Similar to SHIPS, a different set of regression equations for  $\kappa$  was

<sup>\*</sup> Corresponding author address: Mark DeMaria, NOAA/NESDIS/ORA, CIRA/Colorado State University, W. Laporte Avenue, Foothills Campus, Fort Collins, CO 80523-1375; e-mail: Mark. DeMaria@noaa.gov

developed at each forecast time from 0 to 120 h, so LGEM-MR has almost as many predictors as SHIPS.

A new version of LGEM was developed with a much smaller predictor set by assuming the growth rate is a function of just two variables, the 850-200 hPa vertical shear *S* and a thermodynamic variable *C*. The shear *S* is calculated in the same way as in SHIPS, and *C* is the 0-15 km vertical average of the vertical velocity from an entraining plume model. The variable *C* is a measure of convective instability. The winds for the vertical shear and the temperature and moisture sounding for the plume model are obtained from the NCEP global forecast system (GFS) (Yang et al 2006). With this assumption,  $\kappa$  is given by

$$\kappa = a_1 S + a_2 C + a_3 S C + b \tag{4}$$

The  $a_3$  term is included in (4) to allow for interactions between the vertical shear and convective instability. With the reduced predictor set the parameter estimation problem is reduced to the specification of the six constants  $\beta$ , *n*,  $a_1$ ,  $a_2$ ,  $a_3$  and *b*.

Applying techniques from variational data assimilation and parameter estimation (e.g. Zhu and Navon 1999), a method was developed to find the values of the above six constants that minimize the error of the fit of LGEM to the maximum winds from the NHC best track. The adjoint of LGEM allows for a straightforward calculation of the gradient of the mean square intensity error of the model with respect to these constants. A gradient descent method is then used to find the optimal values of the constants in an iterative procedure.

# **3. APPLICATIONS**

LGEM has a number of potential applications including simulation of individual storms, operational intensity forecasting, the application of (2) and (4) for model interpretation, short term intensity prediction with satellite retrievals, and the coupling of LGEM with global forecast and climate models. These applications are briefly summarized in this section.

# 3.1 Simulation of Individual Storms

As described in DeMaria (2008), the variational parameter estimate procedure was first tested by finding the values of the six constants  $\beta$ , *n*,  $a_1$ ,  $a_2$ ,  $a_3$  and *b* that provide the best fit to the simulations of the full life cycles of individual storms. This procedure may be useful for the identification of factors that affect storm intensity other than the vertical shear and thermodynamic structure of the storm environment.

As an example, Fig. 1 shows the NHC best track and the corresponding LGEM simulation that was fit to the NHC best track for Hurricane Rita from the 2005 Atlantic season. LGEM was initialized on 18 Sept at 00 UTC when Rita first became a tropical depression north of Hispaniola and was run for 192 h when the storm dissipated over the central U.S. after becoming a category 5 hurricane in the central Gulf of Mexico. For this simulation, the observed SST and vertical shear along the observed storm track was used. The fitting procedure reduced the mean absolute error of the simulated maximum wind to just 6 kt and the simulation reproduced most aspects of the observed intensity changes. This result shows that the SST, vertical shear and thermodynamic structure of the storm environment exerted strong controls on the intensity changes of Hurricane Rita. The deviations from the LGEM fit from 24 to 60 h may be related to the internal organization during the initial development of Rita and the variations near the time of maximum wind might be related to inner core processes such as eyewall replacements that would not be taken into account in the simple LGEM simulation. Thus, this fitting technique has the potential for separating internal and environmental effects on TC intensity changes.



Figure 1. The maximum winds from the 192 h fitted LGEM simulation and the corresponding values from the NHC best track for Hurricane Rita (2005). The MPI estimated from the SST and storm translational speed and the steady state solution  $V_s$  also show.

#### 3.2 Operational Intensity Prediction

As described in the Introduction, the multiple regression version of LGEM (LGEM-MR) has been run at NHC since 2006 and showed some improvement over SHIPS at 36-120 h. The next step is to run the reduced predictor set version of LGEM in real time. For this purpose, the six constants would need to be determined from a large sample of training cases and then used for the real time runs. Fitting the model to all Atlantic forecasts from 2001-2006 gave the following values of the LGEM constants:  $\beta$ =0.0256 h<sup>-1</sup>, n=2.6, a<sub>1</sub>=-0.0085 h-1, a<sub>2</sub>=.0005, a<sub>3</sub>=-0.0041 h<sup>-1</sup> and b=0.0065 h<sup>-1</sup>. As expected, the growth rate decreases with increasing shear and increases with convective instability since a<sub>1</sub> is negative and a<sub>2</sub> is positive.

LGEM will likely be run in parallel during the 2008 hurricane season for comparison with LGEM-MR and SHIPS. On possible limitation of LGEM is that it does not include persistence information. This factor is included in LGEM-MR and SHIPS by using the intensity change information from the 12 h period before the forecast time. The variational parameter estimation technique provides a systematic way to include the entire storm intensity history up to the forecast time in LGEM as described in DeMaria (2008). This technique will be evaluated as part of the tests during the 2008 hurricane season.

# 3.3 Intensity Forecast Products

The LGEM solution is primarily determined by the value of the steady state intensity as defined by (2) and the growth rate defined in (4). These equations can be used to provide products for interpretation of the LGEM solution and observed tropical cyclones. For example, Fig. 2 shows  $\kappa$  as a function of *S* and *C*, where the four constants in (4) are from the fit to the 2001-2006 Atlantic sample. The growth rate is largest for low *S* and high *C*, and becomes negative for very high values of *S*.



Figure 2. The growth rate  $\kappa$  as a function of vertical shear (S) and convective potential (C). The S and C values range from 0 to the mean plus three standard deviations.

Because the growth rate  $\kappa$  is a function of S and C, the behavior of a given storm is determined by its location in the S-C plane. Thus, it should be useful to plot the time evolution of the S and C values. The S and C values as a function of time map a curve in the S-C phase space. Figure 3 shows the phase space diagram for the full life cycle of Hurricane Rita (2005). The C values were above average for the entire lifetime of the storm. The S value started below average, so most of the storm lifecycle was in the upper left quadrant, which corresponds to the largest  $\kappa$  values in Fig. 2. As was shown in Fig. 1, Rita intensified to a category 5 hurricane. Later in the lifecycle of Rita, the trajectory moved into the upper right quadrant of the S-C plane as the shear values increased. This time period corresponds to decreasing intensity of Rita from about 96-144 h in Fig. 1. At 152 h, Rita made landfall near the

Texas-Louisiana border and the intensity decreased more rapidly.



Figure 3. The time evolution of S and C for Hurricane Rita (2005). The start of Rita at 00 UTC on 18 September is indicated by the large open diamond. The 2001-2006 sample mean values of S and C are indicated by the dashed lines.

The phase space diagram in Fig. 3 in combination with Fig. 2 provides an indication of the growth rate of the storm. For positive  $\kappa$  the maximum winds approach the steady state solution defined by (2). Thus, it is also useful to examine  $V_s$  as a function of time. Figure 1 shows  $V_s$  for the Rita case. This value can be thought of as the MPI value modified by vertical shear and instability. Rita made landfall at 152 h. The inland wind decay model (3) also has a steady state solution  $V_b$ . For storms south of 35°N,  $V_b$ =26.7 kt.

# 3.4 Short Term Forecasting with Satellite Soundings

The LGEM prediction requires an initial intensity estimate, a forecast track, an SST analysis and storm environmental soundings of temperature, moisture and wind along the storm track. As briefly described in section 3.2, the previous intensity history up to the forecast time can also be used. For the simulations and predictions described so far, all of the soundings are derived from the GFS model fields. It would also be possible to replace the initial soundings with those from satellite retrievals. The wind profiles could be obtained from feature track winds (e.g. Velden et al 2005) and temperature and moisture soundings are available from a variety of sources such as the new COSMIC radio occultation mission (Anthes et al 2008). This technique may provide improved short term intensity forecasts, and could also be used as a method to evaluate the utility of satellite wind and thermodynamic retrievals.

### 3.5 Coupling with Global Forecast Models

NHC's guidance models are usually run for all existing storms. It would also be possible to combine

LGEM with a global model prediction to make a genesis and intensity forecast. The GFS model includes a "tracker" (Marchok 2008) that uses automated procedures to detect the formation of tropical cyclones and then track them. Once a storm was identified in the model, LGEM could be applied to make an intensity prediction using the GFS model track and forecast fields to determine the necessary predictors.

An application related to genesis/intensity prediction is the use of LGEM as a downscaling procedure in climate simulations. Climate models develop circulations that resemble tropical cyclones, but are less intense than observed storms due to resolution limitations (e.g., Bengtsson, et al 2007). Once a procedure to identify tropical cyclones in the climate model was developed, LGEM could be applied to estimate the intensity given the SST and the atmospheric fields. For this application it would be preferable to replace the empirically based MPI formula that is a function only of SST with a more general formula such as that of Bister and Emanuel (1998) that also takes into account the atmospheric environment.

# 4. CONCLUSIONS AND FUTURE PLANS

A simplified dynamical system for TC intensity prediction based on a Logistic Growth Equation (LGE) was developed. The application of the LGE is based on an analogy with population dynamics, and constrains the solution to lie between zero and an upper bound. The maximum wind evolution over land is determined by an empirical inland wind decay formula and the combined water/land prediction system is referred to as the LGE Model (LGEM). The LGE contains four free parameters, which are the time dependent growth rate and MPI, and two constants that determine how quickly the intensity relaxes towards the MPI. The MPI is estimated from an empirical formula as a function of SST and storm translational speed. A version of LGEM where the remaining parameters were determined by a multiple regression method using a subset of the input to the SHIPS model was run in real time in 2006-2007 (called LGEM-MR). Results showed that the average LGEM-MR forecasts had smaller errors than SHIPS at 36-120 h.

LGEM-MR contains almost as many prediction coefficients as the SHIPS model (296 versus 420). The adjoint of LGEM can be used to provide a more general method for finding the free parameters to make the predictions as close as possible to the NHC best track intensities. Under the assumption that the growth rate is a function the vertical shear (S) and a convective instability parameter (C) determined from an entraining plume model, the adjoint parameter estimation technique was used to develop a version of LGEM with just six coefficents. It was shown that this version can very accurately simulate the life cycles of individual storms when fitted to the observed intensities. For use in a predictive mode, a single set of the six coefficents was determined by fitting all Atlantic cases from the 2001-2006 seasons.

Several potential LGEM applications were briefly described including simulation of individual storms, real time forecasting, the use of the LGEM framework for intensity forecast products, the evaluation of satellite retrievals, coupling with a global model to produce a genesis and intensity forecast, and possible use in a "downscale" mode in climate models to compensate for the lack of horizontal resolution.

The reduced predictor set version of LGEM and the method to incorporate the past history of the storm up to the forecast time will be tested on forecasts from the 2008 season. Plans are also underway to test the version with satellite soundings as input.

# ACKNOWLEDGMENTS

The views, opinions, and findings in this report are those of the author, and should not be construed as an official NOAA and or U.S. Government position, policy, or decision.

## REFERENCES

Anthes, R.A. (with 21 co-authors), 2008: The COMSIC/FORMOSAT-3 Mission early results. *Bull. Amer. Meteor. Soc.*, **89**, 313-333.

Bender, M., I. Ginis, R. Tuleya, B. Thomas, and T. Marchok, 2007: The operational GFDL coupled hurricane-ocean prediction system and a summary of its performance. *Mon. Wea. Rev.*, **135**, 3965-3989.

Bengstsson, L., K. Hodges, M. Esch, N. Keenlyside, L. Kornblueth, J.-J. Luo, and T. Yamagata, 2007: How may tropical cyclones change in a warmer climate? *Tellus*, **59A**, 539-561.

Bister, M., and K.A. Emanuel, 1998: Dissipative heating and hurricane intensity. *Meteor. Atmos. Phys.*, **50**, 233-240

DeMaria, M., 2008: A simplified dynamical system for tropical cyclone prediction. Submitted to *Mon. Wea. Rev.* 

\_\_\_\_\_, and J. Kaplan, 1994: Sea surface temperature and the maximum intensity of Atlantic tropical cyclones. *J. Climate*, **7**, 1324-1334.

\_\_\_\_\_, J.A. Knaff, and C.R. Sampson, 2007: Evaluation of Long-Term Trends in Operational Tropical Cyclone Intensity Forecasts. *Meteor. and Atmos. Phys.*, **59**, 19-28.

\_\_\_\_\_, \_\_\_\_, and J. Kaplan, 2006: On the decay of tropical cyclone winds crossing narrow landmasses, *J. Appl. Meteor.*, **45**, 491-499.

\_\_\_\_\_, M. Mainelli, L.K. Shay, J.A. Knaff and J. Kaplan, 2005: Further Improvements in the Statistical Hurricane Intensity Prediction Scheme (SHIPS). *Wea. Forecasting*, **20**, 531-543. Marchok, T., 2008: NCEP/EMC tracking of cyclogenesis in models. Available from

www.emc.ncep.noaa.gov/gmb/tpm/emchurr/tcgen/ .

Reynolds, R. W., N.A. Rayner, T.M. Smith, D.C. Stokes, and W. Wang, 2002: An improved in situ and satellite SST analysis for climate. *J. Climate*, **15**, 1609-1625.

Surgi, N., R. Tuleya, Q. Lui, V. Tallapragada and Y. Kwon, 2008 : Advancement of the HWRF for the Next Generation Prediction at NCEP's Environmental Modeling Center. 62<sup>nd</sup> Interdepartmental Hurricane Conference, March 3-7, 2008, Charleston, SC. Available from [http://www.ofcm.gov/ihc08/linking\_file\_ihc08.htm]

Thieme, H.R., 2003: Mathematics in Population Biology. Princeton University Press, 41 William St., Princeton, NJ, 08540. ISBN 0691-09290-7, 543 p.

Velden, C. S., J. Daniels, D. Stettner, D. Santek, J. Key, J. Dunion, K. Holmlund, G. Dengel, W. Bresky, and P. Menzel, 2005: Recent Innovations in Deriving Tropospheric Winds from Meteorological Satellites. *Bull. Amer. Meteor. Soc.*, 86, 205-223.

Whitney, L.D., and J. S. Hobgood, 1997: The relationship between sea surface temperature and maximum intensities of tropical cyclones in the eastern North Pacific Ocean. *J. Climate*, **10**, 2921-2930.

Yang, F., H.-L. Pan, S.K. Krueger, S. Moorthi, and S.J. Lord, 2006: Evaluation of the NCEP Global Forecast System at the ARM SGP site. *Mon. Wea. Rev.*, **134**, 3668-3690.

Zhu, Y., and I.M. Navon, 1999: Impact of parameter estimation on the performance of the FSU global spectral model using its full-physics adjoint. *Mon. Wea. Rev.*, **127**, 1497-1517.