12B.2 IMPROVEMENTS TO STOCHASTIC SIMULATION OF TROPICAL CYCLONE TRACKS

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1. MOTIVATION

Basin-wide models for the stochastic simulation of the tracks of tropical cyclones are an important tool for the investigation and assessment of hazards caused by tropical cyclones, see, for example, Emanuel (2006) and Hall (2007). Especially insurance companies are interested in the detailed results of these models. In Rumpf (2006, 2007, 2008), such a model has been developed and described in detail. The goal of the present study was to systematically improve the fit of the synthetic tracks generated by this model to the historical tracks. For this purpose, a procedure was developed to decide whether or not a synthetic track of a tropical cyclone is acceptable (and accepted) as part of the final simulation. This decision is reached in the following way:

- Those synthetic tracks whose characteristics match those of the historical tracks more closely than others will be accepted with a higher probability.
- On the other hand, those synthetic tracks whose characteristics differ more strongly from those of the historical tracks will be rejected with a higher probability.

The details of this acceptance-rejection procedure are described in Sections 2. through 4. Note that in the original model (see Rumpf (2006, 2007, 2008)), the tropical cyclone tracks are split into 6 different classes according to their shapes and locations with the goal of improved homogeneity during the simulation procedure. Therefore, all steps described in the present study are performed separately for the 6 storm classes. Some example results are shown in Section 5. From the comparison with the historical tracks and tracks simulated without the acceptance-rejection procedure, it appears that the new, refined version of the simulation procedure has lead to visible improvements in most areas.

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2. EVALUATION OF TRACKS

To prepare for the evaluation of simulated tracks, the observation window W is first split into disjoint subwindows w_i, each sized $1° \times 1°$. All historical measurements of storm tracks are then assigned to those subwindows they were taken in. Subwindows that contain only $k < 20$ historical measurements are additionally assigned the $20 - k$ measurements that were taken nearest to the center of the respective subwindow.

For each subwindow w_i, densities for various track characteristics are estimated from the at least 20 historical measurements assigned to this subwindow. A non-parametric kernel estimation method is used, where the kernel is chosen to be the Epanechnikov kernel, and the bandwidth is determined by likelihood-cross-validation (see Silverman (1986)). This procedure is performed for the following characteristics:

- the maximum wind speeds a attained by the historical storms (density in the subwindow w_i: $f^{(a)}_{w_i}(x),\ x \geq 0$),
- the directions of travel b of the historical storms (density in the subwindow w_i: $f^{(b)}_{w_i}(x),\ 0 \leq x < 360$),
- the translational speeds c of the historical storms (density in the subwindow w_i: $f^{(c)}_{w_i}(x),\ x \geq 0$),
- the fraction d of the segments of the historical storm track that have already been measured until the current point of measurement (the current segment included); i. e. the i-th point of a storm track with n points of measurement, which is the start of the i-th segment for $i = 1, \ldots, n-1$, will give a value of $\frac{i}{n-1}$ for this characteristic (density in the subwindow w_i: $f^{(d)}_{w_i}(x),\ 0 < x \leq 1$).

Note that the last points of measurement of storm tracks can not be included in this procedure, since direction and translational speed are characteristics that can only be measured along a segment, i. e. between two points.

Another important aspect of evaluating a simulated storm track is the question how frequently historical storms have passed through the same areas as the synthetic track under evaluation. To account for the natural variability among the storm tracks, points of measurement with different positions in their respective tracks need to be investigated separately. Therefore, 20 „fields of cyclone activity" are estimated in analogy to the intensity fields estimated for the simulation of the points of cyclone genesis (see Rumpf (2006, 2007)): Instead of using only the starting points for the estimation of these fields, the j-th activity field uses all points of measurement whose value of d (i. e. the fraction of the tracks measured so far) is in the interval $I_j = (\frac{j-1}{20}, \frac{j}{20}],\ j = 1, \dots, 20$. Heuristically speaking, the activity fields describe how many historical points of measurement can be found at or near a particular location, differentiated according to the stage of development of the respective storm tracks. For the estimation of this field, the density of the 2-dimensional normal distribution with expectation 0 and a variance determined by likelihood-cross-validation is used as the kernel; the estimator is also normalized to integrate to 1. The resulting activity fields will be denoted by $f^{(e)}_{I_j}(x),\ j = 1, \dots, 20$.

3. SCORING

After the preparations described in Section 2., five different scores, s^a, s^b, s^c, s^d, s^e can be calculated for each storm track. Let $X_i, i = 1, \dots, n$ be the i-th point of measurement of the storm track under evaluation, and denote

- by $a(X_i)$ the maximum wind speed attained on the segment beginning at X_i,
- by $b(X_i)$ the direction of this segment,
- by $c(X_i)$ the translational speed of the storm along this segment,
- and by $d(X_i)$ the fraction of segments of the storm track measured so far, i. e. $d(X_i) = \frac{i}{n-1},\ i = 1, \dots, n-1$.

The subwindow that the measurement X_i is located in will be denoted by $w(X_i)$ and its center by $m(w(X_i))$. Then the five different scores of the storm track are calculated as follows:

$$s^a = \frac{1}{n-1} \sum_{i=1}^{n-1} f^{(a)}_{w(X_i)}(a(X_i)) \quad (1)$$

$$s^b = \frac{1}{n-1} \sum_{i=1}^{n-1} f^{(b)}_{w(X_i)}(b(X_i)) \quad (2)$$

$$s^c = \frac{1}{n-1} \sum_{i=1}^{n-1} f^{(c)}_{w(X_i)}(c(X_i)) \quad (3)$$

$$s^d = \frac{1}{n-1} \sum_{i=1}^{n-1} f^{(d)}_{w(X_i)}(d(X_i)) \quad (4)$$

$$s^e = \frac{1}{n-1} \sum_{i=1}^{n-1} f^{(e)}_{I_j(X_i)}(m(w(X_i))), \quad (5)$$

where $I_j(X_i)$ is such that $d(X_i) \in I_j(X_i)$

Note that $f^{(e)}$ is *not* evaluated at the exact location of the measurement X_i, but only at the center $m(w(X_i))$ of its corresponding subwindow. This is necessary due to constraints on the memory usage and the runtime of the scoring procedure.

4. ACCEPTANCE-REJECTION METHOD

The scoring procedure described in Section 3. is finally used in an acceptance-rejection method that is embedded into the process of simulating tracks of tropical cyclones (described in detail in Rumpf (2006, 2007, 2008)) as follows:

- Before starting any simulation, the evaluation procedure described in detail in Section 3. has been applied to all m historical tracks of the class of storm tracks being simulated. The resulting scores are denoted by $s^a_1, \dots, s^a_m,\ s^b_1, \dots, s^b_m$, etc.
- From these scores, five densities $g^{(a)}(x), g^{(b)}(x)$, etc. are estimated by kernel estimation, again using the Epanechnikov kernel with bandwidths determined by likelihood-cross-validation. Also, the global maxima of these density estimates are determined.
- Once these preparations are complete, storm track simulation is started.
- After a complete storm track has been simulated, initially the class that this storm track belongs to is determined by the procedure described in Rumpf (2007, 2008). If this class does not match the class of tracks currently being simulated, the storm track is rejected right away and a new simulation run with the same starting point is initiated.

- Otherwise, the simulated storm track is evaluated, i. e. five different scores s_*^a, s_*^b, etc. are determined for it (see Section 3.).
- Define 5 rejection probabilities p^a, p^b, etc. by

$$p^a = 1 - \frac{g^{(a)}(s_*^a)}{\max_{x \in \mathbb{R}} g^{(a)}(x)} \tag{6}$$

  and analogous equations for p^b, p^c, p^d, p^e.

- Then, a Bernoulli experiment with success probability

$$p = \max\{p^a, p^b, p^c, p^d, p^e\} \tag{7}$$

  is performed. If this random experiment results in „success“, the storm track is rejected and a new simulation run with the same starting point is initiated. Otherwise, the storm track is accepted as part of the final simulation result.

In other words: The characteristics „wind speed“, „direction“, „translational speed“, „fraction of the track“, and „historical cyclone activity along the track“ of a synthetic tropical cyclone track are checked to see how closely they match the corresponding characteristics of the historical tracks. Better matches will result in scores more similar to those of the historical tracks, larger deviations in scores that are different. The likelihoods of the track's final scores in these five characteristics are then determined from the corresponding scores of the historical tracks. Only those tracks where all of the five scores are not „too unlikely“ have a high chance of being accepted. Conversely, an unlikely value in just one of these characteristics will lead to an increased rejection probability.

5. RESULTS

In this section, the differences in the simulation results created by the acceptance-rejection method described in this report are illustrated. For all of the 6 classes, first the historical storm tracks and a sample of storm tracks simulated without the acceptance-rejection method are shown as examples. These pictures are followed by the plots of a sample of storm tracks simulated with the acceptance-rejection method. The visible effects of the acceptance-rejection method can be described as follows:

- For the tracks of storms of **class 0**, the number of simulated tracks that appear too short in comparison with the historical tracks has been reduced. Also, the areas covered by the simulated tracks now match those covered by the historical tracks more closely.
- Simulated tracks of **class 1** storms are now less spread out in a North-South direction than in the original simulations, a feature that fits the historical tracks better. However, the total track density in the Caribbean might still be somewhat too high.
- In the original simulation, too many storm tracks of **class 2** turned northward too early on the open Atlantic. With the acceptance-rejection method, this problem appears to be greatly reduced, if not even solved. Also visibly improved are the lengths of the storm tracks, which had been mostly too short in the original simulation. The feature that historical storms of class 2 move generally parallel to the eastern coast of North America once they have turned towards the northeast is still not represented very well. Also, the total track density in the western part of the Gulf of Mexico might be a little bit too small.
- While the general shape of simulated storms of **class 3** now appears much closer to that of the historical tracks than in the original simulation, there are still visible discrepancies. For example, the lengths of the tracks are not matched very well.
- The simulation of storm tracks from **class 4** appears to have been improved with regard to the shape of the tracks, especially in the western part of the observation window. Also, the fit of the lengths of the storm tracks appears slightly better.
- The fit of simulated **class 5** storms had been very good already in the original simulation. Therefore, only minor improvements resulting from the acceptance-rejection method can be seen. For example, track lengths seem to have a closer match to the historical data, especially with regard to those storms moving into the central United States.

It appears that the acceptance-rejection method has lead to visible improvments in the fit of the simulated storm tracks to the historical storm tracks in most areas. In addition to the visual comparison, also some numerical comparisons in analogy to what was reported in Rumpf (2008) have been conducted, yielding improved fit between simulated and historical data in all areas of testing.

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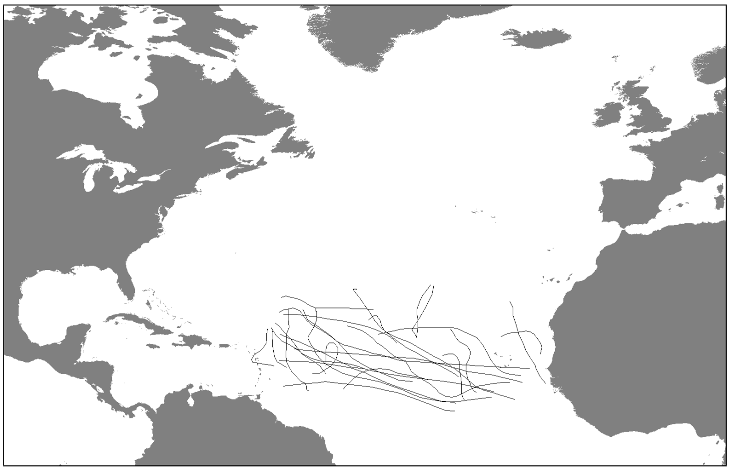

Figure 1: Historical storm tracks of class 0

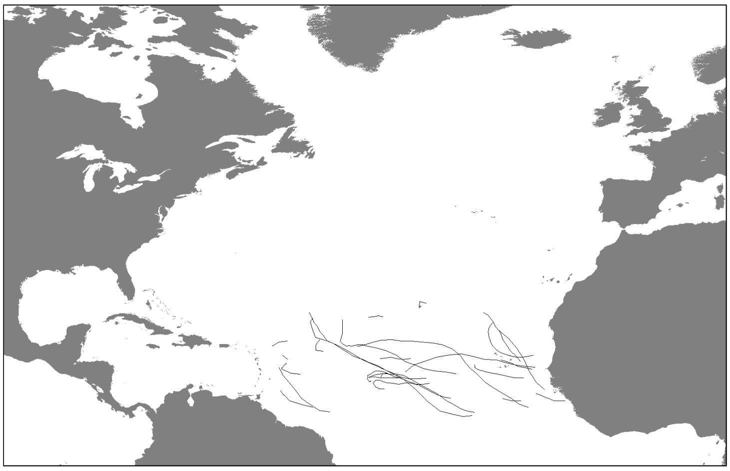

Figure 2: Sample of storm tracks of class 0, **without** acceptance-rejection method

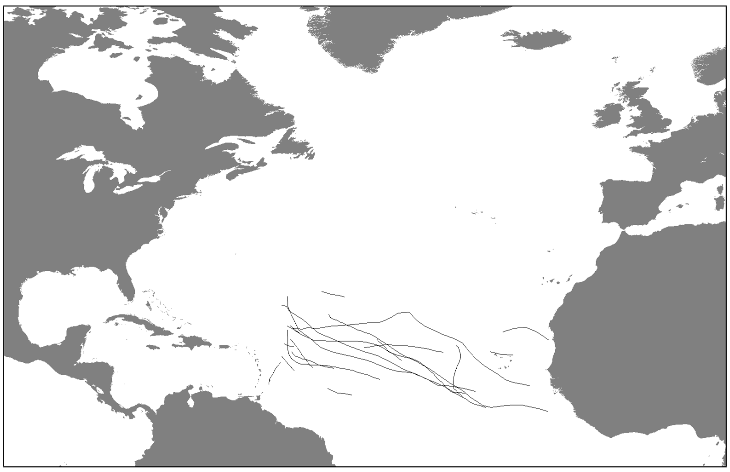

Figure 3: Sample of storm tracks of class 0, **with** acceptance-rejection method

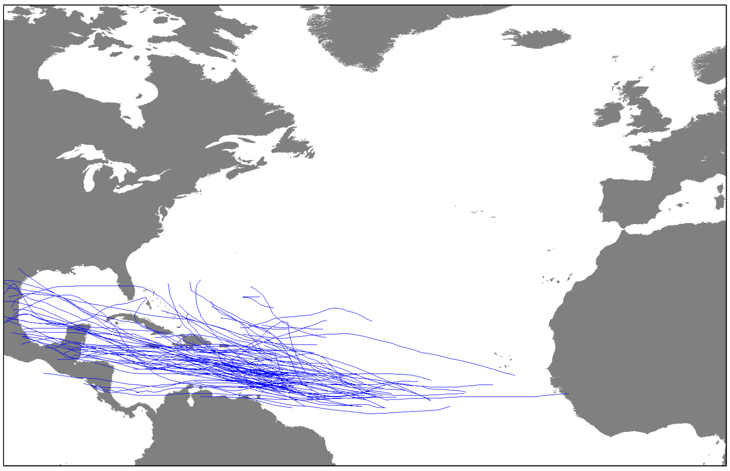

Figure 4: Historical storm tracks of class 1

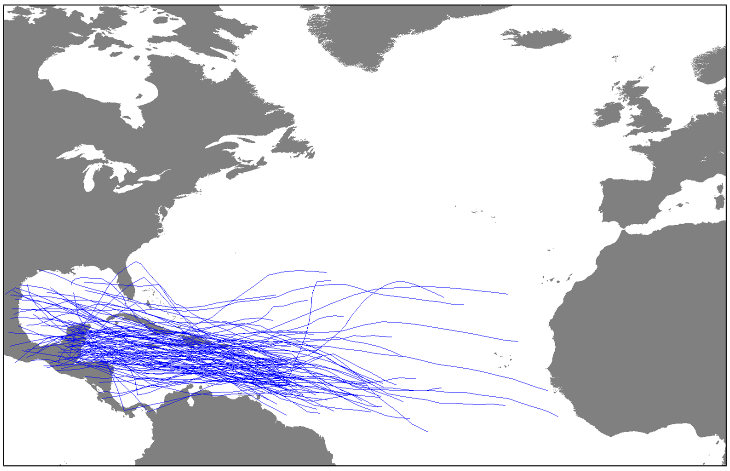

Figure 5: Sample of storm tracks of class 1, **without** acceptance-rejection method

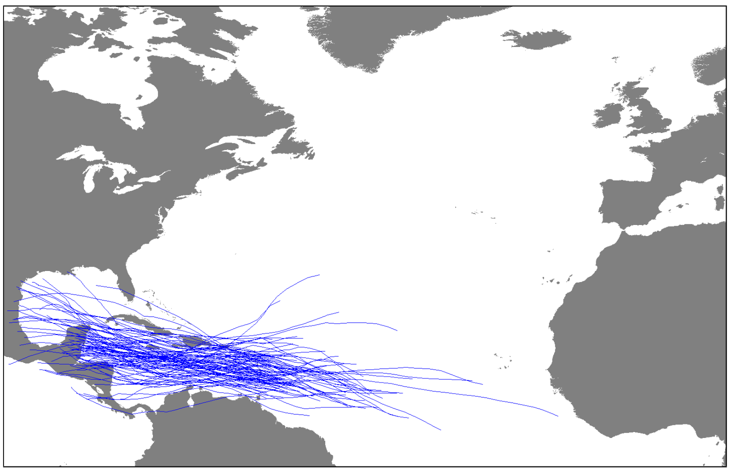

Figure 6: Sample of storm tracks of class 1, **with** acceptance-rejection method

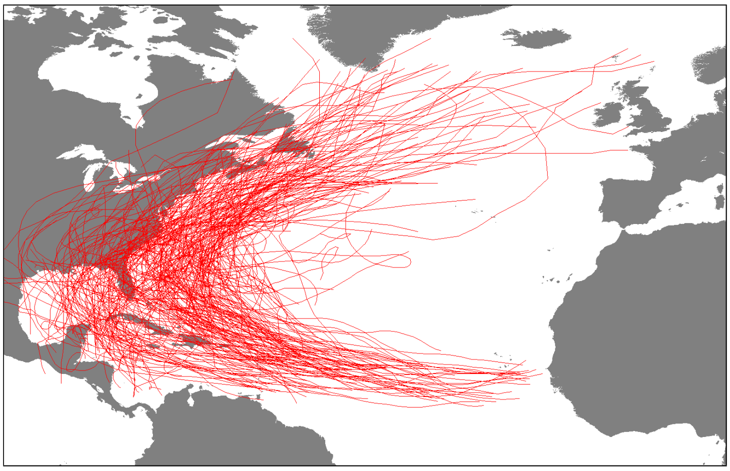

Figure 7: Historical storm tracks of class 2

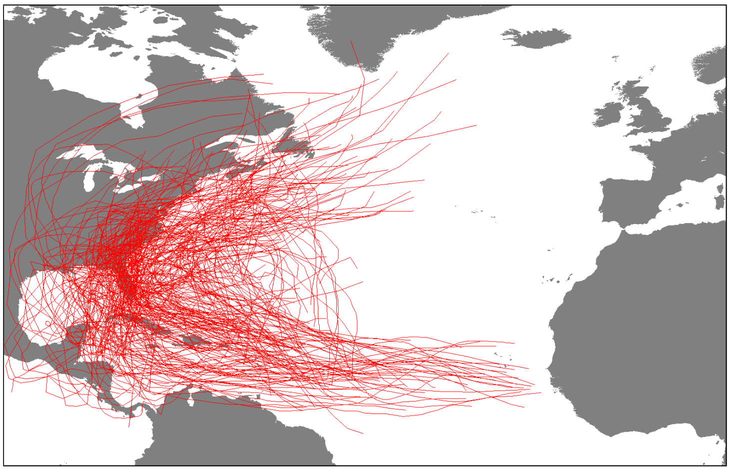

Figure 8: Sample of storm tracks of class 2, **without** acceptance-rejection method

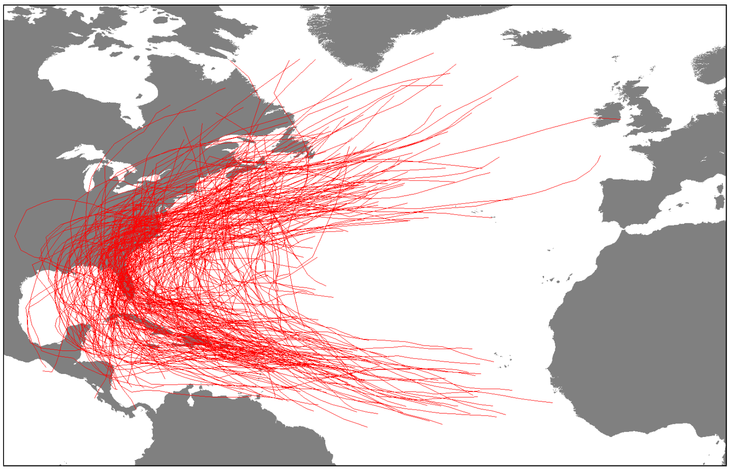

Figure 9: Sample of storm tracks of class 2, **with** acceptance-rejection method

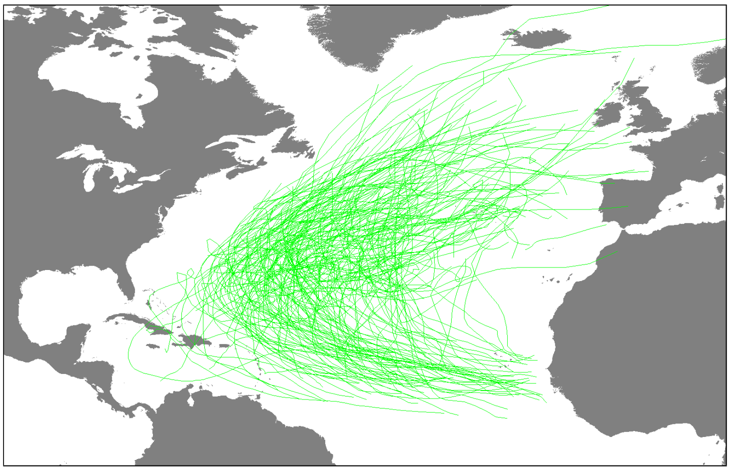

Figure 10: Historical storm tracks of class 3

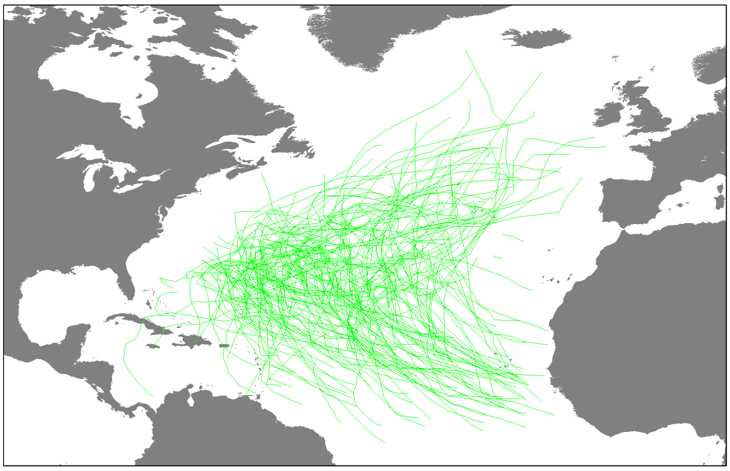

Figure 11: Sample of storm tracks of class 3, **without** acceptance-rejection method

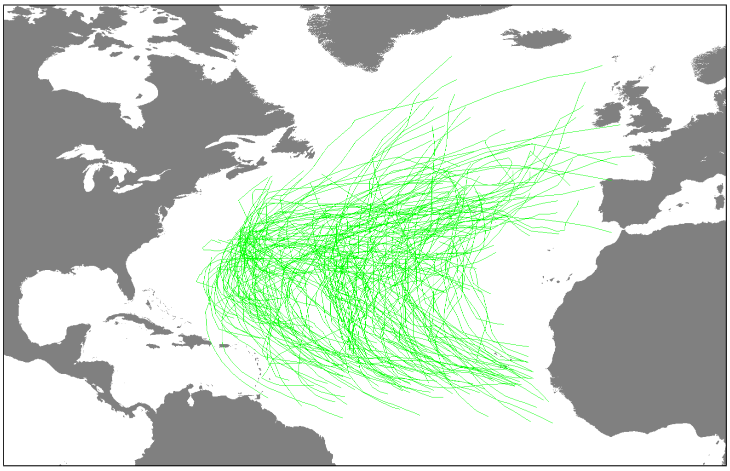

Figure 12: Sample of storm tracks of class 3, **with** acceptance-rejection method

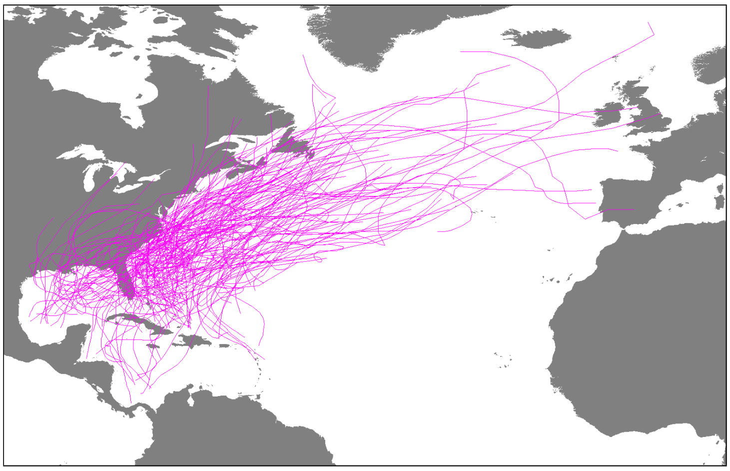

Figure 13: Historical storm tracks of class 4

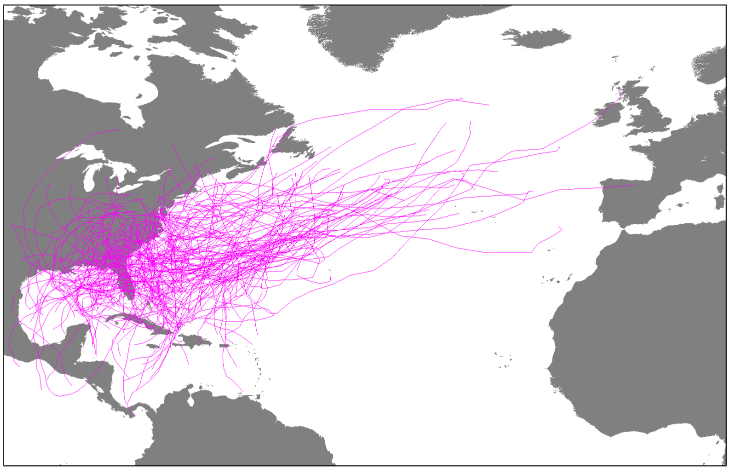

Figure 14: Sample of storm tracks of class 4, **without** acceptance-rejection method

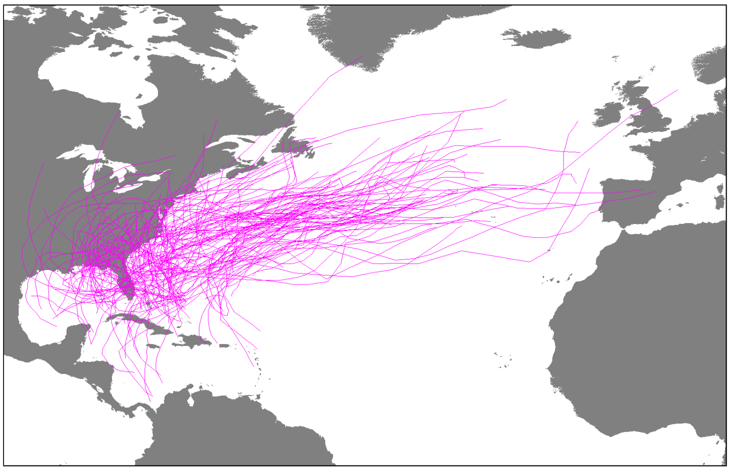

Figure 15: Sample of storm tracks of class 4, **with** acceptance-rejection method

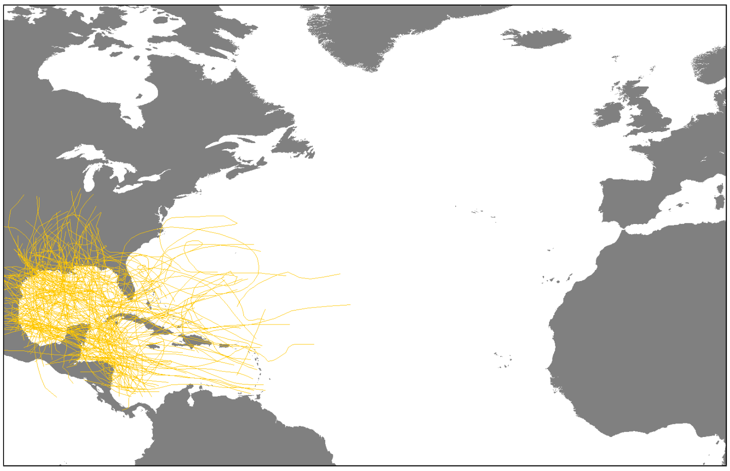

Figure 16: Historical storm tracks of class 5

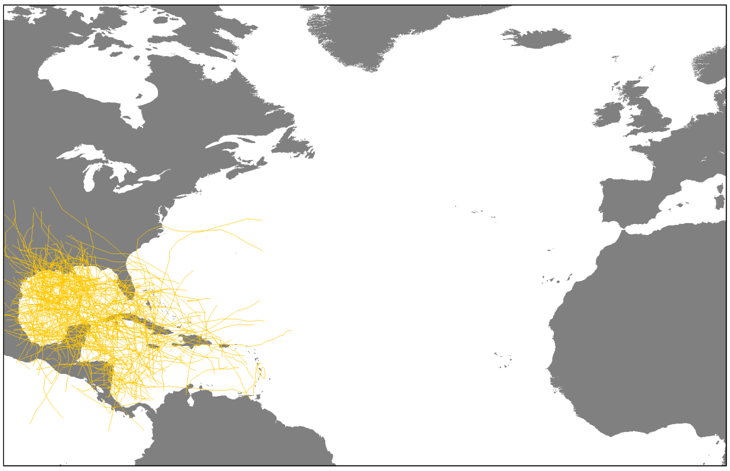

Figure 17: Sample of storm tracks of class 5, **without** acceptance-rejection method

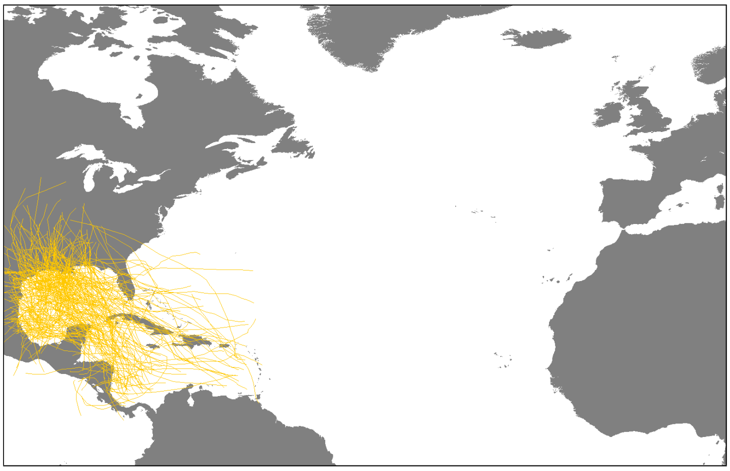

Figure 18: Sample of storm tracks of class 5, **with** acceptance-rejection method