10D.2 Methods for Introducing Vortical Hot Tower Heating in Numerical Models: Retrieving Latent Heat

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1. Introduction

Tropical cyclones (TCs) are fundamentally diabatic systems. The dominant component of diabatic heating is associated with the phase changes of water (latent heat), of which, condensation plays the largest role (Zhang et al. 2002). Therefore, the latent heat of condensation (hereafter latent heating; LH), extracted from the ocean surface and released in convective updrafts within the eyewall, is fundamental to a TC. There are many TC signatures resulting from LH including: (1) the warm-core and (2) the secondary circulation (Shapiro and Willoughby 1982). In this paper, a technique for retrieving LH from airborne Doppler radar is presented with the ultimate goal of inserting vortical hot tower (VHT) heating into an idealized model to understand rapid intensification. Although much effort has been devoted to retrieving LH in global cloud systems from space-based radiometer and radar platforms (Tao et al. 2006), little work has been done in TCs. Deriving LH depends crucially on obtaining fine-scale, three-dimensional winds (especially vertical velocity), which space-based observing systems cannot presently acquire. Thus, the LH fields presented in this study may prove useful for the validation of space-based algorithms and provide motivation for future satellite sensors.

2. Retrieval method

A form of this LH retrieval method was laid out by Satoh and Noda (2001); however significant modifications to the algorithm have been made in this study. To prove the efficacy of our retrieval method, we start by examining the budget of total precipitation mixing ratio from output of a nonhydrostatic, full-physics, cloud-resolving model simulation of Hurricane Bonnie (1998) at 2-km resolution (Braun et al. 2006; Braun 2006). Although we fully realize that the simulated TC does not replicate the actual atmosphere, the dynamically consistent nature of the model budgets allows us to assess the qualitative accuracy of our method. The simplified form of the full model equation for total precipitation mixing ratio (rain, snow and graupel) can be written following Braun (2006) as:

$$\frac{\partial q_p}{\partial t} = -\nabla \bullet \left(q_p \vec{v}\right) - \frac{\partial \left(q_p w\right)}{\partial z} + q_p \left(\nabla \bullet \vec{v} + \frac{\partial w}{\partial z}\right) + \frac{\partial \left(q_p V_t\right)}{\partial z} + Q_+ - Q_- + D + Z, \quad (1)$$

where q_p is the precipitation mixing ratio, V_t is the hydrometeor fallspeed, Q_+ and Q_- are the precipitation sources and sinks, respectively, D is the turbulent diffusion and Z is an artificial model offset for negative mixing ratios. The horizontal winds are storm-relative and all other terms have their standard meanings. Examination of each budget term on the

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convective scale (i.e. a 20 by 15 km mean centered on strong eyewall convection as well as a single grid point within an eyewall convective cell) revealed that the three-dimensional divergence, turbulent diffusion and model offset terms were small and can be safely neglected. Thus, the simplified budget equation used in this study becomes:

$$\frac{\partial q_p}{\partial t} = -\nabla \bullet \left(q_p \vec{v}\right) - \frac{\partial q_p\left(w + V_t^-\right)}{\partial z} + Q_{net}, \quad (2)$$

where the vertical flux divergence of precipitation mixing ratio and the sedimentation of precipitation mixing ratio terms are combined to yield a vertical flux divergence of Doppler velocity. In addition, the sources and sinks of precipitation are combined into a net precipitation mixing ratio source term. Although term two on the right hand side of (2) reduces the error in the budget of precipitation mixing ratio, it can only be used when the radar antenna is positioned in vertical incidence. For our study, using the NOAA P-3 airborne radars during the investigation of Hurricane Guillermo (1997), the antennae were positioned in fore-aft scanning mode and thus the vertical flux divergence term must be separated as shown in (1). By solving for Q_{net} in (2) with the model data, we are able to distinguish the saturation condition of the air (Q_{net} > 0) within a convective cell, which is required before condensation and LH can take place. Once we are able to decipher regions of saturation versus regions of non-saturation, the LH can be calculated according to the entropy form of the first law of thermodynamics:

$$\frac{D\ln\theta}{Dt} \cong \frac{-L_c}{C_p T} w \frac{\partial q_s}{\partial z}, \qquad (3)$$

where q_s is the saturation mixing ratio and all other terms have their standard meanings. With this method, the *magnitude* of the LH is determined from the vertical velocity and the vertical gradient of saturation mixing ratio multiplied by some thermodynamic constants. However, the *structure* of the LH is determined from the saturation condition of the air described above. Figure 1 shows that Q_{net} is very similar to the source of cloud water (condensation) in deep convection within the eyewall of the model TC. Braun (2006) notes that in the azimuthal mean, the source of cloud water in the eyewall is immediately soaked up by precipitation hydrometeors, which is proven here for *convective scales* in Fig. 1 (mean 20 by 15 km box centered on a convective cell as well as for a single grid point within a convective cell, not shown).

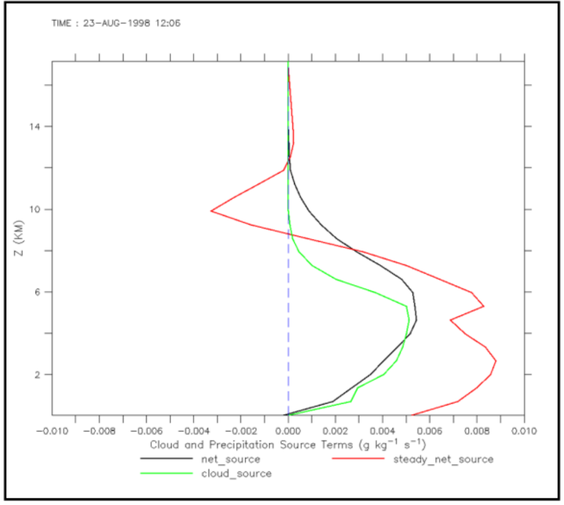

Figure 1. Mean eyewall convective profiles of the net source of precipitation mixing ratio, cloud water source and net source of precipitation mixing ratio computed according to (2) and assuming steady-state.

This means that the signal Doppler radar responds to (precipitating hydrometeors) is very close to the true condensation in convection within the eyewall of a TC.

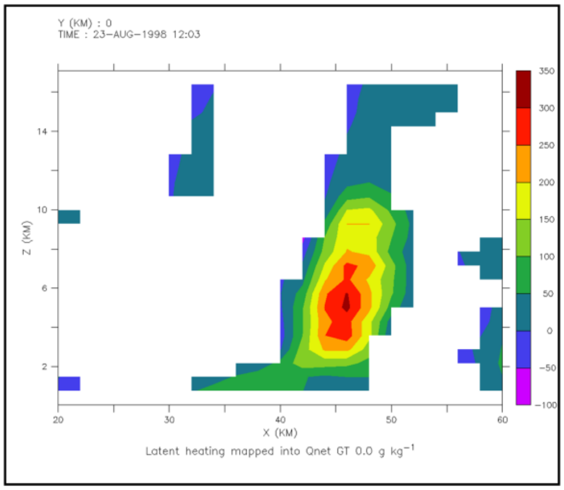

Figure 2. Latent heating computed according to (3) and mapped into saturated air determined through (2) using cloud-resolving model data. The plot shows a slice through deep convection in the eyewall of the model TC. Units of latent heating are K h^{-1}.

There are errors in this interpretation (at the boundaries of the cloud) especially if one uses the steady state assumption to solve for Q_{net} as shown in Fig. 1. However, the LH algorithm presented herein is relatively insensitive to these errors since we only care about the *condition* of saturation and not the *magnitude* of that saturation. Figure 2 shows an example of the LH computed from (3) and mapped into saturated air ($Q_{net} > 0$) for a deep convective cell in the eyewall of the model TC.

3. Doppler radar latent heating

To compute the LH from Doppler radar, knowledge of the total precipitation mixing ratio must be known. In order to derive this quantity, cloud particle data collected by NOAA P-3 aircraft in intense stages of Hurricane Katrina (2005) was analyzed. The cloud particle data was averaged over a period of 6 s to attempt to match the sampling volumes of the particle probe and Doppler radar pulses (Robert Black, personal communication). Through the cloud particle data, radar reflectivity factor (Z) and water content (WC) were computed and relationships constrained to follow the power law of Z = A*WC^B were performed. For 7,067 data points, a RMSE of 0.212 g m^{-3} was found for a relationship of Z = 402*WC$^{1.47}$ (Fig. 3). This relationship was used below the melting layer while Z = 670*WC$^{1.79}$ was used above the melting layer with linear interpolation of the two forms within the melting layer (Black 1990). Equation (2) can now be solved for Q_{net} using the P-3 derived three-dimensional winds, precipitation mixing ratio and hydrometeor fallspeed relations (Heymsfield et al. 1999). As mentioned above, the P-3 radars did not utilize vertical antenna incidence during the penetrations into Hurricane Guillermo (1997) and thus, rely on vertical integration of the anelastic mass-continuity equation which can cause error in the retrieved vertical velocity field (Reasor et al. 2008). Previous studies estimating the water budget of a TC have been unable to calculate the local tendency of precipitation mixing ratio due to inadequate Doppler radar sampling (Gamache et al. 1993).

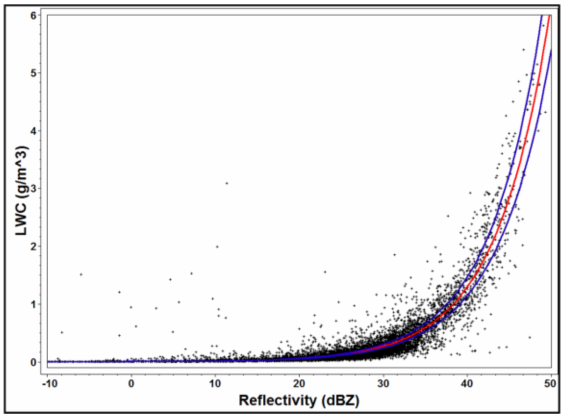

Figure 3. Scatter plot and best fit non-linear model (red curve) using cloud particle data below the melting level gathered during Hurricane Katrina (2005). The region between the two blue curves represents the 95 % confidence interval.

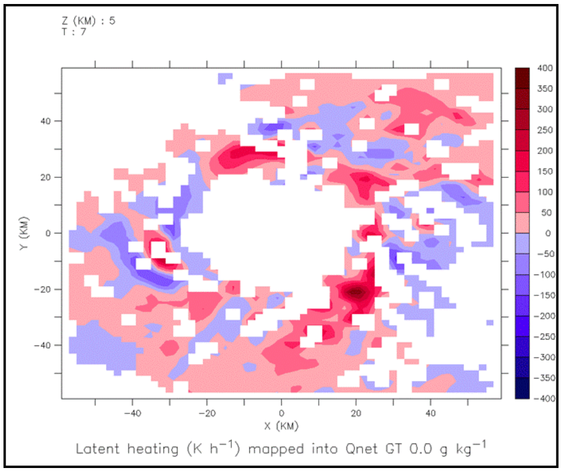

Figure 4. P-3 Doppler radar retrieved latent heating at a height of 5 km in Hurricane Guillermo (1997).

This study is unique in that composite Doppler radar sampling of Hurricane Guillermo was completed on average every 34 minutes for a period of ~5 hours allowing estimation of the local tendency term (Reasor et al. 2008). However, it was found that using a 34 minute Δt in the local tendency term added no more information to the precipitation mixing ratio budget than using a steady-state assumption. In fact, the local tendency term evaluated with 34 minute updates was an order of magnitude smaller than Q_{net}. In addition, by evaluating the local tendency term, some information is lost from the budget due to missing values found (a consequence of the editing procedures) on either side of the time derivative. Thus, at least for examination of convective cloud scales, time sampling finer than 34 minutes is required to add information to the budget.

To compute the vertical gradient of saturation mixing ratio shown in (3), knowledge of the thermodynamic structure of the convection the radar is sampling is required, which is very difficult to obtain. To approximate the thermodynamic structure, a mean dropsonde representative of deep convection in the eyewall of TCs will be used in the continuation of this study. However, for the purposes of the present work, a sounding taken from the eyewall of the model simulation was used as input to the thermodynamic variables in (3) with the vertical velocity taken from the Doppler radar analyses. Figure 4 displays an example of the final LH field shown at a height of 5 km. This snapshot was taken during a period of rapid intensification (12 hPa drop in 6 h) of Guillermo where deep, rotating convective cells were observed in the eyewall (Reasor et al. 2008). Large values of LH are coincident with these cells with values greater than 350 K h^{-1}. Thermodynamic sensitivity in the LH calculation is estimated by using a different eyewall sounding and a sounding taken from the outer portion (~280 km radius) of the simulated TC (Fig. 5). Figure 5 shows that the LH retrieval method does not appear to be appreciably sensitive to the thermodynamics although differences in the peak heating of ~40-50 K h^{-1} are found around 5 km height. Using any one of the soundings shown in Fig. 5 yields a very

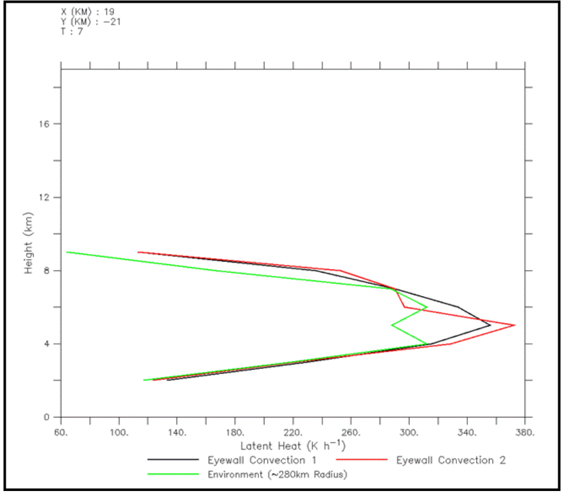

Figure 5. P-3 Doppler radar retrieved latent heating profiles within a deep convective cell in Hurricane Guillermo (1997). The sensitivity to thermodynamic information is shown by using two different eyewall soundings and an outer environmental sounding.

similar structure to the heating, which means that inserting a mean eyewall dropsonde for the thermodynamics in (3) may not be a bad assumption. Sensitivity to the vertical velocity is much greater and is the most important parameter in estimation of LH.

4. Conclusions

A novel method for retrieval of latent heating from Doppler radar is presented. The method relies on use of the precipitation water continuity equation to determine the saturation condition of the air within a radar observed cloud feature. The net source of precipitation mixing ratio is backed out from the water continuity equation, which is shown to be very close to the actual condensation within a cloud-resolving model framework. The local tendency of precipitation mixing ratio estimated from 34 minute sampling of Hurricane Guillermo (1997) is found to add no more information to the budget than using steady-state and can result in loss of data. Once the saturation condition of the air is found, the entropy form of the first law of thermodynamics is employed to compute the LH. Thus, the *magnitude* of the LH is a function of the vertical velocity and thermodynamic information inside the cloud while the *structure* of the LH is determined from the net source of precipitation mixing ratio. The algorithm is insensitive to errors in the budget terms and estimation of water content from radar, provided that the signal (Q_{net}) is substantial. The algorithm is most sensitive to vertical velocity with only small differences noted for a wide variety of thermodynamics. The ultra fine-scale vertical velocity from the ER-2 Doppler radar (EDOP) may provide interesting insights into estimation of LH although the water budget equations cannot be closed with this radar.

Examination of a deep, rotating convective cell during the rapid intensification of Hurricane Guillermo (1997) revealed peak latent heating at 5 km altitude of over 350 K h^{-1}. For future work, the latent heating field of a composite VHT will be inserted into an idealized model and evolved in time to reflect the evolution of diabatic heating found in the model simulation. It is our hope that this observationally motivated simulation will uncover new findings on the rapid intensification of TCs.

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