10B.2 Boundary layer high order concentration statistics

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Abstract

The higher order concentration statistics of a passive tracer in a turbulent Boundary Layer are evaluated by a Fluctuating Plume model. A non stationary Lagrangian Stochastic Model is used to determine the joint evolution of the barycenter of a cloud and a time-dependent filter assures that only the correct portion of the turbulent kinetic energy is considered. The instantaneous dispersion of the plume around its barycenter is parametrized following the inertial range scaling and the concentration of the passive tracer is determined following Gifford (1959). The model has been applied to two different Boundary Layers: a convective boundary layer and a plant canopy. Concentration Fluctuations, Skewness and Kurtosis are presented, in particular their dependence from the concentration PDF relative to the Plume barycenter position inside the Boundary Layer is investigated. Two different PDFs are taken in to account a Gaussian PDF and a Skewed PDF. The comparison is shown and discussed.

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1. Introduction

The dispersion of a passive tracer within the atmospheric boundary layer can be strongly affected by the skewed, non-Gaussian turbulence. The mean concentration fields can be accurately determined by one-particle Lagrangian stochastic models, but they do not provide any information on the concentration fluctuations and the higher order statistics. In Gaussian, homogeneous turbulence the concentration moments can be evaluated by Gifford (1959) meandering plume analytical model. Gifford (1959) suggested to decompose the plume in the meandering of the plume barycenter and the relative-diffusion around it. As long as the scalar fluctuations are produced by the solid movements of the cloud, Gifford (1959) model well predicts the concentration field if the plume meandering is the main cause of the fluctuations, but it completely ignores the effects of the small vortices inside the plume and this causes the model to fail in the far field where the fluctuations are mainly internal and the plume meandering disappears. Yee and Wilson (2000) improved Gifford (1959) model specifying the in-plume fluctuations in terms of a gamma probability density function. Luhar et al. (2000) moved to less idealized turbulent fields applying the meandering plume approach in a convective boundary layer (CBL). In their approach the the probability density function (PDF) of the centroid of the passive tracer plume was evaluated by a Lagrangian stochastic model. Once known the centroid PDF is known the concentration fields can be evaluated parametrizing the cloud dispersion around it. Following these ideas, Franzese (2003) developed a fluctuating plume model where the equations of the centroid motion are derived from the single particle stochastic equations filtering out the turbulent kinetic energy.

Although the motion of the plume centroid can be very well simulated by a Lagrangian stochastic model, the concentration field evaluation is strongly affected by the choice of the PDF of the single particle vertical position relative to the centroid position (Dosio and de Arellano 2006). Luhar et al. (2000) used a PDF depending on the Skewness of plume barycenter, while Franzese (2003) preferred a Gaussian PDF, whose Skewness is only due to the multiple reflections at the boundaries. Later on Dosio and de Arellano (2006) used LES simulations to show that the relative mean particle position PDF must be skewed in order to correctly predict the mean concentrations, especially close to the ground.

In this work we developed a fluctuating plume model (Franzese 2003; Mortarini et al. 2008) to evaluate the concentration statistics in two different turbulent conditions: a CBL (Franzese et al. 1999; Willis and Deardorff 1976, 1978, 1981), and a boundary layer generated by a simulated vegetal canopy (Raupach et al. 1986; Legg et al. 1986). The role of the relative concentration Skewness is investigated and a comparison of the concentration statistics evaluated with a Gaussian PDF and with a skewed PDF are shown.

2. Fluctuating plume model

The basic idea of the fluctuating plume model is to divide the concentration evaluation in two different parts: first we simulate the trajectory of the plume centroid with a Lagrangian stochastic model in a fixed coordinate system and then we parametrize concentration PDF in the relative coordinate system whose origin is located on the plume centroid position. In other words, the absolute dispersion is divided in two components: the meandering of the instantaneous cloud and the relative diffusion of the cloud around its barycenter. The concentration moments in a fixed reference frame are defined as:

$$\langle c^n(x,y,z)\rangle = \int_0^\infty c^n p(c;x,y,z)dc \tag{1}$$

where c is the instantaneous concentration, $p(c;x,y,z)$ is the PDF of the concentration in the fixed system, x is the downwind distance, y the crosswind direction and z is the vertical coordinate. For the sake of simplicity we can assume that the concentration field is independent of the crosswind direction, hence Eq. (1) becomes:

$$\langle c^n(x,z)\rangle = \int_0^\infty c^n p(c;x,z)dc \tag{2}$$

Gifford (1959) hypothesis for the concentration PDF can be written as:

$$p(c;x,z) = \int p_{cr}(c|x,z,z_m)p_m(x,z_m)dz_m \tag{3}$$

where $p_m(x,z_m)$ is the PDF of the position z_m of the plume centroid at a distance x from the source and $p_{cr}(c|x,z,z_m)$ is the concentration PDF relative to z_m, hence $p_{cr}(c|x,z,z_m)$ is the PDF of concentration in the reference frame moving with the cloud centroid, conditional to its location downwind.

After defining the concentration statistic at x relative to z_m, $\langle c_r^n(x,z,z_m)\rangle$, as:

$$\langle c_r^n(x,z,z_m)\rangle = \int_0^\infty c^n p_{cr}(c|x,z,z_m)dc \tag{4}$$

and substituting Eqs. (3) in Eq. (2) we write (Franzese 2003):

$$\langle c^n(x,z)\rangle = \int \langle c_r^n\rangle p_m(x,z_m)dz_m \tag{5}$$

where $\langle c_r^n\rangle$ is expressed by Eq. (4). Equation (5) summarize the principle of the fluctuating plume models, where the concentration field is derived combining the relative concentration statistics $\langle c_r^n\rangle$ and the meandering of the plume centroid, $p_m(x,z_m)$. In this work the centroid velocity and position will be directly modeled by Lagrangian stochastic equations to obtain the PDF, $p_m(x,z_m)$, of the barycenter vertical location, while for relative dispersion statistics $\langle c_r^n\rangle$ two different parametrization of concentration relative to the centroid will be taken into account: a Gaussian PDF and a Gamma distribution.

3. A Lagrangian model for the centroid PDF

Following Thomson (1987) and Franzese (2003) we consider the joint evolution of centroid velocity and position as a Markov process. Hence the following stochastic differential equations can model the trajectories of the plume centroid :

$$dx_m = U(z_m)dt \tag{6}$$
$$dw_m = a_m(t, w_m, z_m)dt + b_m(t, z_m)dW(t) \tag{7}$$
$$dz_m = w_m dt \tag{8}$$

where a_m is the deterministic acceleration term, b_m is a diffusion coefficient, U is the single particle mean wind speed and $dW(t)$ represent the increment of a Wiener process with zero mean and variance dt. Equation (6) is based on the hypotheses that the velocity fluctuations are negligible in the along-wind direction x, and the centroid mean wind speed is equal to the along-wind mean speed.

The stochastic term b_m is derived according to: $b_m = \sqrt{2\langle w_m^2\rangle/T_m}$, where T_m is the analogous of the single-particle Lagrangian time scale for the barycenter.

The drift term a_m is derived as in Franzese (2003) assuming a quadratic functional form:

$$a(w_m, z_m, t) = \alpha_m(z_m, t)w_m^2 + \beta_m(z_m, t)w_m + \gamma_m(z_m, t) \tag{9}$$

with α_m, β_m and γ_m determined using the Fokker-Planck equation associated to the stochastic process (7) and (8):

$$\begin{aligned}
\alpha_m(z_m, t) &= \frac{(1/3)(\partial\langle w_m^3\rangle/\partial t + \langle w_m^4\rangle/\partial z_m)}{\langle w_m^4\rangle - \langle w_m^3\rangle^2/\langle w_m^2\rangle - \langle w_m^2\rangle^2} \\
&- \frac{\langle w_m^3\rangle/(2\langle w_m^2\rangle)\left[\partial\langle w_m^3\rangle/\partial z_m - 2\langle w_m^2\rangle/T_m\right]}{\langle w_m^4\rangle - \langle w_m^3\rangle^2/\langle w_m^2\rangle - \langle w_m^2\rangle^2} \\
&+ \frac{\langle w_m^2\rangle\partial\langle w_m^2\rangle/\partial z_m}{\langle w_m^4\rangle - \langle w_m^3\rangle^2/\langle w_m^2\rangle - \langle w_m^2\rangle^2} \\
\beta_m(z_m, t) &= \frac{1}{2\langle w_m^2\rangle}\left[\frac{\partial\langle w_m^2\rangle}{\partial t} + \frac{\partial\langle w_m^3\rangle}{\partial z_m} - 2\langle w_m^3\rangle\alpha_m\right] + \\
&- \frac{1}{T_m} \\
\gamma_m(z_m, t) &= \frac{\partial\langle w_m^2\rangle}{\partial z_m} - \langle w_m^2\rangle\alpha_m
\end{aligned} \tag{10}$$

We derive the energy of the meandering centroid $\langle w_m^2\rangle$ (and the third and fourth turbulent moment of the centroid vertical velocity) by filtering the turbulent kinetic energy $\langle w^2\rangle$ using:

$$\langle w_m^n\rangle = \langle w^n\rangle\left[1 - \left(\frac{d^2}{d^2 + z_i^2}\right)^{\frac{1}{3}}\right]^{n/2} \tag{11}$$

for $n = 2$, 3 and 4, where z_i is the boundary layer depth and d is proportional to the two-particle relative dispersion $\langle r^2\rangle$ (Franzese and Cassiani 2007) and is parameterized as

$$d^2 = \tfrac{\alpha_d}{3}\langle r^2\rangle = \tfrac{\alpha_d}{3}g\varepsilon\,(t_s + t)^3 \tag{12}$$

where $t_s = \left[\sigma_0^2/(g\varepsilon)\right]^{1/3}$ is related to the finite source size σ_0, g is the Richardson constant and α_d is a parameter assuring that the standard deviation of the barycenter vertical position is null when the plume is well mixed. For the simulation on the vegetal canopy we used $g = .06$, while the Kolmogorov constant for the Lagrangian velocity structure function was $C_0 = 2$ and $\alpha_d = 1$. While for the CBL simulations we preferred $C_0 = 3$, $g = 1.4$ (Franzese 2003) and $\alpha_d = 0.16$ (Franzese 2003). This discrepancy in the g values is not surprising, considering the dumping effect of the canopy on dispersion and the highly inhomogeneity of the canopy itself. We applied perfect reflection to the centroid vertical velocity and position when its distance from the lower boundary is less than $d/2$. This choice was not only made for the sake of simplicity but also to isolate the effect of the skewed turbulence on the concentration statistics on the PDF of the concentration relative to the centroid position.

4. Parameterization of relative concentration PDF

The fluctuating plume model constitutes of two different components: the Lagrangian model, which is used to evaluate the evolution of the barycenter PDF on the horizontal coordinate and the parameterization of relative concentration PDF. Once the centroid PDF is known, we have to give an analytical expression to the concentrations statistics relative to the centroid. Substituting the two different contributes of the concentration PDF into Equation (5), it is possible to have all the moments of the concentration PDF.

In literature it is possible to find different form for $p_{cr}\left(c\left|x,z,z_m\right.\right)$, starting from Hanna (1984), who used an exponential PDF, through Franzese (2003) lognormal PDF, ending with Yee and Wilson (2000) and Luhar et al. (2000) who assumed a Gamma PDF. Dosio and de Arellano (2006) LES simulations showed that the Gamma form is probably the most suitable. Even if this result was obtained for a CBL, we will consider a Gamma PDF for the canopy simulations as well:

$$p_{cr}\left(c\left|x,z,z_m\right.\right)=\frac{\lambda^{\lambda}}{\langle c_r\rangle\Gamma(\lambda)}\left(\frac{c}{\langle c_r\rangle}\right)^{\lambda-1}e^{-\frac{\lambda c}{\langle c_r\rangle}} \quad (13)$$

where $\Gamma(\lambda)$ is the Gamma function, i_{cr} is the relative concentration fluctuation intensity and λ is equal to $1/i_{cr}^2$. Substituting Eq. (13) into Eq. (4) it is possible to have a relation between $\langle c_r^n\rangle$ and $\langle c_r\rangle$:

$$\langle c_r^n\rangle=\frac{\Gamma(n+\lambda)}{\lambda^n\Gamma(\lambda)}\langle c_r\rangle^n \quad (14)$$

Therefore to calculate the concentration field it is necessary to have an explicit form for $\langle c_r\rangle$:

$$\langle c_r\rangle=\frac{Q}{U(z_m)}p_{zr}(x,z,z_m) \quad (15)$$

where Q is the source strength and p_{zr} is the vertical PDF of mean particle position relative to z_m. Combining Eqs. (5), (14) and (15) we can determine the whole concentration field, $c(x,z)$, as:

$$\langle c^n\left(x,z\right)\rangle=\left(\frac{Q}{U}\right)^n\frac{\Gamma\left(n+\lambda\right)}{\lambda^n\Gamma\left(\lambda\right)} \int_0^H p_{zr}^n\left(x,z,z_m\right)p_m\left(x,z_m\right)dz_m \quad (16)$$

Eq. (16) shows that after all our assumptions, $c(x,z)$ only depends on i_{cr} (through λ), $p_m\left(x,z_m\right)$ and $p_{zr}^n\left(x,z,z_m\right)$. We will consider two different shapes for p_{zr}: a simple reflected Gaussian PDF (Franzese 2003) and a skewed PDF obtained as the sum of two reflected Gaussian PDFs (Luhar et al. 2000; Dosio and de Arellano 2006). Our aim is demonstrating that the Skewness produced by the boundary reflections, both in the Lagrangian model and in p_{zr}, it is not sufficient to explain the behavior of the tracer dispersion, especially close to the ground where we expect that not only the vertical position PDF, but also the PDF of the vertical position relative to z_m is skewed.

The Gaussian PDF has the form (Franzese 2003):

$$p_{zr}(x,z,z_m)=\frac{1}{\sqrt{2\pi}\sigma_{zr}}\sum_{n=-N}^{N}\left[e^{-\frac{(z-z_m+2nz_i)^2}{2\sigma_{zr}^2}}+ +e^{-\frac{(-z-z_m+2nz_i)^2}{2\sigma_{zr}^2}}\right] \quad (17)$$

The parametrization chosen for the vertical dispersion coefficient σ_{zr}^2 is:

$$\sigma_{zr}^2=\frac{g_z\varepsilon(t_s+t)^3}{[1+(g_z\varepsilon t^3/\alpha)^{2/3}]^{3/2}} \quad (18)$$

where g_z is the one-dimensional Richardson constant ($g_z=g/6$) and ε is the dissipation of the turbulent kinetic energy. Equation (18) represents the inertial range behavior, $\sigma_{zr}^2=g_z\varepsilon(t_s+t)^3$, for small times and accounts for the boundaries effect that reduces the vertical spreading for large times.

Following Luhar et al. (2000) and Dosio and de Arellano (2006) we chose a skewed PDF of the form:

$$p_{zr}(x,z,z_m)=\sum_{j=1}^{2}\sum_{n=-N}^{N}\frac{a_j}{\sqrt{2\pi}\sigma_j}\left[e^{-\frac{(z-z_m+2nz_i-\bar{z}_j)^2}{2\sigma_j^2}}+ +e^{-\frac{(-z-z_m+2nz_i-\bar{z}_j)^2}{2\sigma_j^2}}\right] \quad (19)$$

where:

$$\begin{aligned}
\bar{z}_1 &=S_{zr}f\sigma_1/\left|S_{zr}\right|\\
\bar{z}_2 &=-\bar{z}_1\sigma_2/\sigma_1\\
\sigma_1 &=\sigma_{zr}\sqrt{a_2/\left[a_1\left(1+f^2\right)\right]}\\
\sigma_2 &=\sigma_1 a_1/a_2\\
a_1 &=\{1-[r/(4+r)]^{1/2}\}/2\\
a_2 &=1-a_1\\
r &=(1+f^2)^3S_{zr}^2/[(3+f^2)^2f^2]\\
f &=\tfrac{2}{3}\left|S_{zr}\right|^{1/3}
\end{aligned} \quad (20)$$

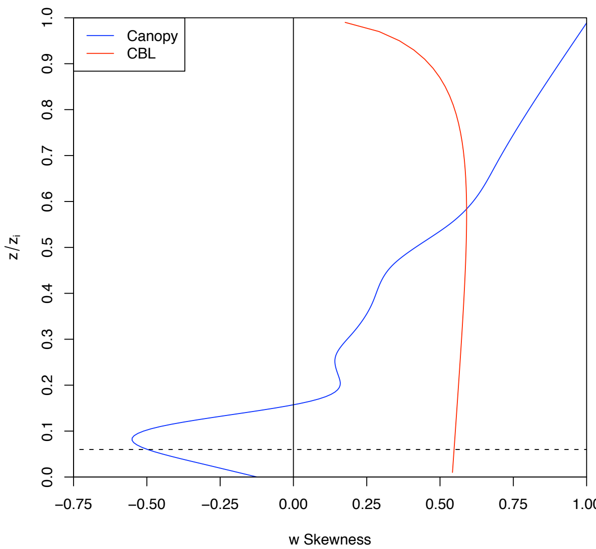


Figure 1: Vertical profile of the vertical velocity Skewness. The blue line represents a spline fit of Raupach et al. (1986) data, while the red line is derived from the analytical expressions of $\langle w^2 \rangle$ and $\langle w^3 \rangle$ in Franzese (2003). The dotted line indicates the canopy height.

and the Skewness S_{zr}, which is given by

$$S_{zr} = \frac{\langle z - \langle z \rangle \rangle^3 - \langle z_m - \langle z \rangle \rangle^3}{\sigma_{zr}^3} \tag{21}$$

is the Skewness of the relative vertical position z_r. To evaluate S_{zr} Luhar et al. (2000) assumed that the Skewness of the single particle S_z was equal to the Skewness of the barycenter, however Dosio and de Arellano (2006) showed that this approximation is valid only close to the source when dispersion is dominated by the meandering, but it is not true elsewhere, hence we decided to parametrize $S_{zr}(x)$ in order to improve the comparison with the measured data.

5. Data and Parameters

We applied the fluctuating plume model to two different turbulent conditions: a Boundary Layer generated by convection and a Boundary Layer developed above a vegetal plant canopy. In the first case (CBL) the model was set to simulate the water tank dispersion experiments of Willis and Deardorff (1976, 1978, 1981) for three different source heights. The turbulent moment used as input of the Lagrangian stochastic model were derived from least-squares fit of several experimental data as described in Franzese et al. (1999) and Franzese (2003). For the canopy case a dispersion experiments from a line source (Legg et al. 1986) was considered. The exact experiment displacement is described in Raupach et al. (1986); Coppin et al. (1986); Legg et al. (1986). The input for the model were provided by polynomial and spline fit of the experimental data (Raupach et al. 1986; Legg et al. 1986; Cassiani et al. 2007) as explained in Mortarini et al. (2008).

For a precise expression of the input turbulence the original works may be consulted, here we would like to stress the main difference in the two turbulence conditions. Although both cases are fairly inhomogeneous and not Gaussian, they present a difference in the Skewness of the vertical velocity profile. As shown in figure 1 the

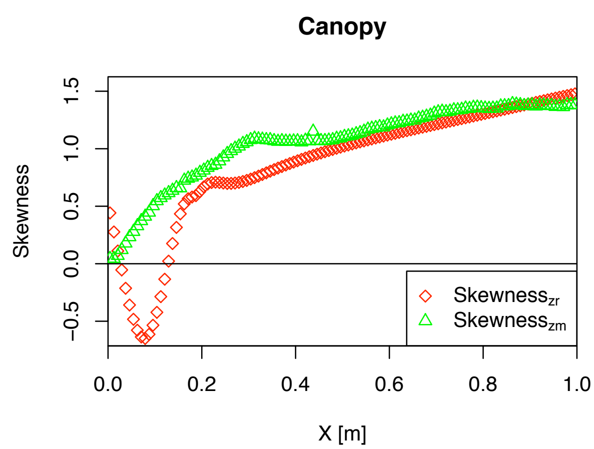


Figure 2: Comparison between the centroid vertical position Skewness (green points) and the assumed Skewness of the vertical position relative to the centroid. The figure refers to the canopy generated turbulence.

two turbulence cases present a strong vertical velocity

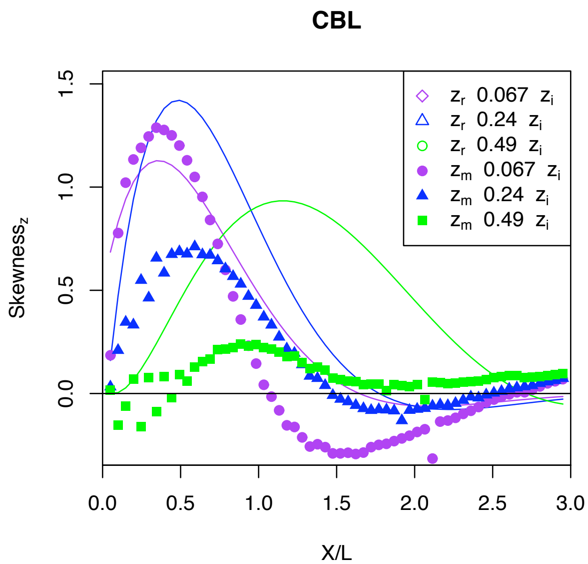


Figure 3: Comparison between the Skewness of z_r and z_m inside the CBL for three different source heights.

Skewness, but the CBL profile is always positive and almost constant in the whole layer, while in the canopy generated w the Skewness inverts it signs at about $0.15z_i$ and is nowhere constant.

6. Skewness of the relative vertical position z_r

Equation (21) states that to correctly determine the skewness of the vertical relative position it is necessary to jointly know the vertical position PDF and the centroid vertical position (Dosio and de Arellano 2006). Our Lagrangian stochastic model only evaluates the barycenter PDF but gives no information about the single particle PDF, therefore we cannot calculate S_{zr}. Hence, in order to use Eq. (21) in the parametrized PDF (19) we have to postulate a form for it. For both the canopy and the CBL cases we determined S_{zr} from the comparison of mean concentration experimental data with the model estimations and then extending the result to the whole domain with spline interpolations.

Figures 2 and 3 show the horizontal evolution of the relative position Skewness and the barycenter Skewness in the two cases. S_{zm} is evaluated from the numerical integration of the simulated barycenter PDF. As it can be seen the two Skewnesses behavior is different. In canopy turbulence S_{zr} is almost always smaller than S_{zm} and, although the latter is always positive, S_{zr} is negative where the plume is close to the ground. Close to the source is therefore difficult to assume an analytical expression relating S_{zm} and S_{zr}. For the CBL the situations is the opposite and the relative position Skewness is larger than S_{zm}. Also in the convective case the major discrepancies between the two profile are observed when the plume reaches the domain bottom. In the CBL a relationship between S_{zm} and S_{zr} is more evident.

The profiles both have a maximum when the plume reaches the ground and decrease to the zero in the far field. The differences between the two S_{zr} profiles can be explained considering that in the CBL the rebound of the plume on the ground is followed by its rise, while inside the canopy the cloud is trapped and tend to stay inside the canopy layer, probably spreading horizontally rather than vertically.

7. Results and comparisons

a. *Mean concentrations*

Throughout this paper, in order to correctly fit the experimental data the concentration is normalized in two ways: for the CBL case we normalize $\langle C \rangle$ asking that when the well-mixed condition is reached the mean concentration is unitary for the whole layer, while for the canopy we evaluate the normalization coefficients θ forcing the equivalence of the areas defined by the experimental and simulated profile. The second method naturally take into account the heat loss measured by Legg et al. (1986).

As for the vegetal canopy simulations, figures 5 and 4 show the normalized mean concentration field. In particular figure 4 represents the comparison among our model results evaluated using the two different parametrization of p_{zr} and Legg et al. (1986). Close to the source (the first two plots of figure 4) equations 17 and 19 are equiv-

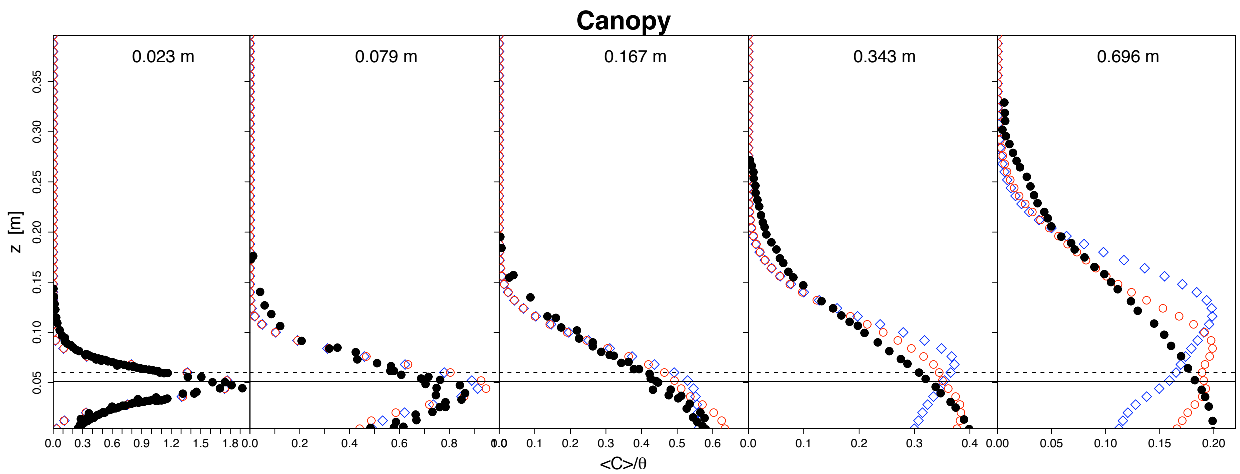


Figure 4: Mean concentration normalized vertical profile evaluated by the fluctuating plume model inside the vegetal canopy: blue diamonds represent Eq. (17), red circles represent Eq. (19), black points Legg et al. (1986) data. The distance from the source is written at the top of each plot.

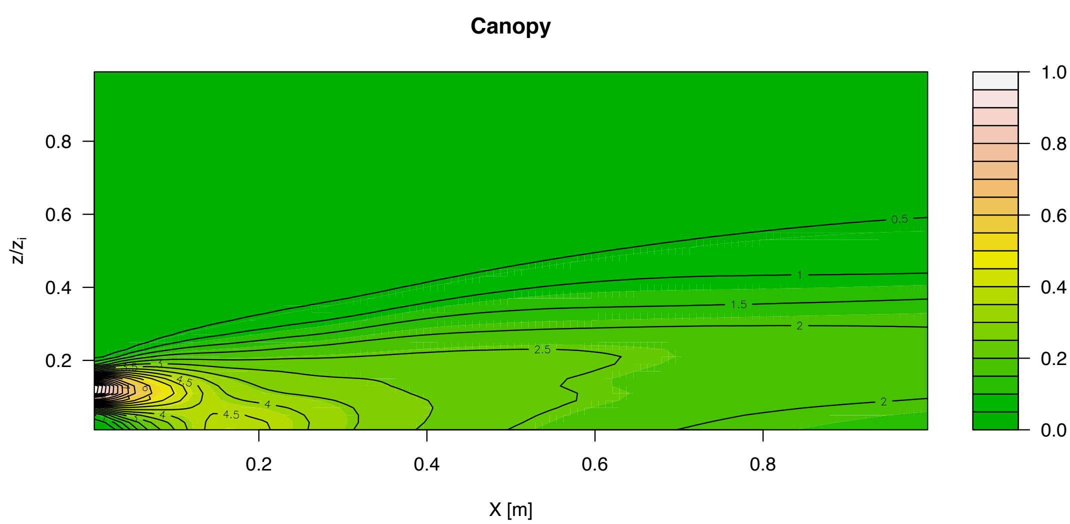


Figure 5: Contourplot of the normalized mean concentration evaluated inside the vegetal canopy applying Dosio and de Arellano (2006) correction to the parametrized PDF of the relative vertical position (equation 19).

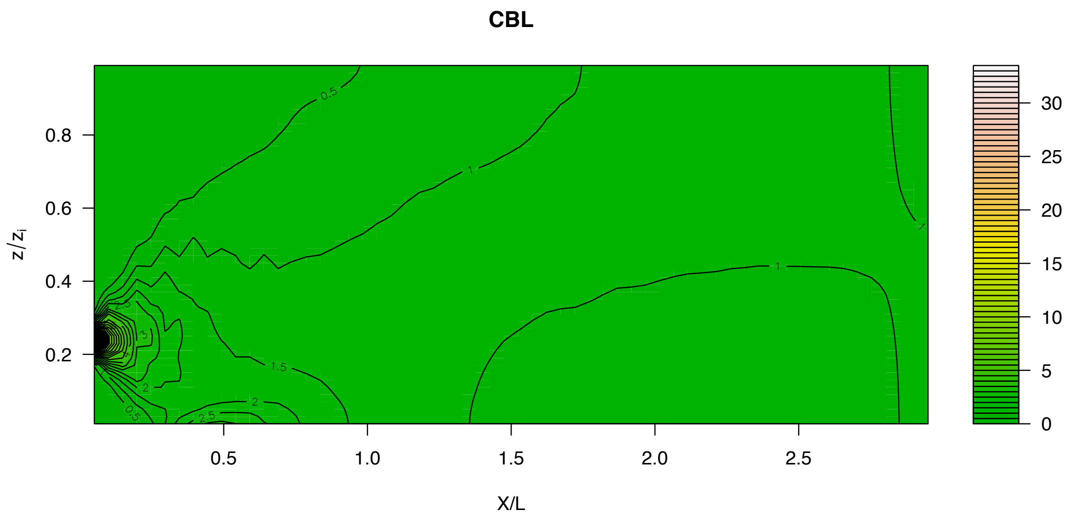


Figure 6: Contourplot of the normalized mean concentration evaluated for the CBL case, applying Dosio and de Arellano (2006) correction to the parametrized PDF of the relative vertical position (equation 19). the source is at 0.24 z_i.

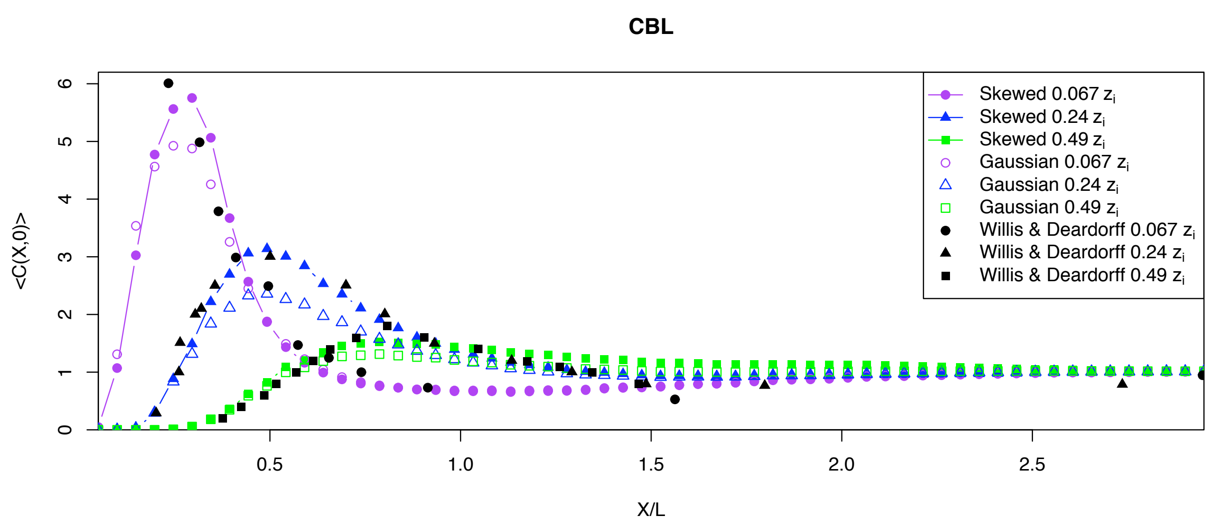


Figure 7: Absolute mean concentration at the ground for the CBL for three different sources height. The colored points refer to the Gaussian form of p_{zr} (Eq. (17)), the colored lines with points refer to the skewed form of p_{zr} (Eq. (19)) and the black points are Willis and Deardorff (1976, 1978, 1981) experimental data.

alent but after the plume reaches the ground the skewed PDF better performance is evident. In the fifth plot both over-estimates the absolute concentration, but it can be explained with the presence in the experiment of an horizontal recirculating flow caused by the canopy elements (Coppin et al. 1986), hence an underestimation close to the ground is perhaps unavoidable (Flesh and Wilson 1992).

Figure 5 represents the contourplot of the mean concentration field for canopy turbulence calculated with Eq. (19). The Skewness effect can be seen close to the ground ($X \sim .2m$) in the elongated shape of the contour. As a consequence of the small value of the Richardson's constant g the plume is very narrow and slow in developing.

Figures 6 and 7 show the normalized mean concentration of the CBL simulations. Figure 6 shows the overall behavior of the concentration field evaluated with the skewed p_{zr} (19). As for the canopy case, the elongated profile along the ground reflects the effects of the Skewness, but here dispersion around the cloud barycenter is faster. The absolute mean concentration at ground level, $\langle C(X/L, 0)\rangle$ is depicted in figure 7 for the two different parametrizations of p_{zr} and for the three different source heights. The agreement between simulated data and the experimental data of Willis and Deardorff (1976, 1978, 1981) is satisfying. It is easy to see that taking the Skewness in to account (Eq. (19)) eliminates the underestimation close to the source found with Eq. (**??**)Eq: FluctpzrGauss improving the maxima mean concentration close to the ground as stated by Dosio and de Arellano (2006).

b. *Concentrations fluctuations*

In order to evaluate the concentration fluctuations profile it is necessary to have a correct parametrization for the relative intensity of concentration fluctuations i_{cr} (Eq. (16)). For the canopy case we used the form proposed by Mortarini et al. (2008), obtained comparing the measured concentration fluctuations profile with the ones evaluated by a fluctuating plume model, while for the CBL we adopted the analytical expression given in Franzese (2003).

Figure 8 shows the comparison between the concentration fluctuations evaluated by the fluctuating plume model in its two versions (Eq. (17) and Eq. (19)) and the experimental data of Legg et al. (1986). Near the source, the model reproduces satisfyingly well the peak location, but the intensity cannot be compared due to missing data. In the second plot ($X = .079\ m$) the profile obtained with the skewed PDF shows a very good agreement with the data, while the Gaussian form of p_{zr} produce a spurious maximum close to the ground. For the three remaining plots it is difficult to say which PDF gives better results, although at .696 m from the source the skewed model better fits the maximum of the curve. As it can be seen from figure 9 the concentration fluctuations evaluated by the fluctuating plume model presents a maximum close to the ground for intermediate distances from the source. This can be possibly explained with the simplified treatment of the boundaries in the Lagrangian Stochastic Model or maybe with the one-dimensionality of the model.

Figure 10 reproduces the contour plot of the concentration fluctuations in the convective case. Unfortunately no data are available for a quantitative comparison, therefore only this qualitative plot is shown. The CBL model shows larger fluctuations than the canopy model and this is probably due to the different turbulent conditions and to the larger value of the Richardson's constant g. The fluctuations in the CBL presents the same behavior of the canopy ones close to the source, with maxima near the grounds. In the far field the profiles tend to a constant value.

c. *Concentrations high-order moments*

The evaluation of the high-order concentration statistics, namely the determination of Skewness and Kurtosis, deeply depends on the knowledge of i_{cr} and the lack of experimental data make the comparison of the second moments of the concentration fields the only way to correctly estimate i_{cr}. As long as we were not able to test our concentration fluctuations evaluation with experimental data for the CBL case, here we prefer to present only the concentration Kurtosis and Skewness for the canopy case.

Figure 11(a) shows the Skewness vertical profile at $.696m$ from the source compared with the experimental data of Coppin et al. (1986). It is a different experiment from the one considered in this work (Legg et al. 1986), but it was carried out in the same plant canopy considering a multiple-line source instead of a single line source. Even if the sources are different we expect the same behavior in the far field, that is why we showed only the farthest

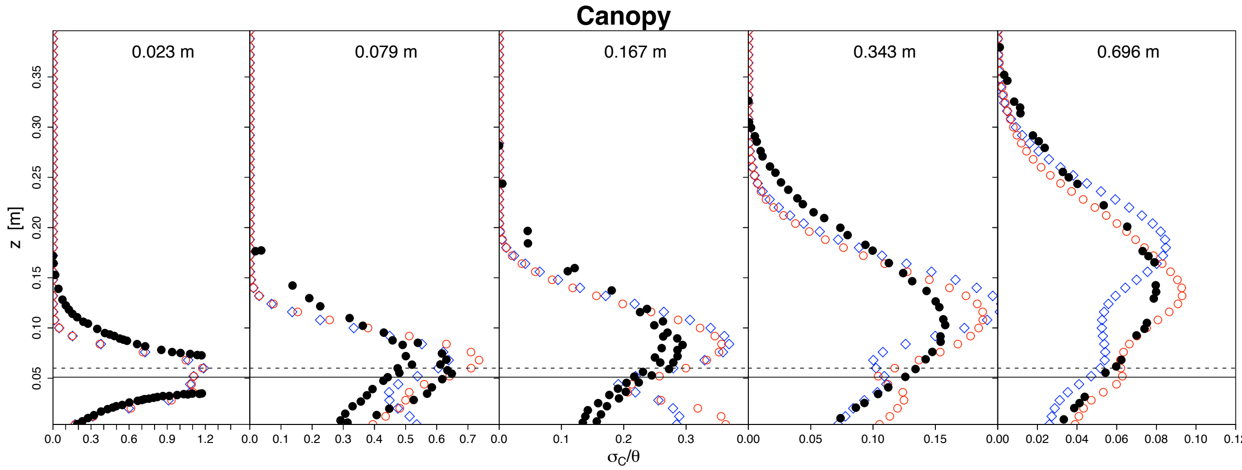


Figure 8: Normalized concentration fluctuation vertical profile evaluated by the fluctuating plume model inside the vegetal canopy: blue diamonds represent Eq. (17), red circles represent Eq. (19), black points Legg et al. (1986) data. The distance from the source is written at the top of each plot.

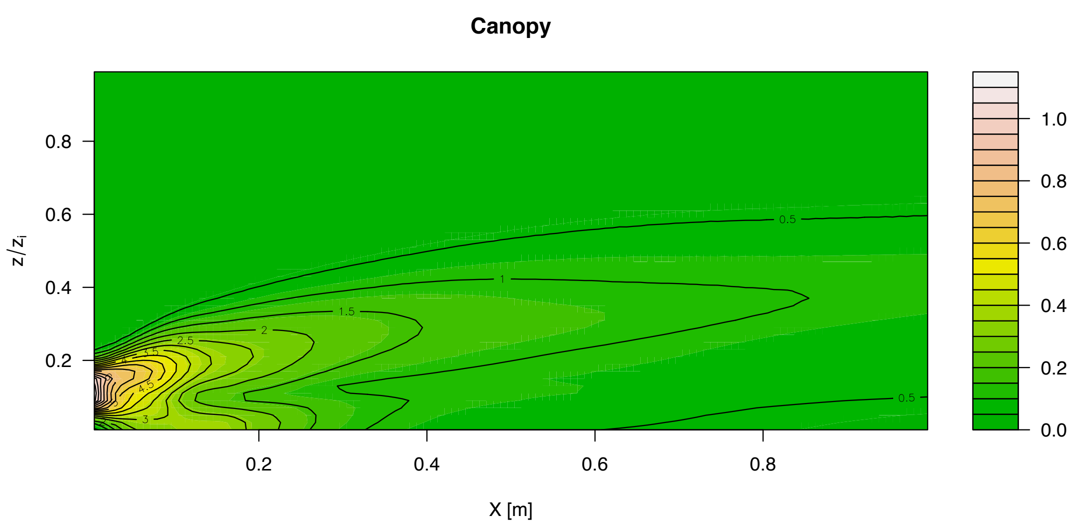


Figure 9: Contourplot of the normalized concentration fluctuations evaluated inside the vegetal canopy applying Dosio and de Arellano (2006) correction to p_{zr} (equation 19).

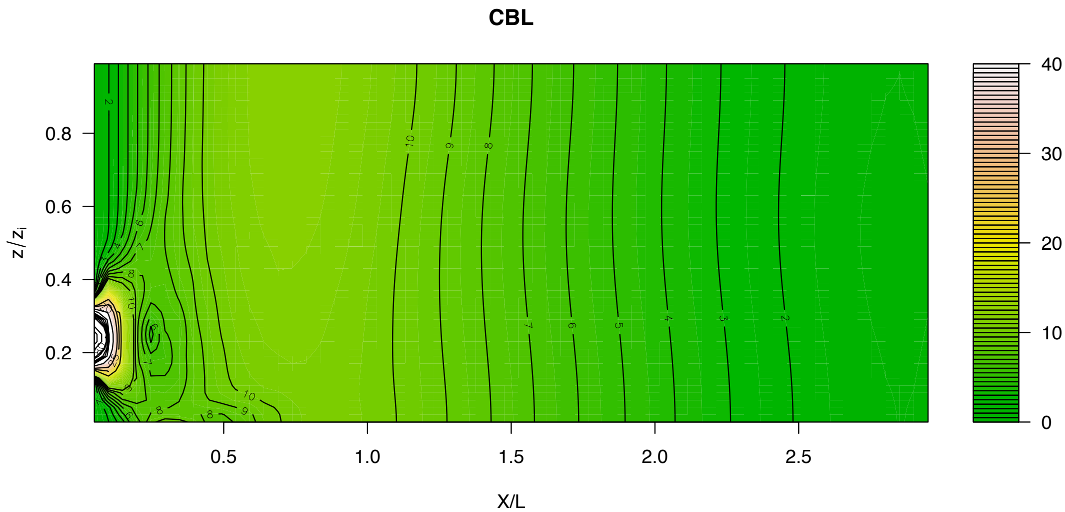


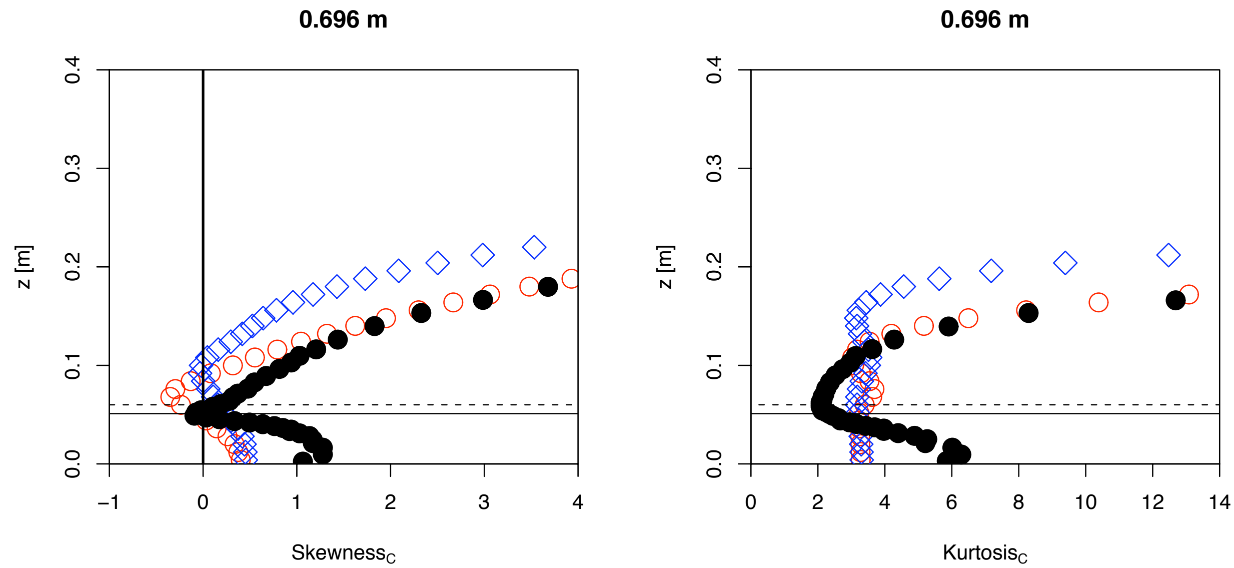
Figure 10: Contourplot of the normalized concentration fluctuations evaluated for the CBL case, applying Dosio and de Arellano (2006) correction to the parametrized PDF of the relative vertical position (equation 19). The source height is $0.24\ z_i$.

profile. A part for a small underestimation close to the ground the profile evaluated with Eq. (19) has a very good agreement with the experimental data above the canopy. Figure 12 shows the concentration Skewness field in the whole simulation domain.

The same conclusions can be applied to the Kurtosis profile (figure 11(b)). The Kurtosis profile evaluated with the Gaussian form of p_{zr} (Eq. (17)) over-estimates the measured profile (Coppin et al. 1986) over the canopy, while both the profiles evaluated by the fluctuating plume present Gaussian values near the ground. Figure 13 shows the concentration Kurtosis field in the whole simulation domain.

8. Conclusions

The fluctuating plume model proved to be a very good investigation tool to simulate relative dispersion in inhomogeneous planet boundary layers. No assumption was made for the PDFs of the turbulent velocity field, a quadratic form for the acceleration in the Langevin equation was considered and all the measured turbulent moment were naturally taken in to account. The model showed to be very flexible and adaptable to the two different turbulent conditions considered. In particular the skewed formulation of the model, i.e. the one where the PDF of the vertical position relative to the barycenter is skewed (Eq. (19)), performed very well in the evaluation of the mean concentration fields both in the CBL and in the canopy cases. In particular it was able to reproduce the maxima of concentration on the ground, while the profile evaluated with a Gaussian form of the PDF of the vertical position relative to the centroid, p_{zr}, over-estimated the concentration maxima height, predicting a faster plume rise. The skewed formulation also improved the model accuracy in estimating the concentration fluctuations, even if a spurious maximum near the ground was produced in some case, probably due to the one-dimensionality of the model. The statistics of Skewness and Kurtosis showed very good results, especially above the canopy.



(a) Concentration Skewness vertical profile inside the vegetal canopy at .696m from the source: blue diamonds represent Eq. (17), red circles represent Eq. (19), black points Coppin et al. (1986) data.

(b) Concentration Kurtosis vertical profile inside the vegetal canopy at .696m from the source: blue diamonds represent Eq. (17), red circles represent Eq. (19), black points Coppin et al. (1986) data.

Figure 11: (a) and (b)

References

Cassiani, M., A. Radicchi, and J. Albertson, 2007: Modelling of concentration fluctuations in canopy turbulence. *Bound.-Lay. Meteorol.*, **122**, 655–681.

Coppin, P., M. Raupach, and B. Legg, 1986: Experiments on scalar dispersion within a plant canopy, part ii: the turbulence structure. *Bound.-Lay. Meteorol.*, **35**, 167–191.

Dosio, A. and J. V.-G. de Arellano, 2006: Statistics of absolute and relative dispersion in the atmoshperic convective boundary layer: A large-eddy simulation study. *J. Atmos. Sci.*, **63**, 1253–1272.

Flesh, T. and J. Wilson, 1992: A two-dimensional trajectory-simulation model for non-gaussian, inhomogeneous turbulence within plant canopies. *Bound.-Lay. Meteorol.*, **61**, 349 – 374.

Franzese, P., 2003: Lagrangian stochastic modeling of a fluctuating plume in the convective boundary layer. *Atmos. Environ.*, **37**, 1691–1701.

Franzese, P. and M. Cassiani, 2007: A statistical theory of turbulent relative dispersion. *J. Fluid Mech.*, **571**, 391–417.

Franzese, P., A. Luhar, and M. Borgas, 1999: An efficient lagrangian stochastic model of vertical dispersion in the convective boundary layer. *Atmos. Environ.*, **33**.

Gifford, F., 1959: Statistical properties of a fluctuating plume dispersion model. *Advances in Geophysics*, **6**, 117–137.

Hanna, S. R., 1984: Concentration fluctuations in a smoke plume. *Atmos. Environ.*, **18**, 1091–1106.

Legg, B., M. Raupach, and P. Coppin, 1986: Experiments on scalar dispersion within a plant canopy, part iii: an elevated line source. *Bound.-Lay. Meteorol.*, **35**.

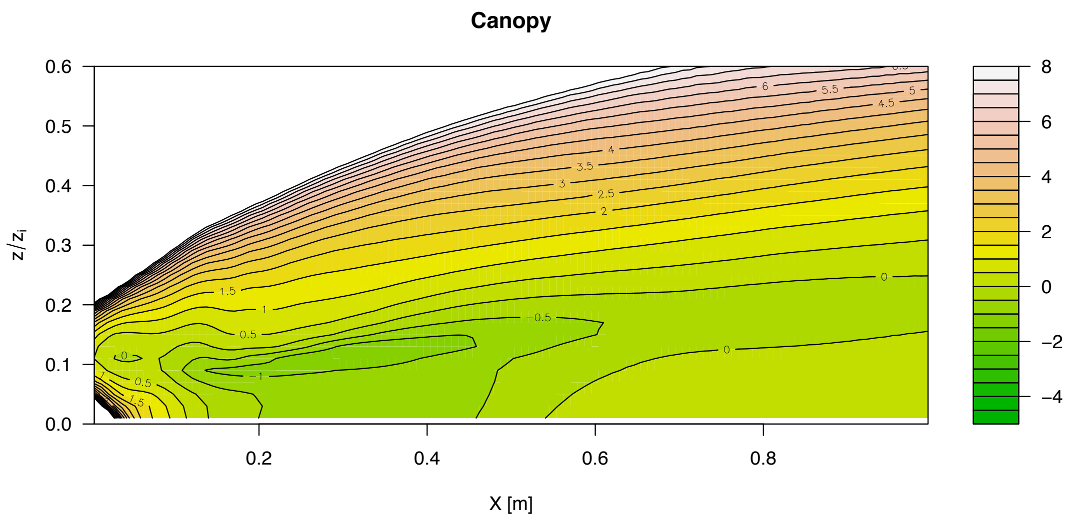


Figure 12: Contourplot of the concentration Skewness evaluated inside the vegetal canopy applying Dosio and de Arellano (2006) correction to p_{zr} (equation 19).

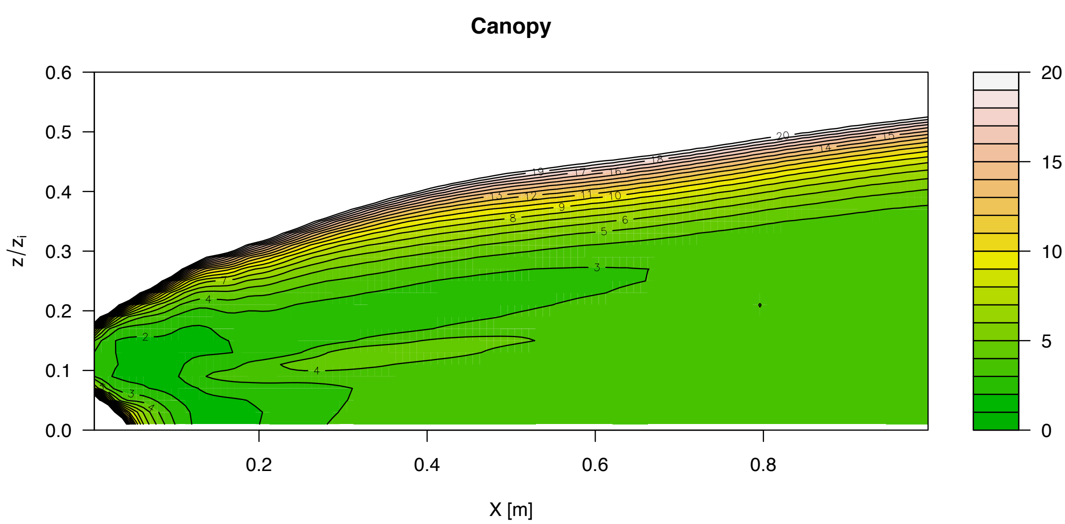


Figure 13: Contourplot of the concentration Kurtosis evaluated inside the vegetal canopy applying Dosio and de Arellano (2006) correction to p_{zr} (equation 19).

Luhar, A., M. Hibberd, and M. Borgas, 2000: A skewed meandering-plume model for concentration statistics in the convective boundary layer. *Atmos. Environ.*, **34**, 3599–3616.

Mortarini, L., P. Franzese, and E. Ferrero, 2008: A fluctuating plume model for concentration fluctuations in a plant canopy. *Submitted to Atmos. Environ.*.

Raupach, M., P. Coppin, and B. Legg, 1986: Experiments on scalar dispersion within a plant canopy, part i: the turbulence structure. *Bound.-Lay. Meteorol.*, **35**, 21–52.

Thomson, D. J., 1987: Criteria for the selection of stochastic models of particle trajectories in turbulent flows. *J. Fluid Mech.*, **180**, 529–556.

Willis, G. E. and J. W. Deardorff, 1976: A laboratory model of diffusion into the convective planetary boundary layer. *Q.J.R. Meterorol. Soc.*, **102**, 427–445.

— 1978: A laboratory study of dispersion from an elevated source within a modeled convective planetary boundary layer. *Atmos. Environ*, **12**, 1305–1311.

— 1981: A laboratory study of dispersion from a source in the middle of the convectively mixed layer. *Atmos. Environ.*, **15**, 109–117.

Yee, E. and D. Wilson, 2000: A comparison of the detailed structure in dispersing tracer plumes measured in grid-generated turbulence with a meandering plume model incorporating internal fluctuations. *Bound.-Lay. Meteorol.*, **94**, 253–296.