10B.3 A new scaling term for use in roughness sublayers

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1. Abstract

Sonic anemometer data from the inertial sublayer (ISL), usually located several roughness heights above a canopy, are easier to interpret than data from within the roughness sublayer (RSL) (Grimmond *et al*. 1998). Some of the problems of using data from within the roughness sublayer include the fact that the data are representative of only a local area, flows are typically not fully adjusted to the heterogeneous surface properties, and the presence of obstacles makes tilt correction extremely problematic. However, knowledge of flow and fluxes within the RSL will improve our understanding of urban and forest transport.

A substitute for u_* is proposed as a tool for use in scaling schemes within the RSL. Traditional u_* calculations can be distorted by the influence of nearby vertical surfaces such as building walls or flow around the sides of trees, as well as being sensitive to sonic anemometer tilt. As a scaling term, u_* is used to create non-dimensional terms such as $\mathrm{C_D}$, z/L, φ_m and T_*, as well as to non-dimensionalize the standard deviations of the three components of the wind vector.

This proposed u_* substitute utilizes invariants of the Reynolds stress tensor which are independent of the choice of coordinate system, making it more reliable for use in the roughness sublayer. It has the added advantage of approximating standard u_* values when applied to data from the ISL. The empirical formula is based on data from lab flows as well as atmospheric field data from CASES99 and JU2003.

2. Introduction

This work was initially motivated by the problem of how to correct for sonic anemometer tilt in the RSL (Klipp 2004). Some aspects of the problem are the presence of multiple wall normal directions and the fact that most RSL flows are not fully adjusted to the surface conditions at the instrument location. For flow past an obstacle, $\hat{y}$ becomes the wall normal instead of $\hat{z}$, making $\overline{u'v'}$ the primary surface stress term instead of $\overline{u'w'}$ (Fig 1). More difficult to define is a total surface stress or an effective surface stress for a location equidistant from two differently oriented walls. Also undefined is the surface stress a short distance downstream from such a situation.

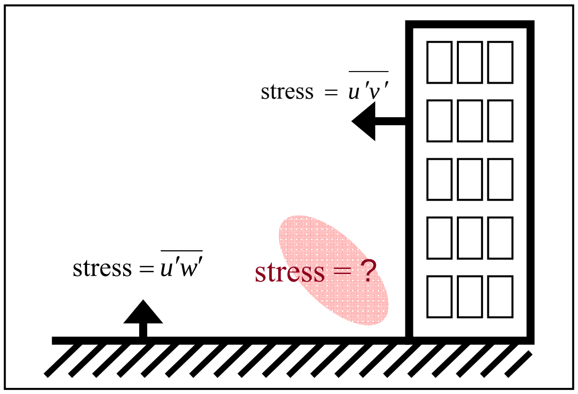

Figure 1 Wall normal is no longer parallel to the gravity vector near buildings, making stress difficult to define

Since $\overline{u'v'}$ becomes important in the urban RSL (Fig 2), one could add a third term to redefine u_* as $u_*^2 = \left(\overline{u'w'}^2 + \overline{v'w'}^2 + \overline{u'v'}^2\right)^{1/2}$, but this is not mathematically consistent with the tensor qualities of the terms. Tensor invariants are a mathematically sound choice. In addition, the invariants are independent of the sonic anemometer orientation, thereby eliminating the need for tilt correction (Wilczak *et al*. 2001).

3. Reynolds stress tensor

There are an infinite number of sets of three mathematically independent invariants for a second rank tensor in three dimensions such as

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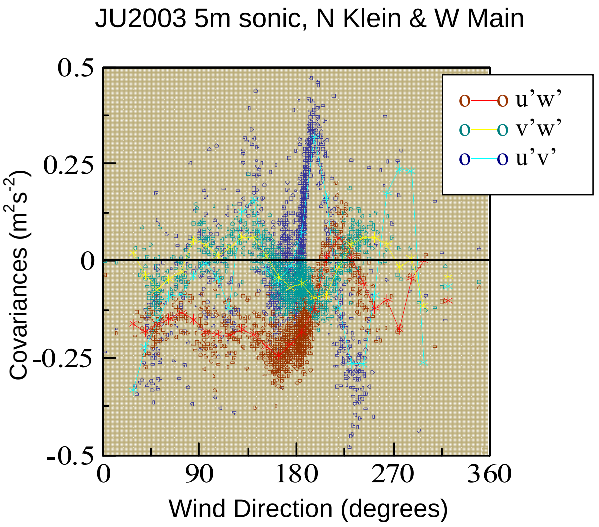

Figure 2 Values for the three covariances at an urban location in OKC. Note the organized behavior of u'v' for flow from either side of a tree near the tower as well as reduced u'w' values. The slight change in sign for u'w' is consistent with flow under the canopy.

the Reynolds stress tensor (Arfken 1985). For this application it makes sense to use the eigenvalues and corresponding eigenvectors. The eigenvectors form an orthogonal coordinate system which defines the fundamental directions for the wind fluctuations as expressed in the Reynolds stress tensor, and the eigenvalues are the corresponding fundamental variance values. In other words, the usual boundary layer coordinate system of along wind, cross wind and vertical directions is not the primary coordinate system for the turbulent fluctuations.

In laboratory flows (Liberzon *et al.* 2005, Hanjalic and Launder 1972) and in the boundary layer over flat terrain in near-neutral and shear dominated cases (Klipp 2007), this primary coordinate system is nearly aligned with the usual boundary layer coordinate system, but is rotated approximately 17° around the cross stream axis in the direction of the mean wind (Fig 3), resulting in an approximately 73° angle between the mean wind direction and the eigenvector associated with the smallest eigenvalue.

4. Scaling term using tensor invariants

For flow inside the RSL, this simple relationship between the two coordinate systems no longer holds, however, the Reynolds stress tensor can still be diagonalized. The resulting diagonal matrix can be rotated 17° in the opposite sense to create a value in the upper right and lower left locations, where $\overline{u'w'}$ would be (Eq 1).

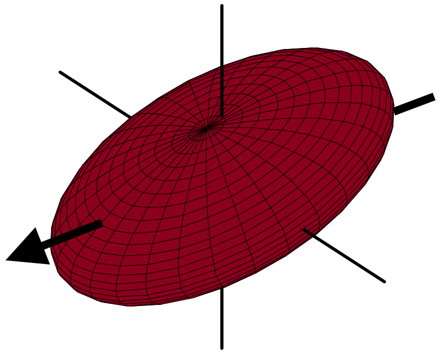

Figure 3 Idealized TKE ellipsoid based on lab flows

$$\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\lambda_{\mathrm{B}} & 0 & 0\\ 0 & \lambda_{\mathrm{M}} & 0\\ 0 & 0 & \lambda_{\mathrm{S}}\end{pmatrix}\begin{pmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{pmatrix}$$

$$=\begin{pmatrix}\lambda_{\mathrm{B}}\cos^2\theta+\lambda_{\mathrm{S}}\sin^2\theta & 0 & (\lambda_{\mathrm{S}}-\lambda_{\mathrm{B}})\cos\theta\sin\theta\\ 0 & \lambda_{\mathrm{M}} & 0\\ (\lambda_{\mathrm{S}}-\lambda_{\mathrm{B}})\cos\theta\sin\theta & 0 & \lambda_{\mathrm{S}}\cos^2\theta+\lambda_{\mathrm{B}}\sin^2\theta\end{pmatrix}$$

Equation 1

Using $u_{*R}^2=-(\lambda_{\mathrm{S}}-\lambda_{\mathrm{B}})\cos\theta\sin\theta$ is analogous to $u_*^2=-\overline{u'w'}$, and is a good approximation to the actual u_*^2 for neutral flow over open terrain. Using 17° for the rotation angle results in $u_{*R}^2=0.280(\lambda_{\mathrm{B}}-\lambda_{\mathrm{S}})$. It can be seen that over open terrain this substitute for u_*^2 has approximately the same value as u_*^2 calculated with $u_*^2=\left(\overline{u'w'}^2+\overline{v'w'}^2\right)^{1/2}$ (Fig 4).

5. Stability dependence

As seen in the CASES99 data, the relative angle between the eigen coordinate system and the mean wind - cross wind - vertical coordinate (*uvw*) system seems to be stability dependent (Fig 5). This might just be an effect of choosing 10 min flux averaging windows, which may be too short for daytime convective conditions and are too long for very stable conditions (Vickers and Mahrt 2003).

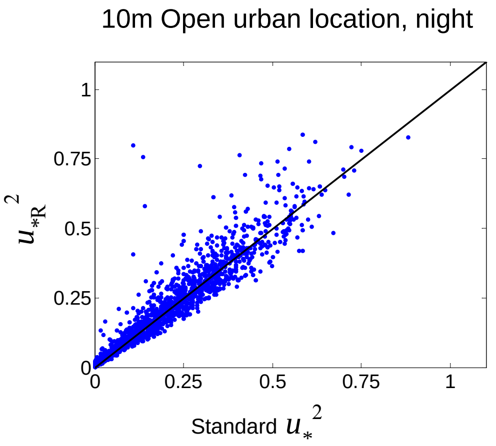

Figure 4 Standard definition $u_*^{\;2}$ compared to idealized $u_{*R}^{\;2}$, open location, night

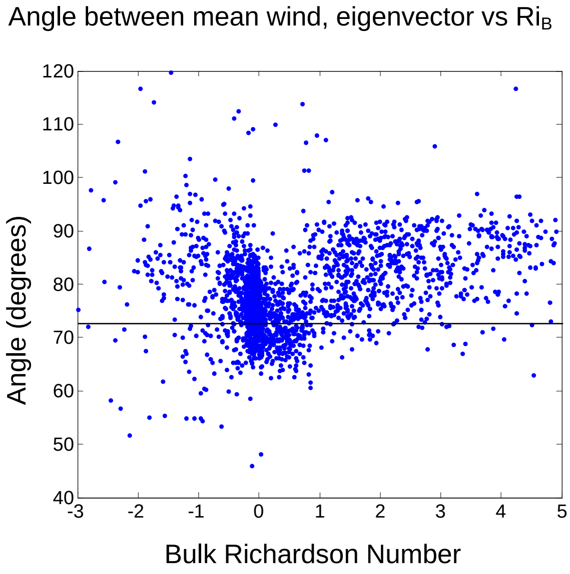

Figure 5 The angle between the mean wind vector and the eigen vector associated with smallest eigenvalue is stability dependent. Neutral conditions as well as stable, but near neutral conditions yield the 73° angle (line) found in laboratory flows. 10min fluxes from 10m CASES99 main tower.

Since many urban flows are shear dominated (Roth 2000), this possible stability dependence should have little influence within the RSL. For the case of low winds or an unusually high degree of stability or convection, the calculation of u_{*R} in urban RSL flows may need to be adjusted for stability. This was not needed for data from July 2003 in Oklahoma City where nearly all flow was shear dominated.

6. Application within RSL

Values of the standard u_* within the RSL can become small as the importance of $\overline{u'v'}$ increases. The Reynolds stress tensor derived u_{*R} values are almost always larger (Fig 6) than the standard u_* values. This results in fewer very large values for terms with u_* in the denominator, such as scaled TKE (Fig 7 and Table 1) and fewer small values for terms with u_* in the numerator such as C_D (Table 2).

Table 1

Scaled	Traditional			New		
TKE	Mean	Median	range	Mean	Median	Range
CASES99 10m	11.1	6.53	1.11–1055	4.37	4.27	1.80–25.2
SPWID 12	7.95	5.70	1.08–155	5.25	4.76	1.81–30.6
SPWID 14	8.94	6.38	2.11–333	4.35	4.07	2.30–17.8

Table 2

C_D	Traditional			New		
	Mean	Median	range	Mean	Median	Range
CASES99 10m	0.0064	0.0034	5e-7–3.01	0.0115	0.0046	9e-5–2.73
SPWID 12	0.064	0.027	5e-4–15.6	0.090	0.032	2e-3–37.0
SPWID 14	0.105	0.080	0.0015 – 5.30	0.179	0.135	0.015–9.75

Since u_{*R} is proportional to the difference between the large and small eigenvalues, and TKE is half the sum of the eigenvalues, scaled TKE using u_{*R} is a measure of isotropy. When the turbulence is nearly isotropic, the difference between the large and small eigenvalues becomes small while the TKE remains finite. This results in a very large scaled TKE which goes to infinity in the limit of pure isotropy. The most extreme example of anisotropy for this case is when two of the eigenvalues nearly vanish. The limiting value for scaled TKE when $\lambda_M = \lambda_S = 0$ is $0.5(\lambda_B + \lambda_M + \lambda_S)/0.280(\lambda_B - \lambda_S) \rightarrow 1.79$.

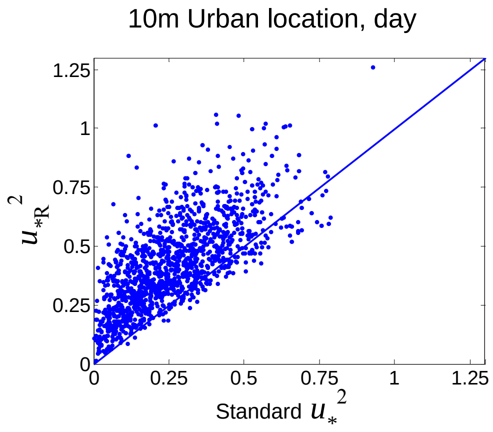
10m Urban location, day

Figure 6 Standard definition u_*^2 compared to idealized u_{*R}^2, urban location, day

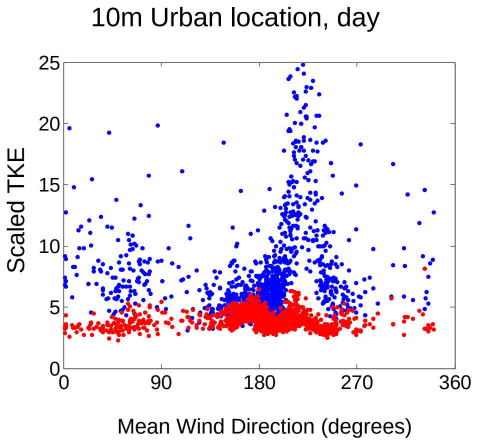
10m Urban location, day

Figure 7 Scaled TKE, urban location, day. Note the significantly smaller values for the coordinate system independent version downwind from a nearby tree at 220°. Blue uses standard u_*, red uses u_{*R}

7. Preliminary results in urban canyon

Of interest within a street canyon is that the new u_{*R} is more consistent from one location to the next compared to standard u_* values (Fig 8). This is an indication that u_{*R} might be a superior choice for scaling terms in the urban canopy.

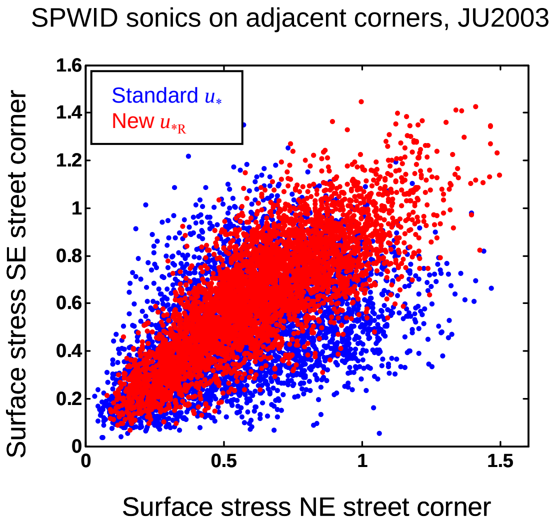
SPWID sonics on adjacent corners, JU2003

Figure 8 Comparison of u_* values at towers located near each other in downtown urban canyon at corner of Park Ave and N. Broadway in Oklahoma City. Surface stress calculated using the standard definition show less agreement from one street corner to the next than the surface stresses calculated using the Reynolds stress tensor.

8. Conclusions

Sometimes one needs to know the momentum flux across a horizontal plane. In those cases, the standard definition of u_* is what is required, not u_{*R}. For most other cases where u_* is used, a scaling factor is what is required. For those situations, the use of tensor invariants to derive a coordinate system independent scaling term produces results more in line with scaled values used by computer models.

This technique has not yet been used on forest RSL data, but should prove to be useful. There is the possibility that subcanopy flow is too close to isotropic, and therefore, the eigenvalues to similar in value, to get meaningful scaling numbers from the Reynolds stress tensor methodology.

Also in need of further investigation is the behavior of u_{*R} in non-neutral conditions to see if there are some scaling applications where u_{*R} might be preferred to the standard u_* even for situations where the standard u_* is meaningful.

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