

## 6A.3 Stably stratified boundary layer simulations with a non-local closure model

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### 1. INTRODUCTION

Simulations of stable boundary layer are nowadays essential in describing many geophysical flows both in the atmosphere and the ocean. Reynolds Averaged Navier-Stokes (RANS) models have been largely developed in the last years for shear-driven and buoyancy-driven boundary layer, while in the stable boundary layer (SBL) RANS models performances are not yet completely assessed. In the present work a third-order turbulence model is used to study the SBL. Turbulence is a complex physical process whose description with statistical methods represents nowadays an open question. Among the several procedures well known and strongly validate in literature, the RANS models, firstly proposed by Reynolds, represent a useful tool to describe many turbulent phenomena. The use of these models has been tested in many fields from stellar astrophysics Canuto (1992) Kupka (2002) Kupka (2003), Kupka (2006) to oceanography Canuto et al. 2001 and 2002, Canuto et al 2007. In particular, Reynolds stress turbulence formalism has been widely used to simulate the planetary boundary layer since the 1970s, when Mellor and Yamada (1974) pioneered this kind of models in geophysical applications. In a "local-model" it is usually assumed a one to one correspondence between turbulent fluxes at a given height and other parameters of the flow, namely the local gradients of the mean fields, at the same position. Such an approximation is justified only when the turbulent mixing length is much less than the length scale of heterogeneity of the mean flow. Observations and Large Eddy Simulations (LES) studies have demonstrated that turbulence in the convective boundary layer is associated with the "non-local" integral properties of the boundary layer (Moeng and Sullivan 1994, Holstag and Moeng 1991).

On the contrary, the SBL has been not yet studied with a third order closure model. The reason is that while for unstably stratified flows a local approximation (e.g. second order models) is not physically justifiable, since eddies sizes under this condition are comparable with the boundary layer depth and thus the computation of vertical fluxes must necessarily encompass non local effects, stably stratified flows are characterized by smaller eddies and thus a local approximation may be more justified (at least as a first approximation) (Canuto et al, 2008).

In this work we use a Third Order Moments (TOMs) model to simulate a stable boundary layer. We include in our analysis the results of a LES carried out by our group, with aim of comparison. Recently, a debate was developed about the role (or the existence) of the critical Richardson number ( $Ri$ ). Two papers, Zilitinkevic et al. (2007) and Canuto et al. (2008), have supported the idea that turbulence can survive also at large  $Ri$ , on the bases of the results of Second Order Moment (SOM) model. The TOMs model is tested in the SBL and the role of the equation for the mean rate of the turbulent kinetic energy ( $\epsilon$ ) dissipation,  $\epsilon$ , is investigated. As a matter of fact this quantity plays a crucial role in the turbulence development, but its dynamical equation is matter of discussion, being merely postulated. We tested a modified form of these equation proposed by Burchard and Baumert (1995).

### 2. THE MODEL

The model (Ferrero and Racca, 2004; Ferrero and Colonna, 2006) consists of a system of 34 dynamical equations derived from the Navier-Stokes equations following the procedure described in Canuto (1992). Horizontal homogeneity is assumed.

The model numerically solves the equations for mean fields, SOMs and TOMs. A closure scheme is required for the fourth order moments (FOMs) appearing in the TOMs equations. In this work we have adopted the Quasi-Normal approximation although it presents some well known shortcomings, divergences of the TOMs in neutral and convective boundary layers and spurious oscillations in the stable regions of the flow (Canuto et al., 2005; Cheng et al., 2005; Moeng, 1984); for this reason a new approach, based on physical grounds, is employed to avoid the anomalous growth of the TOMs due to the use of the Quasi-Normal approximation. As shown in Ferrero and Colonna (2006) the damping of the TOMs is ensured by mean of a proper time scale depending on the length scale of the turbulence.

The model does not include rotation and viscosity effects since their contributions are negligible in the atmospheric boundary layer. Exception is made for the mean wind equations in which rotation effects due to the Coriolis force are included.

In the modified  $\epsilon$ -equation the shear and buoyancy contribution to the production are separated assigning two different constants  $C_{1\epsilon}$  and  $C_{3\epsilon}$  (Burchard and Baumert, 1995):

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$$\partial_t \varepsilon - \partial_z \left( \frac{v_t}{\sigma_\varepsilon} \partial_z \varepsilon \right) = c_{1\varepsilon} \frac{\varepsilon}{e} P + c_{3\varepsilon} \frac{\varepsilon}{e} B - c_{2\varepsilon} \frac{\varepsilon^2}{e}$$

which reduces to the standard equation for  $C_{3\varepsilon}=C_{1\varepsilon}$ . The constant  $C_{3\varepsilon}$  regulates the buoyancy production in  $\varepsilon$ -equation. While the values of the other constants are standard (here we used  $C_{1\varepsilon}=1.4$ ,  $C_{2\varepsilon}=1.85$ ,  $\sigma_\varepsilon=1.3$ ),  $C_{3\varepsilon}$  value is matter in debate. Different values has been proposed by several authors, ranging from 0 to 1.44, for different stability conditions. Burchard and Baumert (1995) suggested negative values for this constant in stable conditions. As a matter of fact, they showed that the gradient Richardson number  $Ri$  can be expressed in term of the model constant as follows (Burchard, 2002):

$$Ri = \frac{c_\mu}{c_\mu''} \frac{c_{2\varepsilon} - c_{1\varepsilon}}{c_{2\varepsilon} - c_{3\varepsilon}}$$

where  $C_\mu$  and  $C_\mu''$  are the structure functions which express the down-gradient approximation for the TKE and heat fluxes respectively. In this work we considered  $C_\mu$  and  $C_\mu''$  constant equal to 0.09 and 0.073 respectively. From the above expression it can be shown that, for  $Ri < 0.25$ ,  $C_{3\varepsilon}$  must be negative (Burchard, 2002). Further, the value of  $C_{3\varepsilon}$  can be estimated from the other constants and from the Richardson number.

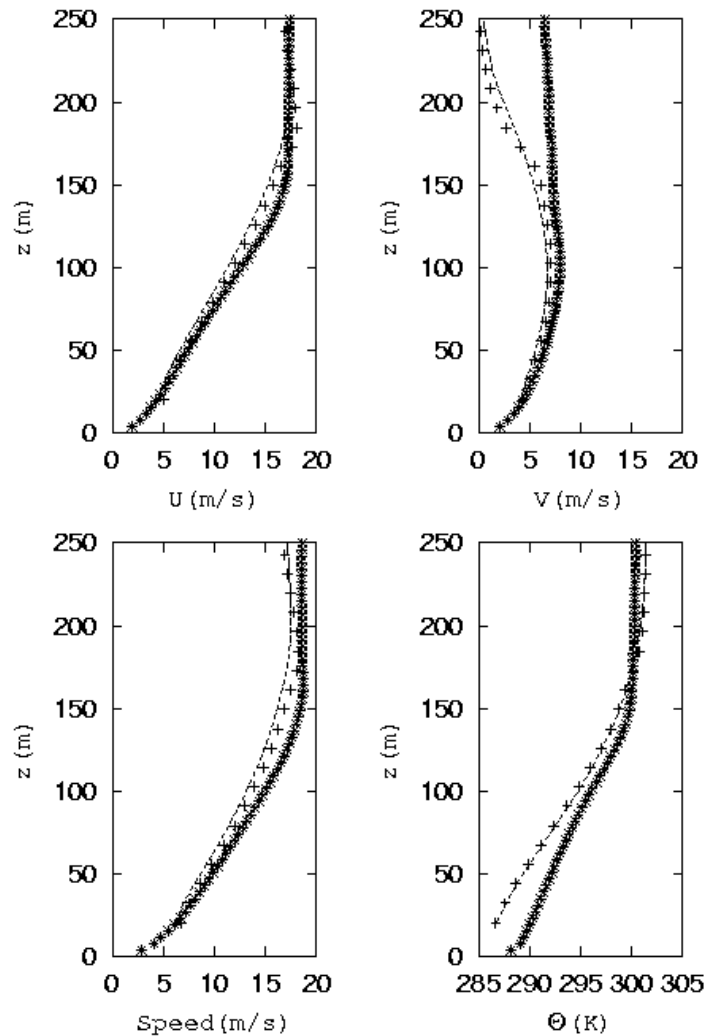


Figure 1: Comparison among mean values vertical profiles predicted by the standard  $\varepsilon$ -equation (---), Burchard and Baumert (1995)  $\varepsilon$ -equation (+++) in the TOM model and LES (\*\*\*).

## 2. SIMULATIONS AND RESULTS

The simulated case reproduces a flow over a flat surface in horizontally homogeneous conditions. The simulation is carried out cooling from below the boundary layer at a constant and uniform rate switched on at the initial time. Turbulence in the SBL is generally considered local (at least as first approximation) while our model, resolving the third order moments (TOMs) dynamical equations, accounts for the non-local transport which, as assessed in several previous works, is generally essential in describing the turbulence flows.

The model employed for the simulation has been tested and validate for shear driven and buoyancy driven boundary layers (Ferrero and Racca, 2004, Ferrero 2005, Ferrero and Colonna 2006).

In this work we present the results of two simulation performed using our TOMs model compared with

the results of a LES appositely carried out by our group. The TOMs simulations were conducted using the standard and the modified  $\varepsilon$ -equation, proposed by Burchard and Baumert (1995), respectively.

In Figure 1, the results of the TOMs simulations are compared with LES data. In the figure the mean quantity (potential temperature, wind speed and its horizontal components) vertical profiles are presented. The agreement of both the TOMs simulations is satisfactory, with a slight underestimation of the wind and overestimation of the potential temperature. Further the version of the model using the  $\varepsilon$ -equation, proposed by Burchard and Baumert (1995), gives slightly better results for the mean velocity.

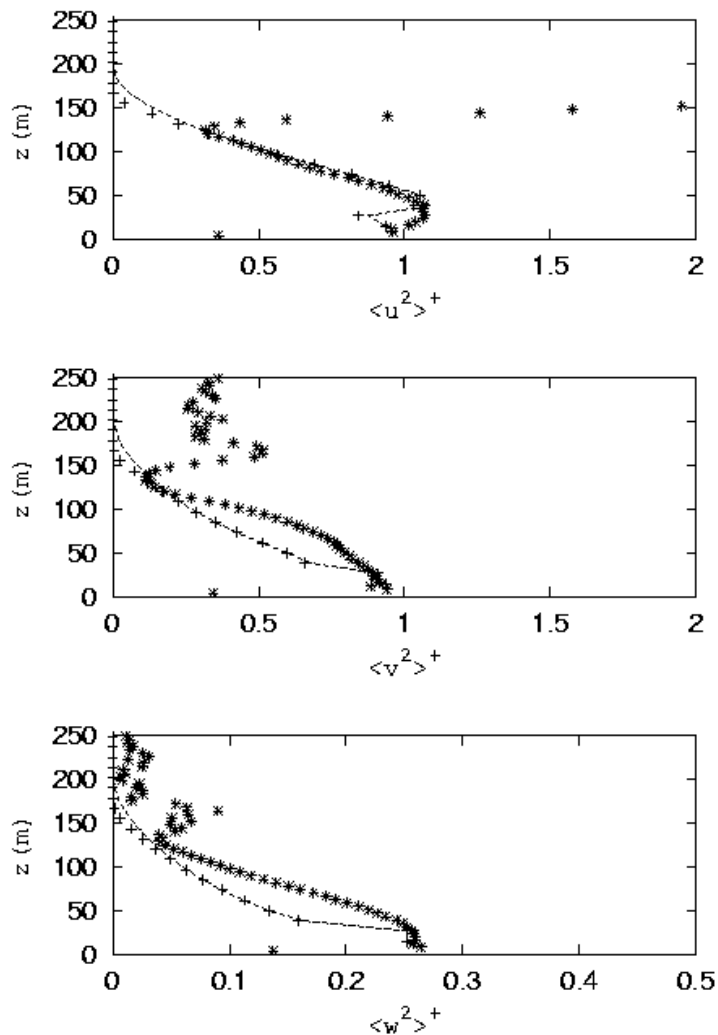


Figure 2: Comparison among normalised variances vertical profiles predicted by the standard  $\varepsilon$ -equation (---), Burchard and Baumert (1995)  $\varepsilon$ -equation (---) in the TOM model and LES (\*\*\*).

In Figure 2 and 3, the results concerning the SOMs, normalised with the velocity and temperature scales, are shown. The heat and momentum fluxes (Figure 2) and the three variances (Figure 3) are correctly reproduced, except for an underestimation of the  $v$  and  $w$  variances. The simulation performed using the standard equation for the dissipation rate is compared with the simulation carried out with the modified equation accounting for the constant  $C_{3\varepsilon}$ . The two simulations point out only some very small differences at the upper levels, where the stability is weak (see Figure 1)

In Figure 4, the comparison between the two models is presented with reference to the normalised TOMs. It can be observed that the model which uses the standard  $\varepsilon$  equation and that accounting for the Burchard and Baumert (1995) equation, predict very similar TOMs profiles giving very small values close to zero as observed in field

measurements by Dias et al. (1995) and recently confirmed by Basu et al.. Nevertheless the presence of the TOMs suggests that turbulent transport is not completely suppressed by the stable stratification.

In figure 5 the gradient Richardson number ( $Ri$ ), the flux Richardson number ( $Ri_f$ ) and the  $C_{3\varepsilon}$  constant vertical trends, calculated by the TOMs model, are depicted. It can be seen that the  $C_{3\varepsilon}$  value is positive and less than 0.5 in the layer of strong stability, while it decreases up to negative values of the order between  $-4$  and  $-2$  in the layer of weak stability ( $Ri < 0.25$ ), in agreement with Burchard (2002), who showed that, based on physical grounds,  $C_{3\varepsilon}$  must be negative in the stable layer.

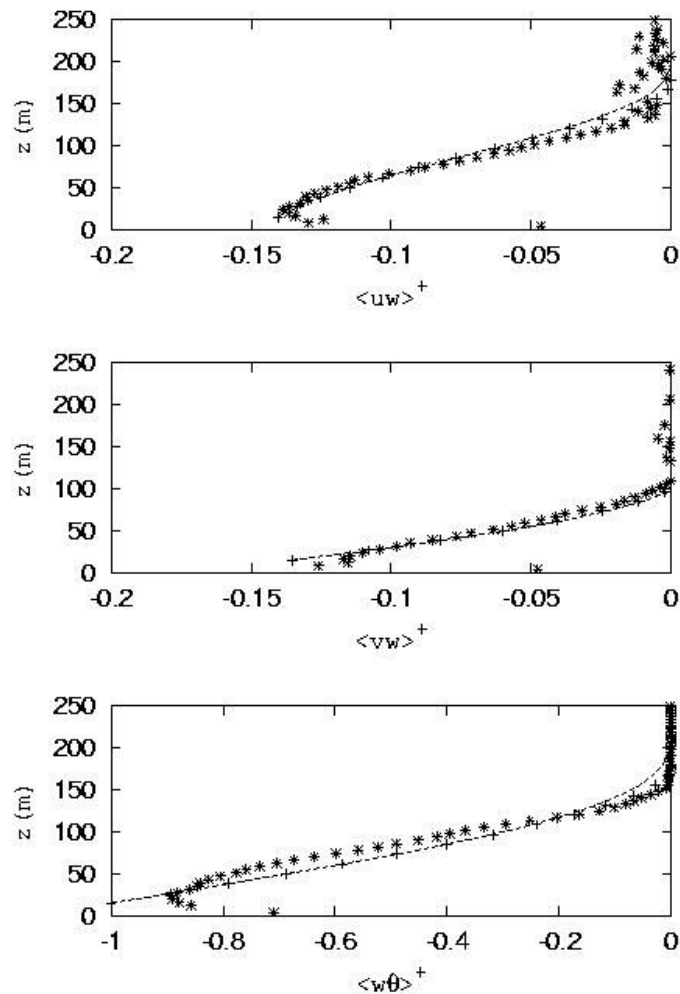


Figure 3 : Comparison among normalised fluxes vertical profiles predicted by the standard  $\varepsilon$ -equation (---), the Burchard and Baumert (1995)  $\varepsilon$ -equation (+++) in the TOM model and LES (\*\*\*)

### 3 CONCLUSIONS

In this work we presented the simulation performed with our TOMs model using two version of the equation for the dissipation rate of the turbulent kinetic energy, of a stable boundary layer. For sake of comparison results of a LES are also reported. The results of our model show a general agreement with LES data and small differences between the two versions of the model. Moreover, the results

support the hypothesis of negative  $C_{3\varepsilon}$ , as proposed by Burchard and Baumert (1995), at least in the layer of weak stability. TOMs are found to be smaller but not null in the layer of strong stability, suggesting that also at higher Ri numbers turbulent transport survives as suggested by Zilitinkevic et al. (2007) and Canuto et al. (2008).

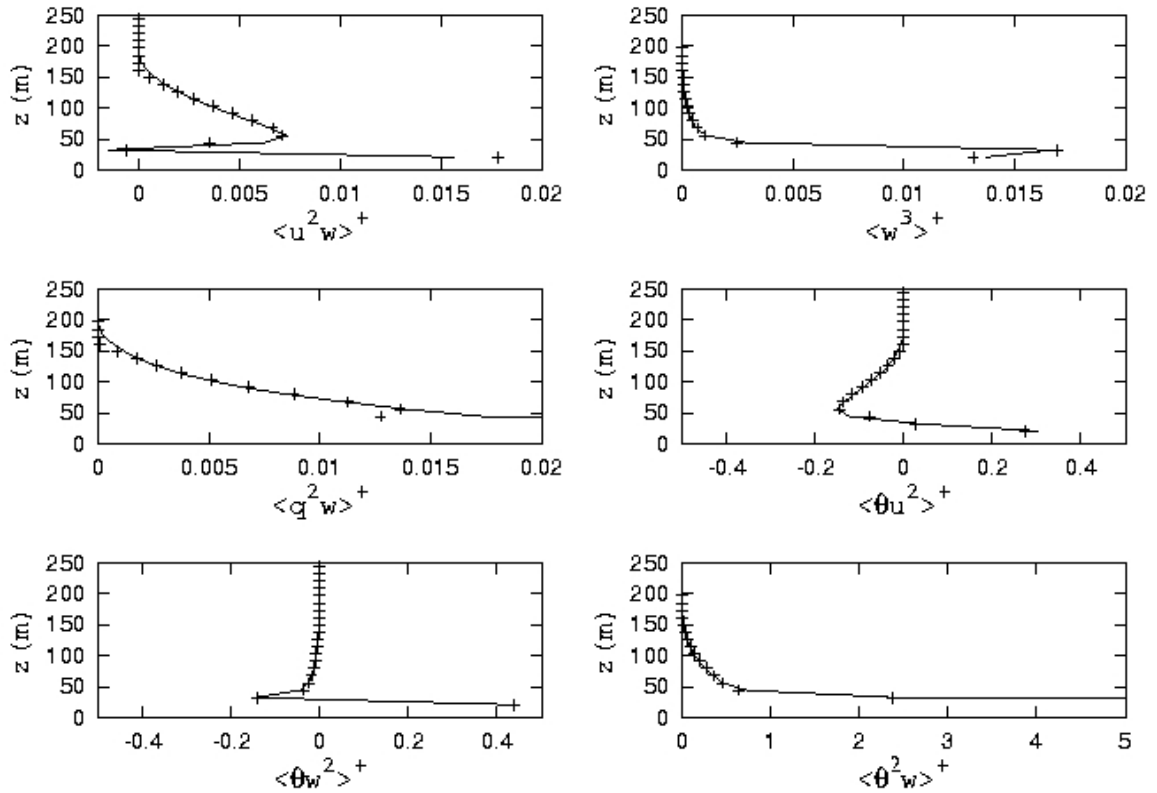


Figure 4: Comparison between normalised third order moments vertical profiles predicted by the standard  $\varepsilon$ -equation (---), Burchard and Baumert (1995)  $\varepsilon$ -equation (+++) in the TOMs model.

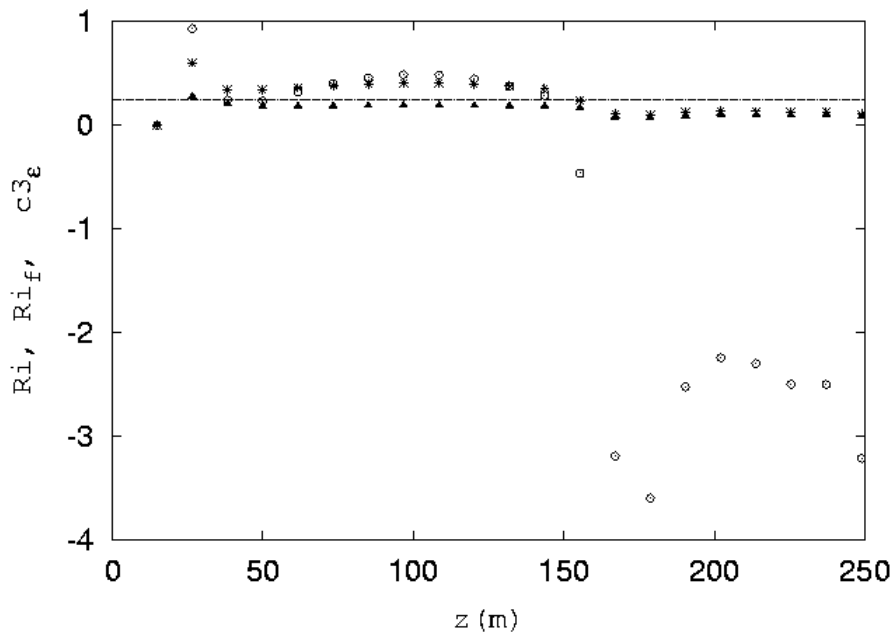


Figure 5: gradient Richardson number (\*), flux Richardson number ( $\blacktriangle$ ) and  $C_{3\epsilon}$  (o) vertical trends; horizontal line indicate the value 0.25.

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