10A. 5 SIMPLE MODIFICATION OF TWO-EQUATION MODELS FOR NON-NEUTRAL FLOW

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1. TWO-EQUATION MODELS

It has been recognized that the turbulent length scale or time scale must be calculated as a dynamic variable in any model that aims to adequately simulate the airflow over heterogeneous surfaces (e.g., Finnigan, 2007). A very promising is the two-equation closure approach based on transport equations for the turbulent kinetic energy (TKE), E, and for some supplementary characteristic, φ:

$$\frac{\partial E}{\partial t}+\left\langle \bar{u}_j \right\rangle \frac{\partial E}{\partial x_j}-\frac{\partial}{\partial x_i}\left(\frac{K}{\sigma_E^{\varphi}}\frac{\partial E}{\partial x_i}\right)= \left(P-\varepsilon\right)+\left[B+S_p-S_d\right] \quad (1)$$

$$\frac{\partial \varphi}{\partial t}+\left\langle \bar{u}_j \right\rangle \frac{\partial \varphi}{\partial x_j}-\frac{\partial}{\partial x_i}\left(\frac{K}{\sigma_{\varphi}}\frac{\partial \varphi}{\partial x_i}\right)= \frac{\varphi}{E}\left(C_{\varphi 1}P-C_{\varphi 2}\varepsilon\right)+\frac{\varphi}{E}\left[C_{\varphi 3}B+C_{\varphi 4}S_p-C_{\varphi 5}S_d\right] \quad (2)$$

The approach does not require a predefined length scale, l, and seems to be naturally suited for a modelling of such flows. The Eqs. (1) - (2) are written in their standard forms (Kantha, 2004; Sogachev and Panferov, 2006). Here K is the eddy viscosity; σ_E^{φ} and σ_{φ} are the Schmidt numbers for E (depending of φ-equation used for closure) and for φ, respectively; P and B represent E-production by shear and by buoyancy, respectively and ε is the dissipation rate of E. S_p and S_d denote correspondingly the wake production and enhanced dissipation of E due to surface drag interactions between phytoelements and the canopy space air. Here, x_i ($x_1 = x$, $x_2 = y$, $x_3 = z$) are the longitudinal, lateral and vertical directions, respectively. u_i ($u_1 = u$, $u_2 = v$, $u_3 = w$) is the instantaneous velocity component along x_i. An overbar denotes time averaging and angle brackets denote horizontal averaging. $C_{\varphi 1}$, $C_{\varphi 2}$, $C_{\varphi 3}$, $C_{\varphi 4}$, and $C_{\varphi 5}$ are model constants. Depending on a choice of supplementary characteristic, φ, the coefficient $C_{\varphi 2}$ should or should not be corrected by a near-wall function $F_{\varphi W}$. For closures based both on ε and ω (where ω is specific dissipation of E, ω=ε/E) equations, such correction is not required. For the reasons explained below, we implement these two kinds of closure only, and that is why we can exclude $F_{\varphi W}$ from following considerations. Also because present work is focussed on the closure issue, the continuity and momentum equations are omitted from consideration as well as the exact expressions for P and B (see Pielke, 2002 for details). The readers are invited to look in (Pope, 2000; Umlauf and Burchard, 2003) to find out how K and l can be derived from E and φ.

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2. UNCERTAINTIES IN TWO-EQUATION CLOSURES

$C_{\varphi 1}$ and $C_{\varphi 2}$ are coefficients selected to be consistent with von Karman's constant and with experimental observations for decaying homogeneous, isotropic turbulence (Wilcox, 1998, Pope, 2000). And although these coefficients vary from one model to another they do not cause large problems in modelling of neutral flow over non-obstructed surface.

The fundamental uncertainty about what is the best way to treat the dissipation mechanisms in the presence of vegetation under conditions of non-neutral air stratification still remains the main problem for development of models based on both ε and ω closures. A number of researchers have emphasized that these models suffer from ambiguities in description of both plant drag and buoyancy effects in E - and φ – equations (e.g. Wilson et al. 1998). For example, the coefficient $C_{\varphi 3}$ is still not defined adequately for φ. For example, only value of the coefficient in dissipation equation, $C_{\varepsilon 3}$, ranges from −1.4 to +1.45 (see Baumert and Peters (2000), for review). Different formulations are given for the terms S_p and S_d and there are uncertainties in coefficients $C_{\varphi 4}$ and $C_{\varphi 5}$ (see Sogachev and Panferov (2006) for review). Very often coefficients $C_{\varphi 3}$, $C_{\varphi 4}$ and $C_{\varphi 5}$ are fitted to agree with one or another experimental data set. Thus, the set of coefficient fitted for certain conditions can not be applied for other. That limits considerably the applicability of two-equation models.

For convenience, we denote the terms in parenthesis on the right side of Eqs. (1) - (2) – P and ε as shear source and shear sink of TKE, as they are well defined by the wind shear. In contrast, the terms in square brackets on the right side of Eqs. (1) - (2) we denote as non-shear sources/sinks because their values can not be derived from the wind shear alone. Although such definition is not absolutely correct, it allows us to distinguish terms in E and φ equations which cause the smallest and the largest uncertainties.

3. TREATMENT OF AERODYNAMIC DRAG

Recently, Sogachev and Panferov (2006) enhanced the description of TKE dissipation mechanism in such models which extended the models generality and applicability to neutral inhomogeneous canopy flow. Assuming the neutral steady-state homogenous shear flow reduces Eq. (2) to (Pope, 2000):

$$C_{\varphi 1}P - C_{\varphi 2}\varepsilon = 0. \tag{3}$$

It means that for such a kind of flow the following ratio should be held:

$$\frac{P}{\varepsilon} = \frac{C_{\varphi 2}}{C_{\varphi 1}} \tag{4}$$

The modification proposed by Sogachev and Panferov (2006) is due to the fact that the model constants estimated experimentally for 'free-air' flow do not allow for adequate reconstruction of the ratio between the production and dissipation rates of turbulent kinetic energy, ε, (Eq. 4) in the vegetation canopy:

$$\frac{P + S_p}{\varepsilon + S_d} = \frac{C^*_{\varphi 2}}{C_{\varphi 1}} \tag{5}$$

and have to be adjusted. Assuming that additional production and the enhanced dissipation of E due to the interaction of air flow with leaves compensate each other as $S_p = S_d$, authors corrected the coefficient $C_{\varphi 2}$ as

$$C^*_{\varphi 2} = C_{\varphi 2} - \frac{\left(C_{\varphi 2} - C_{\varphi 1}\right)}{\varepsilon} S_d \tag{6}$$

It resulted in removing of terms S_p and S_d from Eq. (1) and (2) and eliminated any uncertainties about $C_{\varphi 4}$ and $C_{\varphi 5}$. However, some assumption about S_d had to be made and this was provided and tested by Sogachev and Panferov (2006). The suggested modification of two-equation models to account for plant drag was found robust. It is quite universal, i.e. of the same type for all two-equations models considered, and performs well for wide range of canopies. Authors suggested also that E - ε and E - ω schemes were more promising than the E - El scheme for canopy flow simulation since they were not limited by the need to use a wall function.

4. GENERAL RULE OF TREATMENT OF NON-SHEAR SOURCES IN TWO-EQUATION MODELS

Sogachev and Panferov (2006) used in their model the corrected coefficient $C^*_{\varphi 2}$ as it was given by Eq. (6). But if we put $C^*_{\varphi 2}$ in Eq. (2) directly we can see that it is still in initial form but the coefficients $C_{\varphi 4}$ and $C_{\varphi 5}$ are now:

$$C_{\varphi 4} = 0 \text{ and } C_{\varphi 5} = \left(C_{\varphi 2} - C_{\varphi 1}\right) \tag{7}$$

Thus the Eq. (2) can be rewritten as (buoyancy effect is omitted for a while):

$$\frac{\partial \varphi}{\partial t} + \left\langle \overline{u}_j \right\rangle \frac{\partial \varphi}{\partial x_j} - \frac{\partial}{\partial x_i}\left(\frac{K}{\sigma_\varphi}\frac{\partial \varphi}{\partial x_i}\right) = \frac{\varphi}{E}\left(C_{\varphi 1}P - C_{\varphi 2}\varepsilon\right) + \frac{\varphi}{E}\left(C_{\varphi 1} - C_{\varphi 2}\right)S_d \tag{8}$$

The question arises now whether it is possible to get this expression directly without introducing any correction coefficient. Let us consider the Eq. (1) again with additional source/sinks due to interaction with vegetation only. Lacking the exact expression for canopy source/sinks (denoted as S) we can write its right side as

$$... = P - \varepsilon + S \tag{9}$$

Because left sides in Eqs. (1)-(2) are unchanged we do not show them in the following discussion. Let us assume that the interaction with vegetation could lead both to production and to losses of TKE. In other words it can change magnitudes of both P and ε and these changes would be equal to each other and proportional to S with some arbitrary chosen coefficients α. So we obtain:

$$... = \left(P + \alpha S\right) - \left(\varepsilon + \alpha S\right) + S \tag{10}$$

Eq. (10) is the same as Eq. (9), but allow us to introduce new terms for total production $P^* = \left(P + \alpha S\right)$ (by shear plus by non-shear) and for total dissipation $\varepsilon^* = \left(\varepsilon + \alpha S\right)$. Furthermore it is assumed that exactly these production and dissipation should be presented in supplementary equation and noting else. Then for the right side of Eq. (2) we have

$$... = \frac{\varphi}{E}\left(C_{\varphi 1}P^* - C_{\varphi 2}\varepsilon^*\right) \tag{11}$$

In this case the following ratio should be still correct for homogeneous shear flow even with additional sources/sinks

$$\frac{P^*}{\varepsilon^*} = \frac{C_{\varphi 2}}{C_{\varphi 1}}. \tag{12}$$

Thus we don't need to correct any coefficients. Instead we have an alternative representation of production and dissipation terms in supplementary equation:

$$... = \frac{\varphi}{E}\left(C_{\varphi 1}\left(P+\alpha S\right)-C_{\varphi 2}\left(\varepsilon+\alpha S\right)\right) \tag{13}$$

or

$$... = \frac{\varphi}{E}\left(C_{\varphi 1}P-C_{\varphi 2}\varepsilon\right)+\frac{\varphi}{E}\left(C_{\varphi 1}-C_{\varphi 2}\right)\alpha S \tag{14}$$

The right side of Eq. (14) is similar to the right side Eq. (8) except that S in Eq. (14) is not S_d. Moreover, due to the term S is not removed from Eq. (9), it looks like that the parameterization proposed by Sogachev and Panferov (2006) is not quite correct, despite of its successful verification. A new justification of S is required (and this can certainly be done). However, if we apply the assumption $S = S_p - S_d$ (that we have ignored in above discussion), we get

$$... = P-\varepsilon+S_p-S_d \tag{15}$$

$$... = \left(P+S_p\right)-\left(\varepsilon+S_d\right). \tag{16}$$

From which we obtain

$$... = \frac{\varphi}{E}\left(C_{\varphi 1}P-C_{\varphi 2}\varepsilon\right)+\frac{\varphi}{E}\left(C_{\varphi 1}-C_{\varphi 2}\right)S_d, \tag{17}$$

when $S_p = S_d$. Thus, the parameterization suggested by Sogachev and Panferov (2006) is still correct under the assumption mentioned above. The equation (14) can be considered as the general form to take into account any non-shear source/sinks of TKE, S in supplementary equation. It is clearly demonstrated when the buoyancy term is present in E-equation, which cannot be zero here (except the case of neutral stratification) as the join effect of S_p and S_d.

5. TREATMENT OF BUOYANCY PRODUCTION

5.1 Modification

In similar way as it was done above for arbitrary source/sink S (Eqs. (9)-(10) and (14)) we corrected the production/dissipation ratio in presence of buoyancy force in E – equation

$$... = P-\varepsilon+B \tag{18}$$

$$... = \left(P+\beta B\right)-\left(\varepsilon+\beta B\right)+B. \tag{19}$$

Here β denotes the fraction of the buoyancy force being converted to the energy of TKE pulsations, or energy's losses. With $P^* = \left(P+\beta B\right)$ and $\varepsilon^* = \left(\varepsilon+\beta B\right)$ we finally get in φ-equation:

$$... = \frac{\varphi}{E}\left(C_{\varphi 1}P-C_{\varphi 2}\varepsilon\right)+\frac{\varphi}{E}\left(C_{\varphi 1}-C_{\varphi 2}\right)\beta B. \tag{20}$$

To test the suggested treatment of non-shear source/sinks the numerical experiments were carried out using E-ω model. The Eqs. (1)-(2) under the assumption made above take the form

$$\frac{\partial E}{\partial t}+\left\langle \overline{u}_j\right\rangle\frac{\partial E}{\partial x_j}-\frac{\partial}{\partial x_i}\left(\frac{K}{\sigma_E^{\omega}}\frac{\partial E}{\partial x_i}\right)=P-\varepsilon+B \tag{21}$$

$$\frac{\partial \omega}{\partial t}+\left\langle \overline{u}_j\right\rangle\frac{\partial \omega}{\partial x_j}-\frac{\partial}{\partial x_i}\left(\frac{K}{\sigma_\omega}\frac{\partial \omega}{\partial x_i}\right)= \frac{\omega}{E}C_{\omega 1}P-C_{\omega 2}\omega^2+\frac{\omega}{E}\left(C_{\omega 1}-C_{\omega 2}\right)\left(S_d+\beta B\right) \tag{22}$$

To provide a suitable solution in the atmospheric boundary layer (ABL) the coefficient $C_{\omega 1}$ is corrected as suggested by Apsley and Castro (1997):

$$C_{\omega 1}^* = \left[C_{\omega 1}+(C_{\omega 2}-C_{\omega 1})\frac{l}{l_{\max}}\right]. \tag{23}$$

Where $l_{\max}$ is the maximal value of l at the upper border of neutral ABL estimated using the equation by Blackadar (1962) for neutral stratification as: $l_{\max} = 0.00027G/f$, where G is the geostrophic wind speed and f is the Coriolis parameter. For original value of coefficients $\sigma_E^{\omega}, \sigma_\omega$, $C_{\omega 1}$ and $C_{\omega 2}$ see Wilcox (1998). Coefficient β was approximated as 1. The Prandtl number as a function of Richardson number was taken as in Sogachev et al. (2002). We demonstrate the results of two numerical experiments.

5.2 Verification

Figure 1 shows a modelled daily cycle of wind field in ABL as well as the main surface characteristics of ABL such as friction velocity, u_* and Monin-Obukhov length scale, L. They were estimated under conditions typical for a summer day on July 1 at the 50^o latitude (temperature and its gradient (273K and 0.0098 K m^{-1}) at the upper border of model domain (about 3 km), geostrophic wind (10 m s^{-1})) above a flat underlying surface with aerodynamic roughness z_0 = 0.03 m. Figure 2 compares the wind field derived by E-ω model to the one derived analytically in atmospheric surface layer. Analytical wind was derived from modelled u_* and L according to expressions suggested by Paulson (1970). Figure 3 compares wind profiles derived by different models at several points of time. One can see that the modelled and analytical profiles are in a good agreement. Some differences between profiles occur during a transition time period when Monin-Obukhov length changes its sign and inversion in atmosphere at some height above the surface is present. It should be noted, however, that analytical solution is based on data estimated for underlying surface only and is unable "to feel" the real atmosphere.

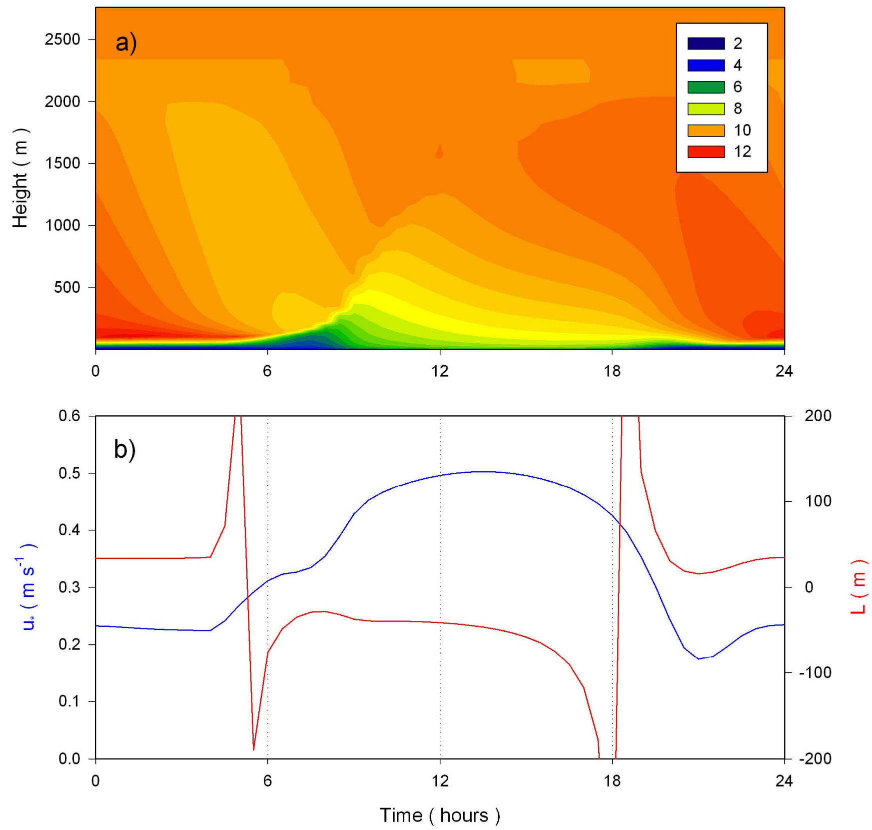

Figure 1. Wind speed (m s^{-1}) (a) and surface ABL characteristics: u (m s^{-1}) and Monin-Obukhov length, L (m) (b) derived by E-ω model above low-roughness surface under conditions typical for a summer day.*

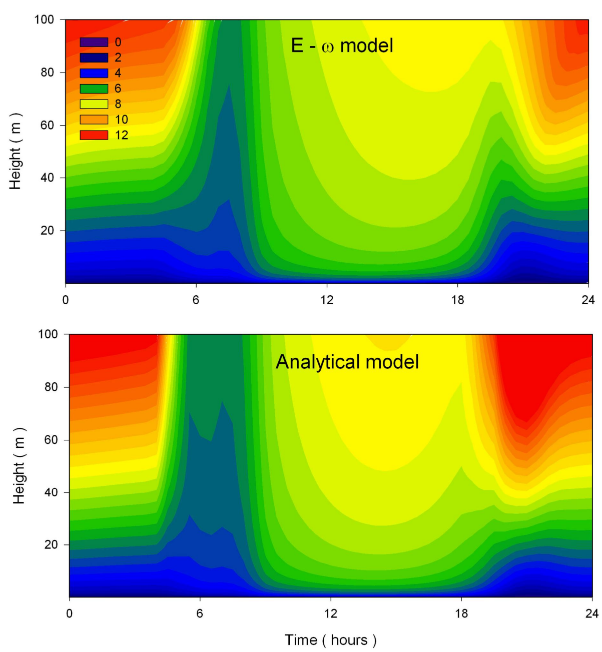

Figure 2. Wind speed (m s^{-1}) derived by E-ω and analytical models in the atmospheric surface layer.

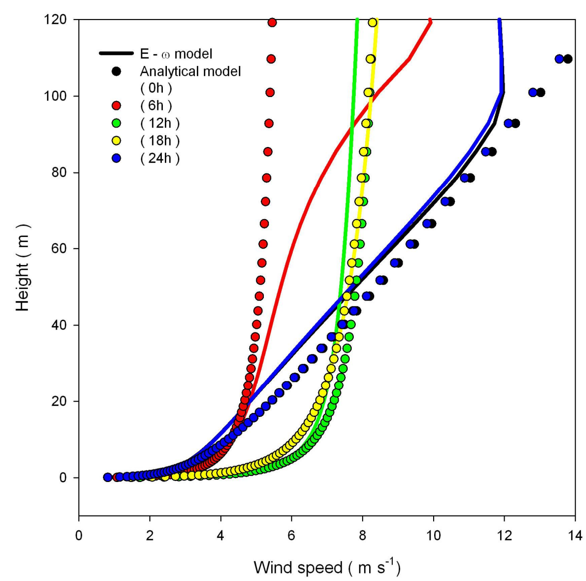

Figure 3. Wind speed profiles derived by E-ω and analytical models in the atmospheric surface layer for different hours. .

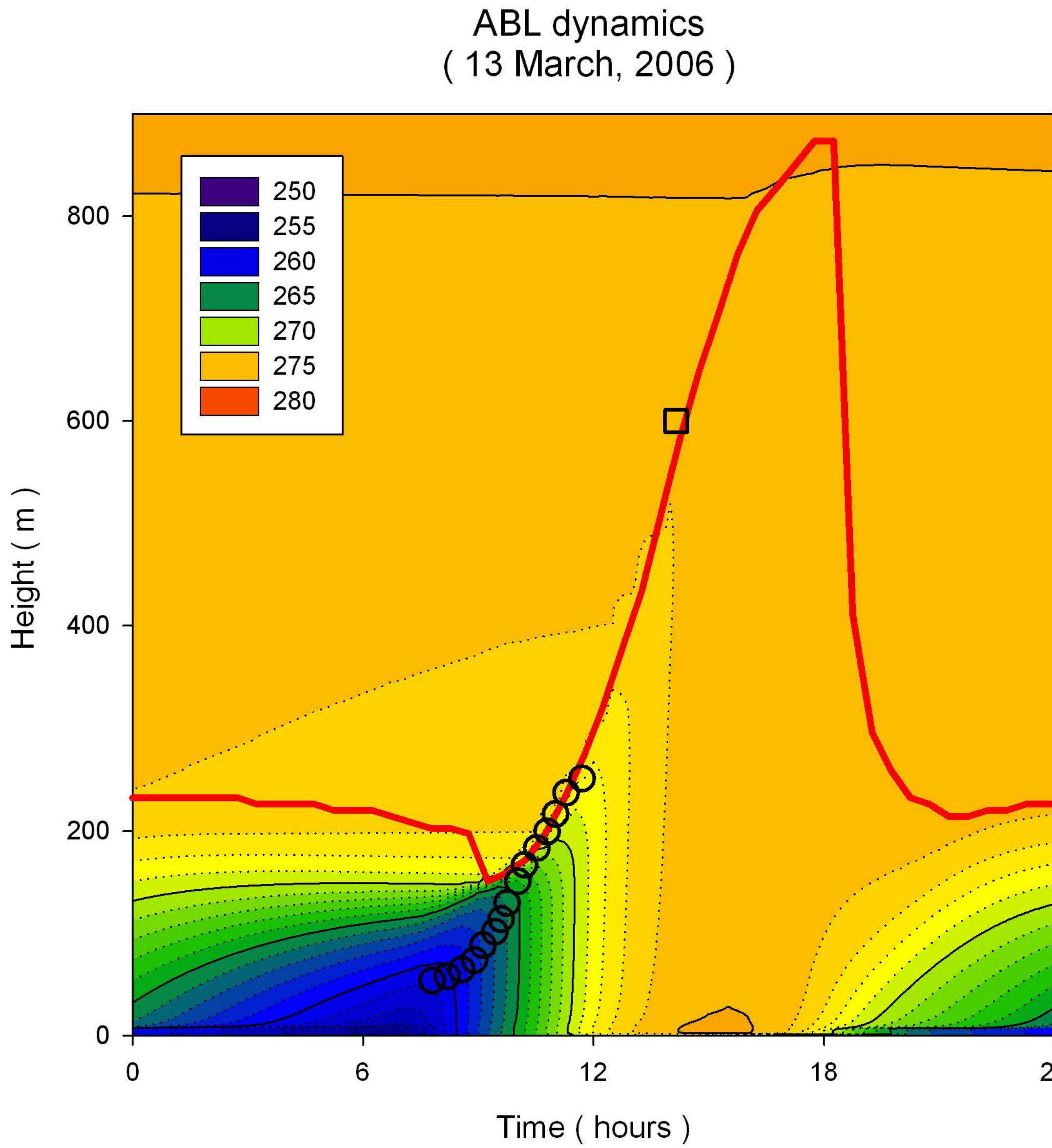

Figure 4. ABL dynamics above forested surface on March 16, 2006. Colour field shows modelled potential temperature, lines – modelled mixing height estimated as a level with Ri = 0.25, open circles and square indicate the mixing height derived from observations of sodar and of air balloon flight, respectively.

Figure 4 compares ABL dynamics represented by changes of modelled potential temperature to that provided by sodar measurements carried out during early morning hours and by air balloon measurements at about 2 pm above Hyytiälä on 13 March 2006 (Laakso et al., 2007). Hyytiälä is a measuring station located in South Finland (62^{o}, forest with height of 15 m and total leaf area index = 7) (Hari and Kulmala, 2005). Modelled data were derived with input for radiation provided from Hyytiälä station and upper border conditions for wind and temperature provided from soundings data of meteorological station located about 100 km from Hyytiälä during whole March. Figure indicates that diurnal dynamics of modelled ABL height estimated as a level where Ri number exceed 0.25 are in a good agreement with observation.

6. CONCLUSIONS

The results showed that the suggested modification of two-equation models for non-neutral flow performs well. No additional model coefficients except well-treated $C_{\varphi 1}$ and $C_{\varphi 2}$ are needed. Thus, we hope that the potential of two-equation models, which are already fully implemented in engineering, could be realized with the same success in the environmental research.

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