P.2 A MULTIVARIATE ANALYSIS OF SUMMARY-OF-THE-DAY SNOWFALL STATISTICS VS. SAME-DAY WATER PRECIPITATION AND TEMPERATURE RECORDINGS

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1. INTRODUCTION

A familiar summary-of-the-day statistic included in Local Climatological Data and U.S. Cooperative station monthly summaries is snowfall. In the COOP's, particularly, only temperature and water-equivalent precipitation observations likely appear more frequently, snowfall less so only because of seasonal and geographical constraints.

Surface snowfall measurements differ somewhat from those of temperature and water-precipitation in that they are more inexact and subjective, especially when the snow is very light, dry, mixed with rain/semi-frozen precipitation, or accompanied by high winds. In short "the measurement of snowfall is difficult at best" [University of Wyoming, 2007]. In addition, surface snowfall measurements and the mix, if any, with non-snow types are influenced by the thermal structure aloft, the exact character of which is not directly quantifiable from surface observations. This complicates the overall statistical relationships among the three, and if for some reason, daily snowfall estimations were to be attempted as a function of surface temperature and water-equivalent precipitation, inaccuracies would undoubtedly result, the errors' character for all cases, however, not known.

The purpose of this study is to investigate the nature of the statistical relationships between same-day surface temperature (daily max, min, average, or range); and water-equivalent precipitation (independent variables) vs. recorded snowfall (dependent variable). The empirical regression models will be analyzed and tested as prediction and reconstruction tools in the estimation of daily, monthly, and seasonal snowfall totals that might have been excluded during certain periods of record. An example of this is some of the pre-1900 Army Signal Corps era years, in which daily snowfall measurements sometimes did not accompany those of water precipitation and temperature.

Such reconstructions, of course, would only be valid if the procedures and instrumentation for snowfall measurement and water precipitation were comparable across different eras, another relative unknown.

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2. DATA AND PROCEDURES

The empirical data were LCD daily temperature, precipitation, and snowfall recordings for Minneapolis-St. Paul, MN, covering the period October 1964 thru April 2007, excluding the 2000-01 thru 2003-04 seasons. Over the latter period, official snowfall measurements were temporarily moved to the nearby Chanhassen station. Snowfall data for the months October through April were analyzed as one unit, the assumption being that the regression relationships were relatively constant across calendar month, and it made for a larger sample size to work with.

Initial trial and error regression experimentation using daily mean temperature and precipitation as predictors determined that owing to the zero-bounded/positively-skewed nature of precipitation, and the critical 32 F phase-change temperature, a single all-inclusive regression model would be ineffective, the analysis more tractable if the data were partitioned into various water-equivalent precipitation/mean temperature subclasses. These five groups were: "light" precipitation/"cold" mean temperature, "moderate" precipitation./"cold" means, "heavy" precipitation./"cold" means, "light" precipitation./"mild" means, and ""non-light" precipitation./"mild" means. The "light" precipitation category extended from .01" to .06", "moderate" from .07" to .42"; "heavy" from .43" and above, and the "non-light" from .07" and above. The "cold" mean category encompassed daily average temperatures of 27.5 F or lower, the "mild" category 28 F or higher. In addition, a 33 F upper-limit constraint on daily minimum temperatures was imposed as well as a 46 F upper-limit on daily maxima. "Trace" water precipitation cases, by necessity, were also excluded.

This particular scheme, of course, was not necessarily the most optimal. A larger data base, including observations from other stations, might have resulted in different, more refined delineations.

3. DATA-PARTITIONED REGRESSION RESULTS AND MODELS

First partitioning of the data set involved attempting to segregate as best as possible snow-only cases from the more mixed (snow/rain, sleet, or rain-only) varieties. To this end, a 27.5 F daily mean temperature threshold was established, means at or below that level assigned to the "cold" group, the 28.0 F or higher cases to the "mild" group. Further trial-and-error regression iterations resulted in more breakdowns by water-equivalent precipitation.

3.1 – "Light" Water-Equivalent Precipitation (.01" to .06")/ "Cold" Mean Temperature (<=27.5 F) Regression

This subcategory was created in response to preliminary results in which low magnitude water-equivalent precipitation observations resulted in too high snowfall estimates, the over-influence of heavier precipitation observations on model coefficients (regression "leverage").

Table 1 - Regression Statistics for "Light" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Estimation Model (Precip.<=.06", Mean Temp <=27.5 F)

Dep Var: SNOWFALL n: 566 Multiple R: +.771					
Adj Sq Multiple R: 0.593 Standard Error: 0.210					
Effect	Coeff	Std Error	Toler -ance	t-value	P two-tail
CONSTANT	0.259	0.038		6.825	.000
PRECIP	15.413	0.554	1.000	27.803	.000
AVGADJ	-0.007	0.001	1.000	- 7.532	.000

Analysis of Variance					
Source:	Sum-of-Squares	df	Mean Sq	F-Ratio	P
Regression	36.291	2	18.146	412.320	.000
Residual	24.777	563	0.044		

Based on an n=566 sample size, model results (Table 1) had both water-equivalent precipitation and daily mean temperature as significant explanatory variables (regression coefficient t-values each significant beyond the .0005 level). Multiple correlation coefficient was +.771, standard error 0.210 in., and the F-statistic (412.320), significant beyond the .0005 level also. Tolerance statistics were at a maximum (1.000), indicating no multicollinearity (intercorrelation) complications with the predictor variables.

The regression model expression was SNOWFALL= 0.259 + (15.413*PRECIP) + (-.007*AVGADJ), where "SNOWFALL" was the estimated snowfall (in.) for the day, "PRECIP" the day's water-equivalent precipitation (in.), and "AVGADJ" the mean temperature (deg F), scaled by adding 20 F to insure that all the daily means would be positively signed.

The slightly negative coefficient for AVGADJ indicates that warmer (colder) daily means had a lessening (enhancing) effect on estimated snowfall amounts.

This model, like the other four, was the result of two runs, observations from the first that generated software-flagged extreme leverage and standardized residuals' statistics thrown out. In this instance, twelve cases or 2.1% were removed for run two.

3.2 – "Moderate" Water-Equivalent Precipitation (.07" to .42")/ "Cold" Mean Temperature (<=27.5 F) Regression

This "Cold " mean temperature subcategory covered water-equivalent precipitation cases between 0.07" and 0.42". Trial-and-error regression model fitting identified three independent variables with coefficient magnitudes beyond the .0005 level of significance: water-equivalent precipitation ("PRECIP"), daily mean temperature +20 F ("AVGADJ"), and a new variable, "RANGE05", the square root of the daily temperature range (daily maximum less daily minimum).

Table 2 - Regression Statistics for "Moderate" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Estimation Model (Precip.>=.07" & <=.42", Mean Temp <=27.5 F)

Dep Var: SNOWFALL n: 390 Multiple R: +.856					
Adj Sq Multiple R: 0.730 Standard Error: 0.611					
Effect	Coeff	Std Error	Toler -ance	t-value	P two-tail
CONSTANT	2.081	0.219	.	9.498	.000
PRECIP	12.331	0.388	0.981	31.821	.000
AVGADJ	-0.031	0.004	0.882	- 8.312	.000
RANGE05	-0.186	0.033	0.896	- 5.569	.000

Analysis of Variance					
Source:	Sum-of-Squares	df	Mean Sq	F-Ratio	P
Regression	394.365	3	131.545	352.282	.000
Residual	144.136	386	0.373		

From Table 2, the regression model expression was SNOWFALL= 2.081 + (12.331*PRECIP) + (-.031* AVGADJ) + (-.186*RANGE05). Multiple correlation was +.856, standard error 0.611 in., and F-statistic 352.282, all significant beyond the .0005 level.

The coefficient sign for AVGADJ was again negative, the same true for RANGE05, the latter indicating that higher (lower) daily ranges contributed to lower (higher) snowfall amounts, probably reflecting the inverse association between diurnal temperature spread and cloudiness (or precipitating time). Tolerance statistics again indicated negligible multicollinearity.

Sixteen cases or 3.9% of the original sample were removed after the first run.

3.3 – "Heavy" Water-Equivalent Precipitation (>=.43")/ "Cold" Mean Temperature (<=27.5 F) Regression

This group contained the high-end water-equivalent precipitation cases for the cold daily means' group. Initial regression results using precipitation levels above the light category (>=.07") frequently overstated the snowfall amounts at higher levels, and since the errors could be quite large in absolute terms, the decision was made to fit these observations separately.

The model selection process identified water-equivalent precipitation ("PRECIP"), daily mean temperature +20 F ("AVGADJ"), and daily temperature range ("RANGE") as effective explanatory variables, the expression being: SNOWFALL= 19.237+(7.266 * PRECIP) + (-0.346*AVGADJ) + (-0.245*RANGE).

Table 3 - Regression Statistics for "Heavy" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Estimation Model (Precip.>=.43", Mean Temp <=27.5 F)

Dep Var: SNOWFALL	n: 41	Multiple R: +.634			
Adj Sq Multiple R: 0.354	Standard Error: 3.293				
Effect	Coeff	Std Error	Toler -ance	t-value	P two-tail
CONSTANT	19.237	4.004	.	4.805	.000
PRECIP	7.266	2.039	0.921	3.564	.001
AVGADJ	-0.346	0.088	0.906	- 7.930	.000
RANGE	-0.245	0.087	0.900	- 2.808	.008
Analysis of Variance					
Source:	Sum-of-Squares	df	Mean Sq	F-Ratio	P
Regression	270.176	3	90.059	8.306	.000
Residual	401.164	37	10.842		

From Table 3, multiple correlation was +.634, the standard error 3.293 in. The F-statistic (8.306) was significant beyond the .0005 level, and the coefficients, all beyond at least the .008 level. Three or 6.8% of the original 44 observations were excluded for run two.

3.4 – "Light" Water-Equivalent Precipitation (.01" to ".06"/ "Mild" Mean Temperature (>=28.0 F) Regression

As previously stated, the "mild" daily mean temperature category (>=28.0 F) was created to set apart daily observations in which precipitation phase-change events were potentially more likely, influencing snowfall estimation accuracy. Presumably, different regression model types would be suitable. First results confirmed this – daily *maximum* temperature was a better explanatory variable than adjusted daily mean temperature, the latter not statistically significant at a high level.

Like the "cold" daily mean cases, first results also showed frequently overestimated snowfall amounts at low water-precipitation levels. An identical .06" cutoff was therefore established, creating another low-end precipitation sub-grouping.

Table 4 - Regression Statistics for "Light" Water-Equivalent Precipitation & "Mild" Daily Mean Temperature Snowfall Estimation Model (Precip.<=.06", Mean Temp >=28.0 F)

Dep Var: SNOWFALL	n: 254	Multiple R: +.509			
Adj Sq Multiple R: 0.253	Standard Error: 0.172				
Effect	Coeff	Std Error	Toler -ance	t-value	P two-tail
CONSTANT	0.551	0.104	.	5.285	.000
PRECIP	5.017	0.669	0.997	7.498	.000
MAX	-0.014	0.003	0.997	- 5.225	.000
Analysis of Variance					
Source:	Sum-of-Squares	df	Mean Sq	F-Ratio	P
Regression	2.610	2	1.305	43.941	.000
Residual	7.454	251	0.030		

From Table 4, model results had both water-equivalent precipitation and daily maximum temperature as significant (beyond the .0005 level of significance) predictors. Multiple correlation coefficient was +.509, standard error 0.172 in., and the F-statistic (43.941) significant beyond the .0005 level. Tolerance statistics were each at a near optimum level (0.997), indicating no multicollinearity..

The regression model expression was SNOWFALL= 0.551 + (5.017*PRECIP) + (-0.014*MAX), where "SNOWFALL" was estimated snowfall, "PRECIP" the water-equivalent precipitation, and "MAX" the daily maximum temperature (deg F).

The slightly negative coefficient for "MAX" indicated that warmer (colder) daily maxima had a lessening (enhancing) effect on reported snowfall amounts, an intuitively reasonable result.

Two observations were removed after the first run.

3.5 – – "Non-Light" Water-Equivalent Precipitation (>.06")/ "Mild" Mean Temperature (>=28.0 F) Regression

Based on its nature and relatively large sample size, this subcategory was the probably the most problematic for accurate daily snowfall estimation, the higher water-equivalent values creating the potential for relatively frequent and uncertain phase-change issues. Trial and error fitting with both linear and non-linear models identified the expression:

SNOWFALL= -3.563 + (4.346 * SQRT(PRECIP) + (3969.927*MAX^-2) as the best-fit simple model on an F-ratio basis (see Table 5 below).

Table 5 - Regression Statistics for "Non-Light" Water-Equivalent Precipitation & "Mild" Daily Mean Temperature Snowfall Estimation Model (Precip.>=.07", Mean Temp >=28.0 F)

Dep Var: SNOWFALL	n: 308	Multiple R: +.617			
Adj Sq Multiple R: 0.377	Standard Error: 1.235				
Effect	Coeff	Std Error	Toler -ance	t-value	P two-tail
CONSTANT	-3.563	0.408	.	-8.748	.000
PRECIP^0.5	4.346	0.391	0.999	11.123	.000
1/(MAX^2)	3969.927	475.823	0.999	8.343	.000
Analysis of Variance					
Source:	Sum-of-Squares	df	Mean Sq	F-Ratio	P
Regression	286.116	2	143.058	93.732	.000
Residual	465.506	305	1.526		

Coefficient magnitudes for both the PRECIP^0.5 and 1/(MAX^2) terms were significant beyond the .0005 level, the multiple correlation coefficient +.617, and the standard error 1.235 in. The F-statistic (93.732) was also significant beyond the .0005 level, the tolerance statistics at near optimal levels (both 0.999). Fifteen observations (or 4.6%) were removed for the second run.

In summary, partitioning of the original observations into five sub-groupings helped enable generation of regression models with coefficient magnitudes and F-ratios all statistically significant beyond the .0005 level. Standard errors averaged 0.212 in. for the two "light" precipitation categories (results in Tables 1 and 4), 0 866 in. for the "cold" mean temperature, "moderate" and "heavy" categories (Tables 2 and 3), and 1.235 in. for the "mild" mean temperature/"non-light" group (Table 5). The higher figure for the latter, of course, relates to the greater uncertainty regarding snow-only precipitation for those days with means that were near 32 F.

3.6 – Individual Regression Models' Residuals

Next, the models' residuals were evaluated using a variety of diagnostics' tests and graphs. While the models' purpose was to be of point estimation and summation only (no confidence intervals to be constructed, for example), knowledge of these diagnostics is an integral part of regression analysis.

Ideally, model residuals should normally distributed, with equal variances and no dependencies across predicted value ranges. Hopefully, the data partitioning had largely neutralized the effects of the zero-bounded/positively skewed nature of snowfall and precipitation, particularly at the higher magnitude levels.

3.6.1. - "Light" Precipitation/ "Cold" Means Model

Figure 1 below is a cumulative normal p-plot of the residuals generated by the "Light" Water Precipitation/ "Cold" Daily Means' model

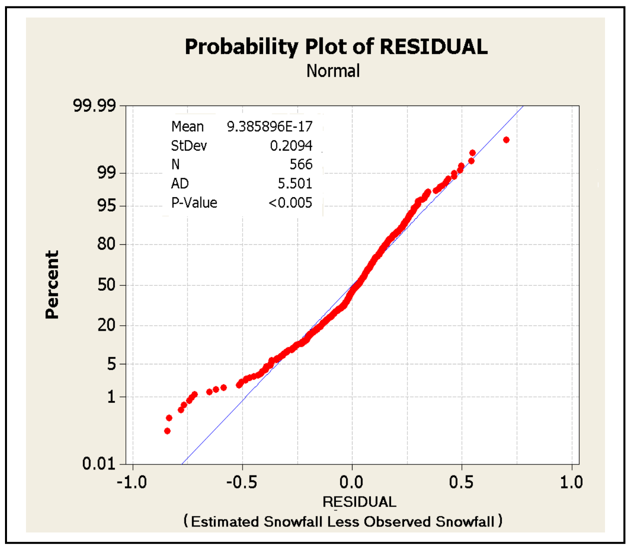

Figure 1 - Normal Probability Plot for Residuals of Light" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Estimation Model

The residuals were statistically non-normal, the Anderson-Darling statistic ("AD" in the inset), rejecting the null-hypothesis ("P-Value") beyond the .005 level. Much of this was attributable to a disproportionate number of cases with relatively large underestimations of actual snowfall amounts (lower-left graph portion). This was likely caused by the fact that in this particular light precipitation category (.06" or less melted), actual snowfall measurements were relatively unbounded.

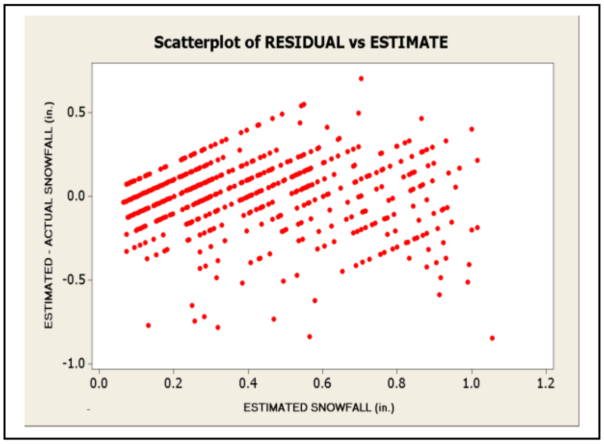

Figure 2 - Scatterplot of Light" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Model Residuals vs. Model Estimated Snowfall

on the high side compared to the lower, zero-bounded one. This was possible at lower temperatures levels where appreciable snowfall of a low water-content could be received. Practically speaking though, these error outliers made up less than 5% of the total sample (n=566), and due to the constraints of the category, their magnitudes were seldom more than 0.5" in absolute terms.

Figure 2 is scatterplot of the residuals as a function of predicted snowfall amount. Again, reflecting to some extent the zero-bound constraint, the points are not randomly distributed. The zero-bound precludes points from appearing in the graph's upper-left corner (low magnitude, highly overestimated predictor region), and the few highly negative residual points stretching across the chart's lower portion reflected the lower-left tail configuration (underestimated predictions) shown in Figure 1.

3.6.2 - "Moderate" Precipitation/ "Cold" Means Model

Figure 3 is the cumulative normal p-plot of the residuals generated by the "Moderate" Water Precipitation/ "Cold" Daily Means' model. Except for the extreme negative and positive ends of the x-axis, the residuals conformed quite well to the normality diagonal, the overall departure pattern reflecting "stretched-out tails", or a relatively greater proportion of both high-end overestimates and underestimates. The Anderson-Darling statistic rejected normality at the .014 level, but these extreme-end nonconformance cases were associated with perhaps 8% or less of the 390 cases.

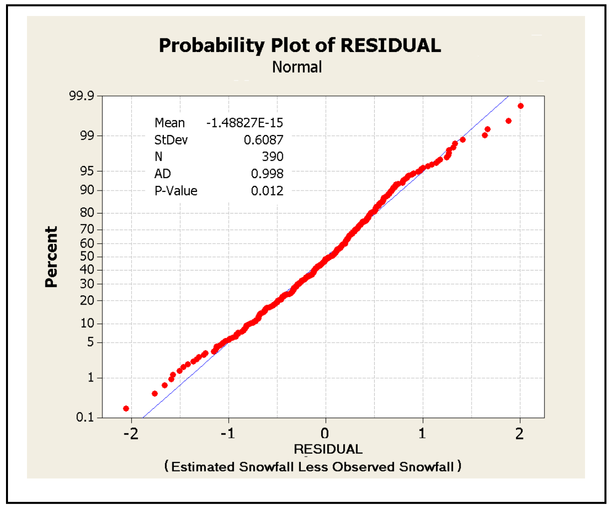

Figure 3 - Normal Probability Plot for Residuals of "Moderate" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Estimation Model

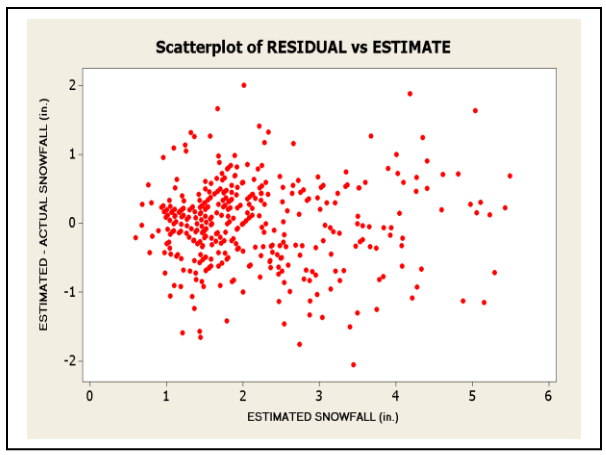

Figure 4 - Scatterplot of "Moderate" Water-Equivalent Precipitation & "Cold" Daily Mean Temperature Snowfall Model Residuals vs. Model Estimated Snowfall

In the residuals versus estimated snowfall scatterplot (Figure 4), the points' also seem to be well distributed over the x-axis range, reinforcing the notion that this model, from a practical point estimation and summation standpoint was satisfactory.

3.6.3 - “Heavy” Precipitation/ “Cold” Means Model

Figure 5 below shows the distribution of residuals for the “Heavy” Water-Equivalent Precipitation & ”Cold” Daily Mean Temperature Model

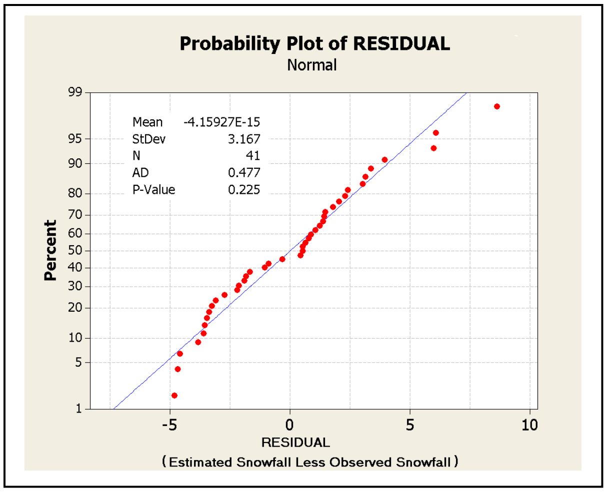

Figure 5 - Normal Probability Plot for Residuals of “Heavy” Water-Equivalent Precipitation & ”Cold” Daily Mean Temperature Snowfall Estimation Model

The points, just 41 in number, show a mostly good fit with the normality diagonal, the Anderson-Darling statistic significant at just the .225 level.

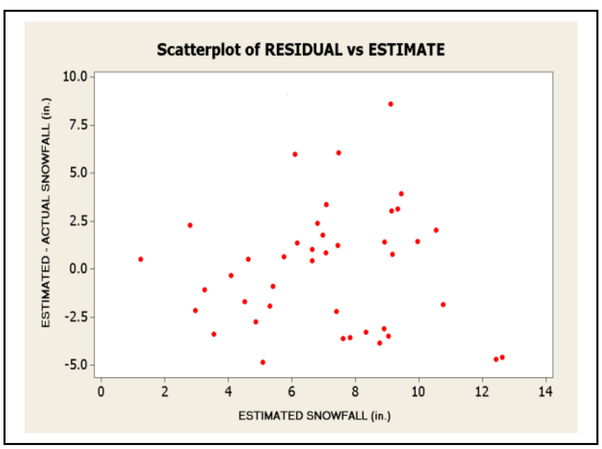

Figure 6 - Scatterplot of “Heavy” Water-Equivalent Precipitation & ”Cold” Daily Mean Temperature Snowfall Model Residuals vs. Model Estimated Snowfall

In Figure 6, the most noticeable feature is the increased variance over the 6”-10” range on the estimated snowfall axis. Some of the errors are rather large.

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3.6.4 - “Light” Precipitation/ “Mild” Means Model “

Figure 7 shows the ordered residuals for the “Light” Water-Equivalent Precipitation & ”Mild” Daily Mean Temperature Model

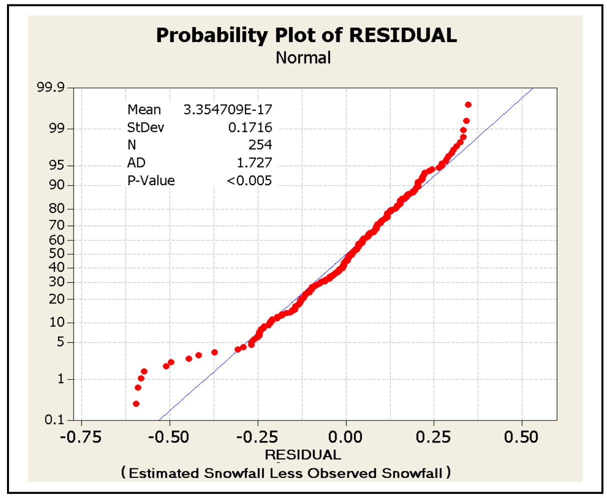

Figure 7 - Normal Probability Plot for Residuals of “Light” Water-Equivalent Precipitation & ”Mild” Daily Mean Temperature Snowfall Estimation Model

Similar to the “Cold” mean temperature counterpart model, there was a disproportionate number of relatively pronounced underestimations of snowfall (lower-left of graph), the orientation of the opposite tail, however, “compressed”, reflecting fewer than expected relatively high overestimates. These combined features caused the hypothesis of normality to be rejected at the .005 level, but the absolute number of these “suspect” cases (~15) is only about 6% of the sample size (n=254).

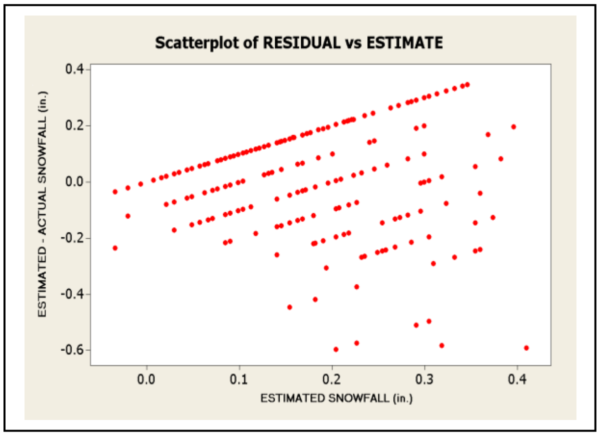

Figure 8 - Scatterplot of Light" Water-Equivalent Precipitation & "Mild" Daily Mean Temperature Snowfall Model Residuals vs. Model Estimated Snowfall

In the residuals versus estimated chart (Figure 8), the points, like that for the counterpart "cold" mean temperature model are non-randomly distributed and quantized. There is also the arrangement of highly negative residual points extending across the chart's lower portion, from about 0.15'" and higher.

A small number of slightly negative snowfall estimates (points to the left of the x=0 position) were also produced. In actual practice, these would be set to zero.

3.6.5 - "Non-Light" Precipitation/ "Mild" Means Mode"

Figure 9 shows the normal distribution p-plot of the "Non-Light" Water-Equivalent Precipitation and "Mild" Mean Temperature model residuals.

Repeating the features shown in Figures 1 and 4, there was a tendency for a disproportionate number of appreciable underestimations of snowfall (lower-left configuration of points), contributing to rejection of the normality hypothesis at the .006.level. These errors, of course, were more significant in absolute magnitude than those depicted in Figures 1 and 4, the latter associated with the "light" water-equivalent precipitation models. Total frequencies, however, only numbered about 15, less than five percent of the n=308 sample size.

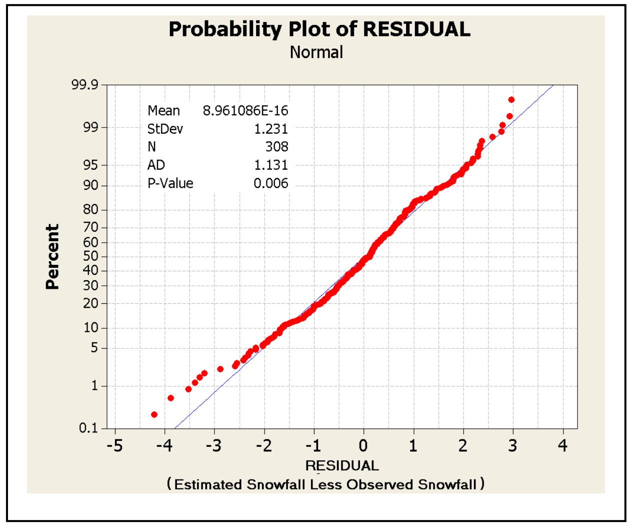

Figure 9 - Normal Probability Plot for Residuals of "Non-Light" Water-Equivalent Precipitation & "Mild" Daily Mean Temperature Snowfall Estimation Model

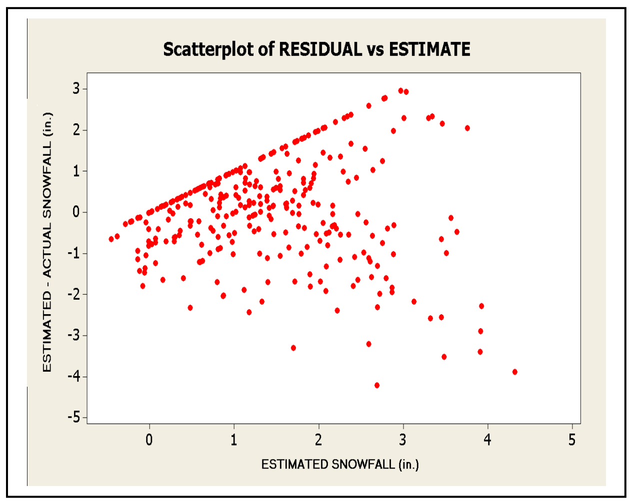

Figure 10 - Scatterplot of "Non-Light" Water-Equivalent Precipitation & "Mild" Daily Mean Temperature Snowfall Model Residuals vs. Model Estimated Snowfall

The error distribution across estimated snowfall amounts (Figure 10) was decidedly non-random, with some large underestimations of actual snowfall, especially beyond the 2 inch estimated snowfall level. This property was reflected in Figure 9 as the non-conformance to normality in the lower-left portion of the graph. In addition, there were 19 negative snowfall estimations, or 6.2 % of the sample size. Again, in actual application, these would be set to zero. In this particular sample, if the negative estimations were reset to zero, and the residuals recalculated, the standard deviation of the "new" residuals (i.e.., the standard error) would be reduced to from 1.235 to 1.211.

In summary, the residual diagnostics' charts showed varying degrees of non-conformance to idealized properties such as normality, equal variances, and non-dependence vs. estimated values. The two light water-equivalent precipitation models showed deficiencies, but the associated error magnitudes would likely have slight cumulative impact on seasonal snowfall estimation. The "moderate" and "heavy" precipitation models for the "cold" daily means had residuals' patterns not far removed from the ideal, but those for the "non-light, mild" model confirmed it to be the most problematic.

Most of the above results seemed to point to a likelihood of slight underestimation rather than overestimation, a preferred outcome. The exclusion of daily minima >=34 F and/or daily maxima >=47 F cases from consideration further enhanced this probability that, all other things being equal, reconstructed seasonal snowfall totals would not be excessive.

3.7. - Combined Overall Performance of the Models

3.7.1 - Individual Daily Basis

Applying the five models together on the original data (including those cases that had been excluded after "run 1"), the resulting overall "standard error" (standard deviation of all the models' errors as one grouping) was 1.0". Mean and median absolute errors (irrespective of sign) were 0.5" and 0.2", respectively.

Table 7 - Ten most pronounced cases of over or under-estimated daily snowfall amounts (ranked by absolute magnitude)

Date	Max (F)	Min (F)	Prcp (in.)	Est Snow (in.)	Actual Snow (in.)	Error (in.)
31 Mar1985	33	27	1.25"	4.9"	14.7"	-9.8"
1 Mar 1965	36	15	1.62"	10.1"	1.2"	+8.9"
14 Apr 1983	37	28	1.56"	4.8"	13.6"	-8.8"
24 Jan 1967	32	19	1.21"	9.1"	0.5"	+8.6"
25 Dec 1982	38	17	1.35"	7.5"	1.4"	+6.1"
17 Jan 1996	34	20	0.90"	6.1""	0.1"	+6.0"
17 Nov 1978	37	27	0.73"	3.0"	8.3"	-5.3"
13 Mar 2006	34	18	0.78"	9.9"	5.1"	+4.8"
1 Mar 2007	33	24	0.90"	4.2"	9.0"	-4.8"
20 Jan 1982	14	2	0.80"	12.4"	17.1"	-4.7"

Table 7 lists the ten most extreme daily snowfall estimation errors for the period of record. Five each are positive and negative. Topping the list is 31 March 1985, a day with 1.25" water-equivalent precipitation, a daily mean temperature 30 F, daily maximum temperature, 33 F, estimated snowfall 4.9", but actual measured snowfall: 14.7". Assigned to the "mild" daily mean, "non-light" precipitation group, model-estimated snowfall amounts for this day would be lower than otherwise because of the closeness of the daily means to 32 F, and the resulting higher likelihood that at least some portion of the overall precipitation was non-snow. But in actuality, 30 March 1985's precipitation was predominantly (or exclusively) snow, the ratio of snow to water-equivalent precipitation (12:1), more typical of lower temperature conditions.

The second most extreme case was 1 March 1965, a day with 1.62" water-equivalent precipitation, a daily mean of 26.5 F, an estimated snowfall of 10.1", but in reality just 1.2". Having been assigned to the "cold" daily mean, "heavy" precipitation group, the bulk of the precipitation was expected as snow, but the actual ratio was only 0.7:1, indicative of a decidedly non-snow event.

Seven of the remaining eight cases were variations on these two themes, the under or over-estimation of snowfall amounts owing to an uncharacteristic proportion of non-snow. The only exception was 20 January 1982, ranked tenth on the list. On this cold day (mean temperature 8 F, some 17.5 F lower than the list's next coldest, 0.80" of water-equivalent precipitation was received. The "cold" daily mean, "heavy" precipitation model estimated 12.4" snowfall (snow to water-precipitation ratio 15:1), but 17.1" was officially measured (19:1 ratio), a 4.7" underestimation.

In spite of these extreme cases, from the "standard error", mean, and median absolute error statistics cited at the beginning of this section, typical individual days' estimating precision seemed to be reasonable. Also, on a more aggregate monthly or seasonal basis, the cumulative canceling out effect of the errors' positive and negative signs would hopefully result in proportionately accurate estimates for these longer periods.

3.7.2 - Seasonal Basis (1964-5 through 2006-07)

Next, the estimated daily snowfall measurements were tallied for each of the 39 seasons and compared with official recordings.

To make the comparisons as valid as possible, two correction factors were also applied to each year's estimated totals. The first was the mean seasonal snowfall resulting from Trace amounts of water-equivalent precipitation, and the second, the mean seasonal snowfall from days that had measurable precipitation, daily minimum temperatures >=34 F and/or daily maximum temperatures >=47 F. As described earlier, observations exhibiting the second set of conditions had been deliberately excluded from direct model consideration because the actual number of snowfall cases in these instances were so few.

The adjustments were 0.5", and 0.9" respectively, a total correction of 1.4", or about 2.6 % of the mean.

Table 8 - Models-estimated seasonal snowfall vs. Actual totals – Minneapolis- St. Paul, MN (1964-65 through 2006-07 seasons, excluding 2000-01 through 2003-04)

SEASON	EST. SNOW (in.)	ACT. SNOW (in.)	EST. MINUS ACT. ERROR (in.)	EST. MINUS ACT. ABS ERROR (in.)
1964-65	92.1	73.7	+18.4	18.4
1965-66	41.7	36.1	+ 5.6	5.6
1966-67	88.5	78.4	+10.1	10.1
1967-68	21.2	17.5	+ 3.6	3.6
1968-69	65.6	68.1	- 2.5	2.5
1969-70	52.1	63.4	-11.3	11.3
1970-71	56.6	54.7	+ 1.9	1.9
1971-72	58.3	64.4	- 6.1	6.1
1972-73	37.3	41.7	- 4.4	4.4
1973-74	45.1	51.2	- 6.1	6.1
1974-75	64.7	64.2	+ 0.5	0.5
1975-76	54.5	54.5	0	0
1976-77	37.6	43.6	- 6.0	6.0
1977-78	44.8	50.7	- 5.9	5.9
1978-79	66.1	68.4	- 2.3	2.3
1979-80	45.0	53.3	- 8.3	8.3
1980-81	27.0	21.1	+ 5.9	5.9
1981-82	82.9	95.0	-12.1	12.1
1982-83	64.3	74.4	-10.1	10.1
1983-84	90.1	98.4	- 8.3	8.3
1984-85	59.4	72.7	-13.3	13.3
1985-86	71.7	69.5	+ 2.2	2.2
1986-87	20.2	17.4	+ 2.8	2.8
1987-88	42.0	42.4	- 0.4	0.4
1988-89	67.7	70.1	- 2.4	2.4
1989-90	37.7	35.5	+ 2.2	2.2
1990-91	46.6	43.6	+ 3.0	3.0
1991-92	88.8	84.1	+ 4.7	4.7
1992-93	52.0	47.4	+ 4.6	4.6
1993-94	49.2	55.7	- 6.5	6.5
1994-95	36.2	29.6	+ 6.6	6.6
1995-96	55.7	55.8	- 0.1	0.1
1996-97	86.2	73.6	+12.6	12.6
1997-98	53.0	45.0	+ 8.0	8.0
1998-99	63.4	56.6	+ 6.8	6.8
1999-00	31.6	36.2	- 4.6	4.6
2004-05	36.4	25.5	+10.9	10.9
2005-06	39.9	44.4	- 4.5	4.5
2006-07	39.1	35.5	+ 3.6	3.6
Mean	53.98	53.96	+0.00	5.78
Median	52.06	53.90	+0.24	5.15

Table 8 compares, by season, the Models-estimated seasonal snowfall totals ("Est. Snow") versus those actually recorded ("Act. Snow"), together with two error statistics (estimated less actual, and the absolute value of the estimated less actual). Roughly half of the absolute errors are 5 inches or less, the mean figure (5.78") about 10.7% of the long term mean (each 54.0" for the estimated and actual).

The 18.4" overestimation for the 1964-65 season was attributable mostly to the cumulative effects (+21") of six cases in which the relatively heavy precipitation on those days was consistently and uncharacteristically non-snow (prime example: second ranked case in Table 7). The 13.3" understatement for 1984-85 was due primarily to the big snowfall event on 31 March (number one ranked case in Table 7).

With respect to the year-by-year seasonal errors' patterns, it was of interest to determine if they were randomly distributed, year-by-year. Application of the Wald-Wolfowitz runs test on the "estimated minus actual error" signs led to rejection of the null-hypothesis (two-tail) at just the .55 level. An identical test on the errors as absolute (sign-free) values using their median value (5.15") as "cutoff", yielded similar results -- rejection at the .35 level. Thus, the hypothesis that the models'' performance was uniform over the period of record was not rejected.

In addition, the correlation between the estimated snowfall totals and their (signed) errors was just +.088, indicative that there was essentially no statistical association between the two.

3.7.3 - Seasonal Basis (1944-5 through 1963-4)

Analysis of the combined models' 1964-65 to 2006-07 error statistics gave a sense on how they might perform on an individual seasonal basis in reconstructing snowfall totals. The results, however, were somewhat biased because they originated from the very same data from which the models were generated. Therefore it would be additionally important to test them on other periods of record in which snowfall data were available.

To this end the five models were reapplied on Minneapolis-St. Paul daily snowfall data for the 20 previous seasons, 1944-45 through 1963-64.

Table 9 compares the estimated seasonal totals, the actually recorded ones, and the two error statistics.

Table 9 - Models-estimated seasonal snowfall vs. Actual totals – Minneapolis- St. Paul, MN (1944-45 through 1963-64 seasons)

SEASON	EST. SNOW (in.)	ACT. SNOW (in.)	EST. MINUS ACT. ERROR (in.)	EST. MINUS ACT. ABS ERROR (in.)
1944-45	40.5	33.5	+7.0	7.0
1945-46	42.1	35.7	+6.4	6.4
1946-47	31.7	24.5	+7.2	7.2
1947-48	50.2	49.1	+1.1	1.1
1948-49	41.6	36.2	+5.4	5.4
1949-50	57.5	49.1	+8.4	8.4
1950-51	92.6	88.9	+3.7	3.7
1951-52	82.3	78.2	+5.1	5.1
1952-53	50.0	42.8	+7.2	7.2
1953-54	35.7	25.4	+10.3	10.3
1954-55	39.4	33.5	+5.9	5.9
1955-56	51.0	44.3	+6.7	6.7
1956-57	37.1	38.7	-1.6	1.6
1957-58	23.8	21.1	+2.7	2.7
1958-59	24.0	18.8	+5.2	5.2
1959-60	33.8	31.2	+2.6	2.6
1960-61	44.7	39.7	+5.0	5.0
1961-62	81.7	81.0	+0.7	0.7
1962-63	33.1	34.2	-1.1	1.1
1963-64	28.4	25.4	+3.0	3.0
Mean	46.07	41.57	+4.50	4.77
Median	41.06	35.95	+5.14	5.14

Results were less accurate than those in Table 8. Each of the first twelve seasons, through 1955-56, had their seasonal snowfall totals overestimated - by an average 6.3” or about 14%. The remaining eight seasons’ estimates were more reasonable, the last three quite close, but mean overall error for the 20-year period was still +4.5”, median absolute error 5.1” or about 12% of the actual long-term average for the period (41.6”). As with the 1964-65 to 2006-07 data, there was no significant statistical association between the errors and estimated snowfall, the correlation only -.058. Also incorporated into the estimates were the two climatological correction factors (combined magnitude 1.1 F), generic to the period.

In comparing Tables 8 and 9, it was readily observed that mean seasonal snowfall for the 1964-65 through 2006-07 era (54.0”) was much higher (+30 %) than that for 1944-45 through 1963-64 (41.6”). But additional analysis determined that mean seasonal snow-day water-equivalent precipitation for the 1964-65 through 2006-07period (4.85”) was only 18% higher than that for 1944-45 through 1963-64 (4.11”). Thus, the more recent era got more snow per unit of water-equivalent precipitation - a snowfall to water-equivalency ratio issue.

Investigating this aspect in more detail, mean equivalency ratios for each of the five partitioned groups were tested for statistical significance across the two eras (Table 10 below).

Table 10 - Comparative Snowfall to Equivalent Water Precipitation Ratios, by Model Type Grouping - Minneapolis- St. Paul, MN (1944-45 through 1963-64 seasons versus 1964-65 through 2006-07 seasons)

Model Type	Mean Snowfall to Water-Equivalent Precipitation Ratio (1944-45 through 1963-64 seasons)	Sample size (1944-45 through 1963-64 seasons)	Mean Snowfall to Water-Equivalent Precipitation Ratio (1964-65 through 2006-07 seasons)	Sample size (1964-65 through 2006-07 seasons)
"Cold" Daily Means & "Light" Precipitation	13.84 : 1	n=351	15.99 : 1	n=578
"Cold" Daily Means & "Moderate" Precipitation	12.48 : 1	n=170	14.79 : 1	n=406
"Cold "Daily Means & "Heavy" Precipitation	10.56 : 1	n=25	10.25 : 1	n=44
"Mild" Daily Means & "Light" Precipitation"	6.45 : 1	n=123	6:54 : 1	n=323
"Mild" Daily Means & "Non-Light" Precipitation	6.17 : 1	n=135	7:00 : 1	n=256

The most significant changes from the older to more recent era were the increases in the average ratio magnitudes for the “Cold” Daily Means/”Light” Precipitation and the “Cold” Daily Means/”Moderate” Precipitation groups. The first experienced a 16 % increase from 13.84:1 to 15.99:1, the second a 19 % increase from 12.48:1 to 14.79:1. The latter, of course, would have more absolute impact on snowfall estimation because the water-precipitation amounts were greater (.07” to .42”).

Employing the two-sample t-test, the inter-era ratio difference for the “Cold” Daily Means/”Light” Precipitation group was significant at the.001 level (t=+3.337), that for the “Cold” Daily Means/”Moderate” Precipitation group significant beyond the.0005 level (t=+5.16). None of the other three groups’ differences approached statistical significance.

So for whatever reason (procedural?), the models’ general overestimation of individual seasonal snowfall totals for 1944-45 to 1963-64 appeared to be attributable, at least in part, to the significantly lower snowfall to water-precipitation ratios for days with moderate to light water-equivalent precipitation (<=0.42”) levels and “cold” daily mean temperatures (<=27.5 F)

Taking these results further, the estimated 1944-45 to 1963-64 daily snowfall amounts that were associated with these two groups were then adjusted for these lesser ratios (multiplied by 13.84/15.99 or .866 for the light precipitation group, and 12.48/14.79 or .844 for the moderate precipitation group). The recalculated estimates are shown in Figure 11.

Table 11 - Adjusted, Estimated Seasonal Snowfall Totals for Minneapolis- St. Paul, MN (1944-45 through 1963-64 seasons) versus Actual Totals.

SEASON	EST. SNOW (in.)	ACT. SNOW (in.)	EST. MINUS ACT. ERROR (in.)	EST. MINUS ACT. ABS ERROR (in.)
1944-45	36.0	33.5	+2.5	2.5
1945-46	38.6	35.7	+2.9	2.9
1946-47	28.0	24.5	+3.5	3.5
1947-48	47.0	49.1	-2.1	2.1
1948-49	39.9	36.2	+3.7	3.7
1949-50	53.6	49.1	+4.5	4.5
1950-51	83.8	88.9	-5.1	5.1
1951-52	75.1	78.2	-3.1	3.1
1952-53	45.2	42.8	+2.4	2.4
1953-54	33.4	25.4	+8.0	8.0
1954-55	34.9	33.5	+1.4	1.4
1955-56	45.5	44.3	+1.2	1.2
1956-57	33.9	38.7	-4.8	4.8
1957-58	22.4	21.1	+1.3	1.3
1958-59	21.3	18.8	+2.5	2.5
1959-60	30.4	31.2	-0.8	0.8
1960-61	42.2	39.7	+2.5	2.5
1961-62	75.0	81.0	-6.0	6.0
1962-63	29.3	34.2	-4.9	4.9
1963-64	26.9	25.4	+1.5	1.5
Mean	42.13	41.57	+0.56	3.24
Median	37.30	35.95	+1.49	2.74

Results show that the estimated long-term average snowfall figure (plus the two correction factors) was reduced from 46.07" to 42.13" the mean error to +0.56". On an individual seasonal basis, the new median absolute error was 2.74", just 6.6 % of the actual long-term average (41.57").

While it was stated at the outset that a workable reconstruction methodology of this kind would have to depend on similarities in procedures and instrumentations across eras, and a difference *was* identified between 1944-45 to 1963-64 and 1964-65 to 2006-07, its effective and simple correction seemed to indicate that the model variable selection was probably reasonable, and that the basic multi-model approach could be applicable across different periods of record, allowing for additional snowfall to water precipitation ratio adjustments. One might have to decide on which snowfall to water-equivalent ratio was "representative", an older-era statistic or a more recent one.

The Minneapolis daily snowfall record extends back into the early 1890's, and it would have been invaluable to analyze daily snowfall data back to that period. But unfortunately, computation of snowfall to water-equivalent precipitation ratios would have been compromised by the fact that in nearly all of the years prior to the 1943-44 season, precipitation was measured on a midnight-to-midnight basis, snowfall on a differing 24-hour interval: for example 7:30 PM on the previous day to 7:30 PM on the officially designated day of record. It might also be inferred from this that in those earlier years the time intervals between snowfall measurements were longer. This would have given the snow more time to settle [and/or melt], which has the effect of lessening measurements when they are taken [University of Wyoming, 2007].

Koonce [1996] wrote that "in the 1940s the Weather Bureaus Form 1009 gave straightforward but vague instructions for measuring new snow that had fallen in the preceding 24 hours" a measuring stick with an average calculated from several points of least drift". As of 1996, the instructions had become more "sophisticated", the measuring stick still used, but instructions "now advis[Ing] on factors affecting sample points, particularly where drifting ha[d] occurred. It was recommended to seek flat areas away from buildings and trees, and use "an average of places where the snow is more evenly distributed"

4. - Estimated Daily and Seasonal St. Paul, MN Snowfall Totals for the Great "Snow Winter" of 1880-81

With the combined models' generally estimating precision determined (~10 % mean errors on a year-to-year basis) and the snowfall to water-equivalent precipitation issue identified and quantified, the next step was to apply the methodology on an example of particular interest, in this case the great "Snow Winter" of 1880-81. St. Paul Army Signal Corps daily observations of temperature and precipitation *but not snowfall* are available for this period from the NOAA National Climatic Data Center.

Table 12 lists the 1880-81 daily maximum and minimum temperatures, precipitation, and reconstructed snowfall estimates for all days in which had the latter. Two separate columns of estimates are presented, reflecting the two different sets of equivalency ratios for the 1964-65 to 2006-07 and 1944-45 to 1963-64 periods of record (see Table 10). The actual daily max/min temperatures and precipitation for 1880-81 were reportedly read a few hours before midnight [St. Martin, 1997].

From the total seasonal water-equivalent precipitation figure in column 3 (14.81"), one could easily deduce that snowfall for this season was extraordinarily heavy. Merely using the 10:1 rule-of-thumb ratio suggests that it was close to 150 inches, exceeding the modern-era 1983-84 official seasonal snowfall record for Minneapolis-St. Paul (98.6") by more than 50 percent.

The 1880-81 snow-season commenced early, in mid-October, with some six inches (estimated) falling in St. Paul over the 16th-18th, severe blizzard-like conditions

prevailing over the western portions of the state [Minnesota Climatological Working Group, 2001].
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Table 12 - Daily Max/Min Temperatures, Precipitation, and Estimated Daily Snowfall Amounts for St. Paul, MN on likely Snow-Days (October 1880 through April1881)

DATE	MAX (F)	MIN (F)	PRECIP (in.)	ESTIMATED SNOWFALL ('64-'65 to '06-'07) Ratios) (in.)	ESTIMATED SNOWFALL ('44-45 to '63-'64 Ratios) (in.)
16 OCT	40	31	1.08	3.4	3.0
17 OCT	33	28	0.30	2.5	2.2
18 OCT	36	23	0.02	0.1	0.1
5 NOV	40	29	0.01	0.0	0.0
10 NOV	42	32	1.05	3.1	2.8
11 NOV	33	24	0.52	3.2	2.8
13 NOV	28	22	0.41	5.3	4.8
14 NOV	28	11	0.03	0.4	0.4
15 NOV	39	18	0.12	0.5	0.5
19 NOV	26	12	0.11	1.5	1.4
20 NOV	20	0	0.12	1.8	1.6
23 NOV	23	0	0.15	2.1	1.9
26 NOV	26	1	0.01	0.2	0.2
30 NOV	22	9	0.18	2.5	2.3
1 DEC	32	17	0.06	0.9	0.8
4 DEC	33	17	0.32	3.9	3.5
5 DEC	24	-2	0.14	1.9	1.7
7 DEC	4	-15	0.14	2.5	2.3
11 DEC	34	20	0.05	0.7	0.6
12 DEC	37	32	0.28	1.6	1.4
13 DEC	38	30	0.04	0.2	0.2
14 DEC	38	27	0.01	0.1	0.1
19 DEC	21	9	0.07	1.2	1.1
20 DEC	21	8	0.07	1.2	1.1
21 DEC	20	15	0.05	0.8	0.7
22 DEC	22	15	0.09	1.5	1.4
23 DEC	21	15	0.02	0.3	0.3
24 DEC	30	16	0.51	4.6	4.8
25 DEC	27	17	0.22	2.9	2.6
26 DEC	20	-2	0.60	8.2	8.4
28 DEC	10	-27	0.02	0.5	0.4
5 JAN	28	19	0.07	1.0	0.9
6 JAN	20	-3	0.01	0.2	0.2
9 JAN	-4	-22	0.02	0.5	0.4
12 JAN	23	0	0.40	5.1	4.6
13 JAN	10	-18	0.32	4.5	4.1
14 JAN	5	-25	0.14	2.5	2.2
15 JAN	12	-1	0.26	3.8	3.4
16 JAN	13	-2	0.20	3.0	2.7
20 JAN	35	2	0.06	0.9	0.8
21 JAN	30	21	0.34	4.3	3.9
22 JAN	31	19	0.50	4.4	4.5
23 JAN	24	9	0.02	0.3	0.3
28 JAN	12	-9	0.21	3.2	2.8
29 JAN	20	11	0.26	3.6	3.3
30 JAN	19	9	0.37	5.0	4.5
31 JAN	16	10	1.16	14.8	15.2
1 FEB	15	1	0.12	2.0	1.8
2 FEB	15	6	0.52	10.3	10.6
3 FEB	20	11	0.06	0.9	0.8
4 FEB	20	4	0.04	0.7	0.6
6 FEB	33	24	0.62	3.5	3.1
7 FEB	38	30	0.60	2.5	2.2
15 FEB	22	10	0.18	2.5	2.3
20 FEB	28	8	0.01	0.1	0.1
24 FEB	23	4	0.40	5.2	4.7
2 MAR	28	17	0.12	1.6	1.5
3 MAR	30	10	0.02	0.3	0.2
4 MAR	31	16	0.04	0.6	0.5
12 MAR	36	23	0.02	0.1	0.1
13 MAR	41	18	0.04	0.2	0.2
14 MAR	35	12	0.02	0.3	0.2
15 MAR	42	32	0.42	1.5	1.3
16 MAR	35	28	0.32	2.1	1.9
31 MAR	36	20	0.06	0.3	0.3
3 APR	34	25	0.01	0.1	0.1
11 APR	40	30	0.01	0.0	0.0
14 APR	45	28	0.04	0.1	0.1
TOTAL:			14.81"	151.6"	141.7"

November was one of the coldest in Minnesota history, 20.6" total snowfall estimated for St. Paul from the models, December likewise very snowy with 33.0 inches approximated. The winter's outstanding month, however, was January, with an estimated 57.1", derived from 4.34" of water-equivalent precipitation. The 57.1" estimated snowfall figure (53.9" using the 1944-45 to 1963-64 equivalency ratios) is more than 10" greater than the current-day Minneapolis-St. Paul individual monthly record of 46.9" for November 1991. January 1881 was a cold month, the mildest daily mean for a snow-day only 25.5 F, so each of the sixteen daily snowfall estimates were produced by the "cold" daily mean models, incorporating the reduced likelihood of mixed snow/non-snow events. In February, another 27.7" of snow "came", March "receiving" 7.0" and April 0.2", respectively.

Total estimated snowfall for the season using the 1964-65 to 2006-07 generated models was 151.6", a somewhat lower 141.7" figure produced adjusting for the 1944-45 to 1963-64 snow to water equivalency ratios.

5. Summary and Conclusion

Utilizing 39 years' of partitioned daily Minneapolis-St. Paul, MN snowfall, precipitation, and temperature observations for the 1964-65 through 2006-07 seasons, the multivariate relationships between daily reported snowfall (dependent variable) versus same day water precipitation and temperature recordings (independent variables) were analyzed. Regression model results were tested for their potential as tools in the reconstruction of daily and seasonal snowfall totals for periods of record that had temperature and precipitation observations but not snowfall. Mean prediction errors on a year-by-year basis were in the 10 % range.

Applying the models on the immediately previous 20-year period of record (1944-45 to 1963-64 seasons) produced generally overstated seasonal snowfall estimates, attributed mostly to the period's significantly lower snowfall to water equivalency ratios for certain combinations of daily mean temperature and water-equivalent precipitation. Possibly a procedural difference between the two eras (less exacting methods for taking snowfall observations and/or longer time intervals between measurements during the older periods), but in any case, adjusting for these differences resulted in the re-estimated snowfall totals being very close to the official ones (~ 6 % errors on a year-by-year basis).

Daily and seasonal snowfall histories were then reconstructed for the famously snowy 1880-81 winter season in St. Paul using the newer era's equivalency ratios as well as the older one's (two separate series). It should be mentioned that at least some North Central States' stations during the 1880-81 season did record snowfall totals. Detroit, MI totaled 93.6", a still-standing all-time record maximum for that locality. In this regard, possibly some useful equivalency ratio information could be derived from Detroit 's data and from others that also recorded daily snowfall in the early 1880's.

More data, of course, would have been desirable for this study, especially in the case of the "cold" daily means/"heavy" precipitation model, in which only 44 cases were considered. The relatively high standard error for the model (3.293") might have been reduced, and there would have been more cases available at extreme high precipitation levels to make for a more all-inclusive regression.

Additional data would also have been useful for the "mild" mean temperature, heavier water-equivalent precipitation cases, permitting a more refined data partitioning. Cases of this kind are probably more frequent at other snow-susceptible continental U. S. stations in winter, most of them warmer than Minneapolis-St. Paul, climatologically.

Recognizing the inherent difficulties in accurately measuring snowfall, and the apparent older-era to newer-era differences in snow to water equivalent precipitation ratios (at least in some cases for the Minneapolis-St. Paul station), a partitioned data set, regression model methodology of this kind might be worthwhile considering as a tool for reconstructing missing or old snowfall histories, assuming the ~10% individual year estimating error encountered in this particular study is typical and tolerable.

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