10.1 EVALUATION OF THE THEORETICAL SPEED AND DEPTH OF GRAVITY CURRENTS USING THREE-DIMENSIONAL NUMERICAL SIMULATIONS

George H. Bryan*
National Center for Atmospheric Research, Boulder, Colorado

1. INTRODUCTION

Severe convective storms often have a deep layer of cold air near the surface. Typically referred to as "cold pools," they play an important role in the propagation, structure, and evolution of mesoscale convective systems.

To help understand their effects, researchers have often studied cold pools in isolation — i.e., in the absence of the processes that create the cold pool (such as evaporation and melting), and assuming an otherwise homogeneous environment. The flows that develop in such conditions are usually referred to as gravity currents (or, sometimes, density currents). The primary advantage of such studies is that analytic solutions for speed and depth can be determined. For simplicity, all cases herein consider motionless, unsheared, and isentropic environments.

The first analytic equation for gravity current propagation speed (C) was derived by von Kármán (1940). His equation can be expressed generally as

$$C^2 = -2 \int_0^h B \, dz \tag{1}$$

wherein C is propagation speed, B is buoyancy (relative to an isentropic environment), and h is the depth of the gravity current. This equation is the most commonly used analytic solution for gravity current propagation speed.

A more comprehensive study of gravity currents was presented by Benjamin (1968). He identified the importance of the channel depth (i.e., the limited vertical domain in which a gravity current propagates) and the dissipation of energy; both effects are essentially neglected in von Karman's solution. Benjamin's solution for propagation speed is more complex, and is difficult to write in a general form such as (1). The most important point to draw from Benjamin's study, in the present context, is that C also varies depending on $\alpha = h/H$, the ratio of gravity current depth (h) to channel depth (H).

Benjamin (1968) also found that the maximum possible value of α for realistic, steady gravity currents is 0.5; that is, the gravity current cannot fill more than one-half of the channel. This is because steady flows with $\alpha > 0.5$ would require an external source of kinetic energy, which is unphysical for the flows being considered herein. Benjamin also speculated that a *likely* maximum value of α for steady flows would probably be 0.347; this depth corresponds to the maximum possible propagation speed as well as the maximum realizable rate of energy dissipation. Benjamin's conjecture has largely been supported by subsequent studies (e.g., Klemp et al. 1994)

One potential problem with the application of the aforementioned analytic solutions to atmospheric phenomenon is that they were derived using the incompressible Boussinesq equations, which are valid only for shallow flows (of order 1 km or less). Cold pools in MCSs are known to be several km deep (e.g., Bryan et al. 2005), and thus the incompressible Boussinesq equations are not entirely appropriate. To address this problem, Bryan and Rotunno (2008) derived analytic solutions using the deep anelastic equations. The resulting analytic equations are more applicable to gravity currents in the Earth's atmosphere, and are thus more applicable to mesoscale convective systems. Their results are expressed in terms of a nondimensional number, H/H_0, which is the ratio of channel depth (H) to the maximum possible channel depth (H_0, the depth at which density is zero in an isentropic atmosphere). Benjamin's (incompressible) results correspond to $H/H_0 = 0$, and the environments of MCSs correspond roughly to $H/H_0 = 0.5$ [see Bryan and Rotunno (2008) for more details]. Bryan and Rotunno (2008) further found that the analytic propagation speed of MCS gravity currents is likely ~25% slower than is suggested by Benjamin's equations (Fig. 1), and the likely maximum depth is ~35% shallower (Fig. 2).

A relatively uncertain aspect of these theoretical results is the role of energy dissipation. Most studies in the severe storms community have used the inviscid equations, which, as discussed earlier, yield the maximum possible depth of a steady gravity current (see solid curve in Fig. 2). However, Benjamin (1968) argued that this state is "difficult, if not impossible" to achieve. Instead, he argued that steady flows would likely be maintained at a much shallower

*Corresponding author: George H. Bryan, NCAR, 3450 Mitchell Road, Boulder, CO 80301. Email: gbryan@ucar.edu

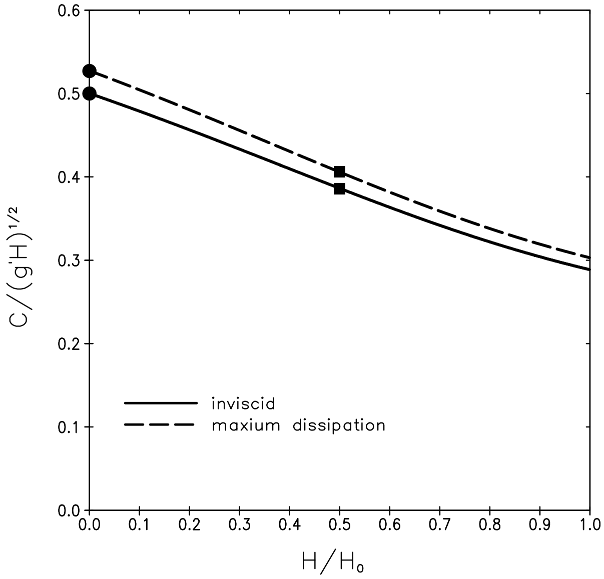

FIG. 1: Analytic solutions by Bryan and Rotunno (2008) for gravity current propagation speed [C, nondimensionalized by $(g'H)^{1/2}$] as a function of channel depth [H, nondimensionalized by the maximum possible value, H_0]. Solid line is for inviscid flow, and the dashed line is the maximum possible propagation speed (at maximum energy dissipation rate). The values for $H/H_0 = 0$ (0.5 and 0.527, denoted by dots) were derived by Benjamin (1968). Values for $H/H_0 = 0.5$ (0.386 and 0.406, denoted by squares) are most likely representative of MCS environments.

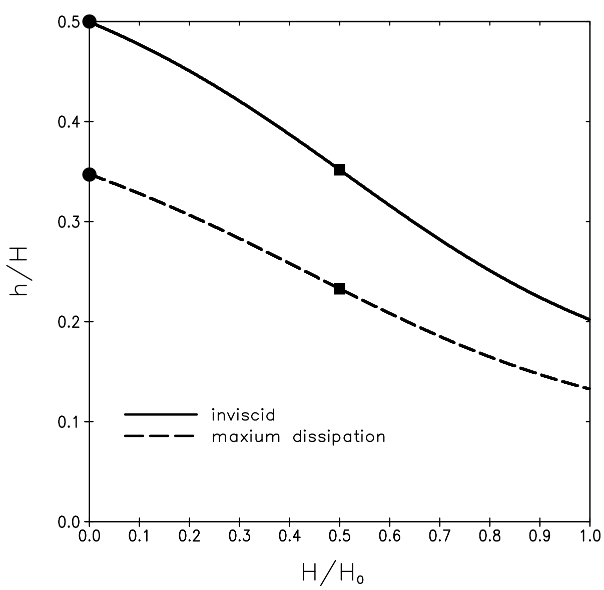

FIG. 2: As in Fig. 1 except for gravity current depth (h, nondimensionalized by channel depth). The values for $H/H_0 = 0$ (0.5 and 0.347, denoted by dots) were derived by Benjamin (1968). Values for $H/H_0 = 0.5$ (0.352 and 0.233, denoted by squares) are most likely representative of MCS environments.

depth (dashed curve in Fig. 2), owing ultimately to energy dissipation. Significant dissipation of kinetic energy seems to be inevitable in gravity currents, which can be inferred from their highly turbulent structure (see, e.g., Simpson 1997).

To provide guidance for more practical applications, I have been conducting numerical simulations of gravity currents to help determine whether the inviscid or maximum-dissipation analytic solutions are more likely. A large number of two-dimensional simulations were analyzed by Bryan and Rotunno (2008); they support the relevance of the maximum-dissipation solution. However, some concern about these simulations is warranted, owing to their two-dimensionality. Two-dimensional simulations cannot realistically simulate turbulent flows. It is well known that most two-dimensional simulations cascade energy artificially to larger scales, whereas three-dimensional simulations can cascade energy to smaller scales (where it is ultimately dissipated by viscous terms). So, in summary, because of the different energetics of two-dimensional simulations, they may not be relevant to natural gravity currents. To provide more realistic guidance, I present herein three-dimensional simulations of gravity currents using large eddy simulation.

2. METHODOLOGY

All simulations use the Bryan Cloud Model (CM1). The setup is the same as the "compressible equation set" simulations presented by Bryan and Rotunno (2008) (hereinafter referred to as BR08), except three-dimensional simulations are included herein. There are 4000 × 100 gridpoints in the horizontal directions, and 100 grid points in the vertical. The nondimensional grid spacing and time step are the same as in BR08. Further details of the model, numerics, and physical parameterizations, are provided in BR08.

As in BR08, simulations of "lock-exchange" flow are conducted. This setup yields the deepest possible cold pools depths, which is of most interest for evaluation of the theoretical problem discussed in section 1. In these simulations, the left half of the domain is filled (from the surface to the domain top) with relatively cold air, and the right half of the domain is simply the reference (isentropic and motionless) environment. The initial conditions are the same as in BR08, except that random, small-amplitude perturbations in temperature and velocity are inserted along the initial interface to allow three-dimensional motions to develop.

The following analysis is focused on two environments. One is $H/H_0 = 0.1$, which is a very shallow flow for which the incompressible equations are probably adequate. The dimensional channel depth is $\sim$3 km, so the resulting cold pools are of order 1 km deep. The second case is $H/H_0 = 0.5$, which BR08 argued was most representative of deep cold pools in MCSs. The dimensional channel depth is $\sim$15 km and, from theory (i.e., Fig. 2), cold pools are expected to be of order 4 km deep. For simplicity, all results are presented non-dimensionally.

Gravity current depth is diagnosed from the model output using the integral of the buoyancy field,

$$H_m = \frac{1}{B_0} \int_0^H B \, dz, \tag{2}$$

wherein B_0 is the initial buoyancy in the cold pool. As in BR08, H_m is averaged in space (for $-3 \leq x/H \leq -1.5$, relative to the surface gust front) to produce a representative value for each simulation.

3. COMPARISON OF 2D and 3D SIMULATIONS

For the sake of reference to previously published results (which have primarily used two-dimensional simulations), I first compare results from two-dimensional (2D) and three-dimensional (3D) simulations. Fig. 3 shows instantaneous snapshots of the potential temperature field using $H/H_0 = 0.1$. As expected, the 2D simulation has much larger turbulent eddies (Fig. 3a), which is typical of the upscale energy cascade that occurs in most 2D simulations. In contrast, the 3D simulation has much smaller turbulent eddies, especially in the region far behind the leading edge of the gravity current (Fig. 3b). Despite these differences in flow structure, the gravity currents propagate at approximately the same speed; the surface gust front is at roughly $x/H = 7$ in both cases, although the three-dimensional case has propagated slightly faster. The average depth of the gravity currents (denoted by dashed lines in Fig. 3) is comparable in the two simulations, although the diagnosed depth is slightly shallower in the three-dimensional case. It is also notable that the evolution towards a steady state is faster in 3D than in 2D (not shown). However, overall, the quantitative information about speed and depth of gravity currents seems to be comparable, even though the turbulent flow features are obviously artificial in the 2D simulation.

The same qualitative conclusions are drawn from simulations using $H/H_0 = 0.5$ (Fig. 4). Again, the gravity current propagates slightly faster and is somewhat shallower in the 3D simulation. Again, the turbulent eddies are artificially large in the 2D simulation.

4. COMPARISON TO ANALYTIC RESULTS

Hereinafter, only the 3D simulations are used. Propagation speed (determined by tracking the average position of the surface gust front over time) is remarkably steady for $t/T > 4$ (i.e., after an initial adjustment period). The average propagation speed is listed as c in Table 1; its nondimensionalized value is given by c^*. The nondimensional analytic solutions for inviscid flow (C^*_{inv}) and for maximum dissipation (C^*_{max}) are also provided in Table 1. The simulated gravity current speeds are slightly less than the two theoretical solutions. This analysis, then, is inconclusive; it does not provide any guidance about which solution might be more applicable to natural gravity currents. However, it is encouraging that the simulated gravity currents do not propagate faster than the theoretical maximum possible propagation speed.

Table 1: Evaluation of gravity current propagation speed: c is the measured value from three-dimensional simulations, and c^* is its nondimensional value; C^*_{inv} is the theoretical nondimensional value for inviscid flow; and C^*_{max} is the theoretical maximum possible value.

H/H_0	c (m s^{-1})	c^*	C^*_{inv}	C^*_{max}
0.1	9.0	0.468	0.479	0.504
0.5	16.2	0.375	0.386	0.406

The diagnosed gravity current depth is shown as a function of time in Figs. 5–6. In both cases, current depth quickly drops below the theoretically maximum possible value (i.e., the solution for inviscid flow; dashed line in these figures). For roughly $t/T > 15$, both cases have an approximately steady value that is very close to the maximum likely current depth (i.e., the solution at maximum dissipation rate; dotted line in these figures). These results support conclusions drawn previously (e.g., Benjamin 1968; Klemp et al. 1994; Bryan and Rotunno 2008); that is, the solution for inviscid flow is not likely to be achievable in realistic steady gravity currents.

5. CONCLUSIONS

These simulations are obviously idealized, and are not directly comparable to natural gravity currents (i.e., cold pools) in the Earth's atmosphere. Nevertheless, it has been well established that observed gravity currents are very turbulent (see, e.g., Simpson 1997, and references therein), particularly along the interface between cold and warm air. The simulated gravity currents are similarly turbulent along the interface. Diagnosis of the subgrid turbulence terms from the numerical simulations (not shown) shows significant dissipation of kinetic energy in the interface, as is expected in a highly turbulent flow.

All these points suggest that analytic solutions using the inviscid equations are not relevant to natural

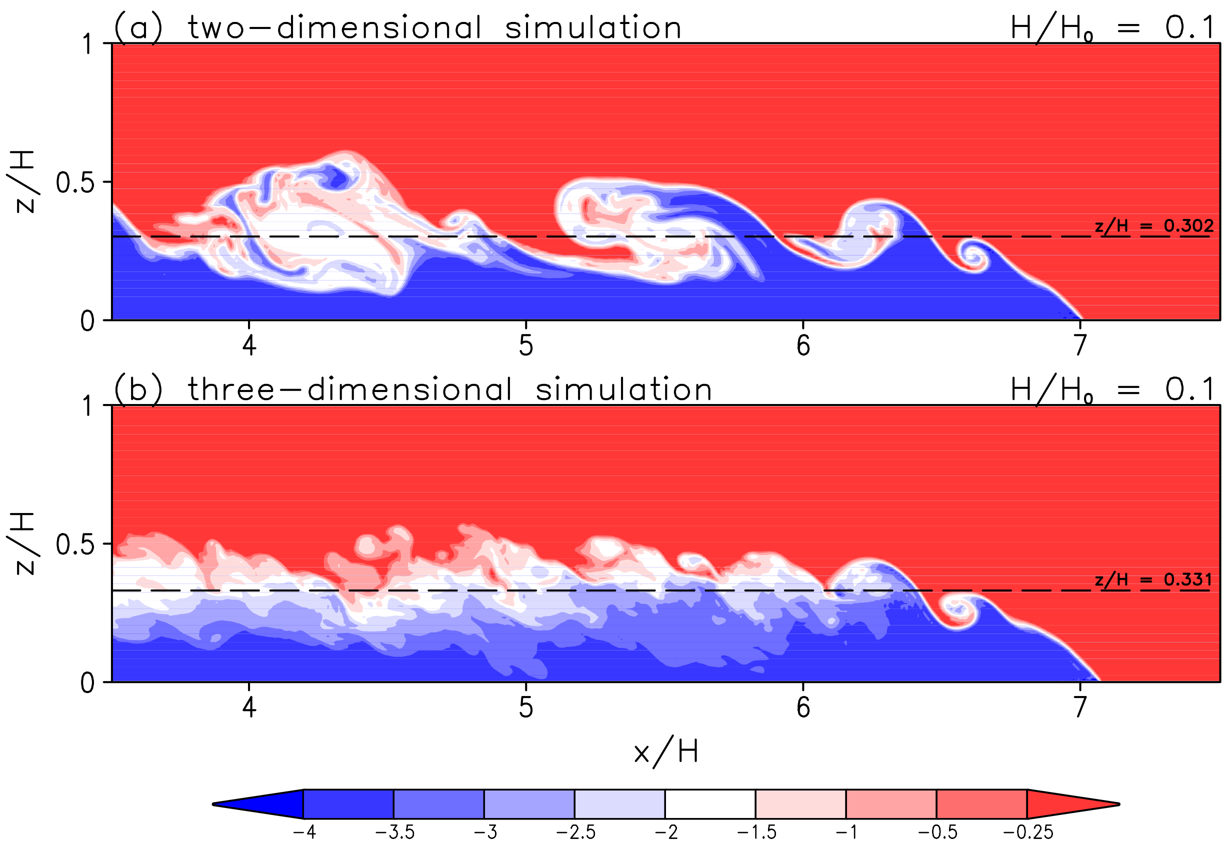

FIG. 3: Instantaneous snapshots of potential temperature perturbation (K) at $t/T = 20$ for $H/H_0 = 0.1$ from: (a) a two-dimensional simulation, and (b) a three-dimensional simulation. The dashed line denotes the average depth of the gravity current, diagnosed as in BR08.

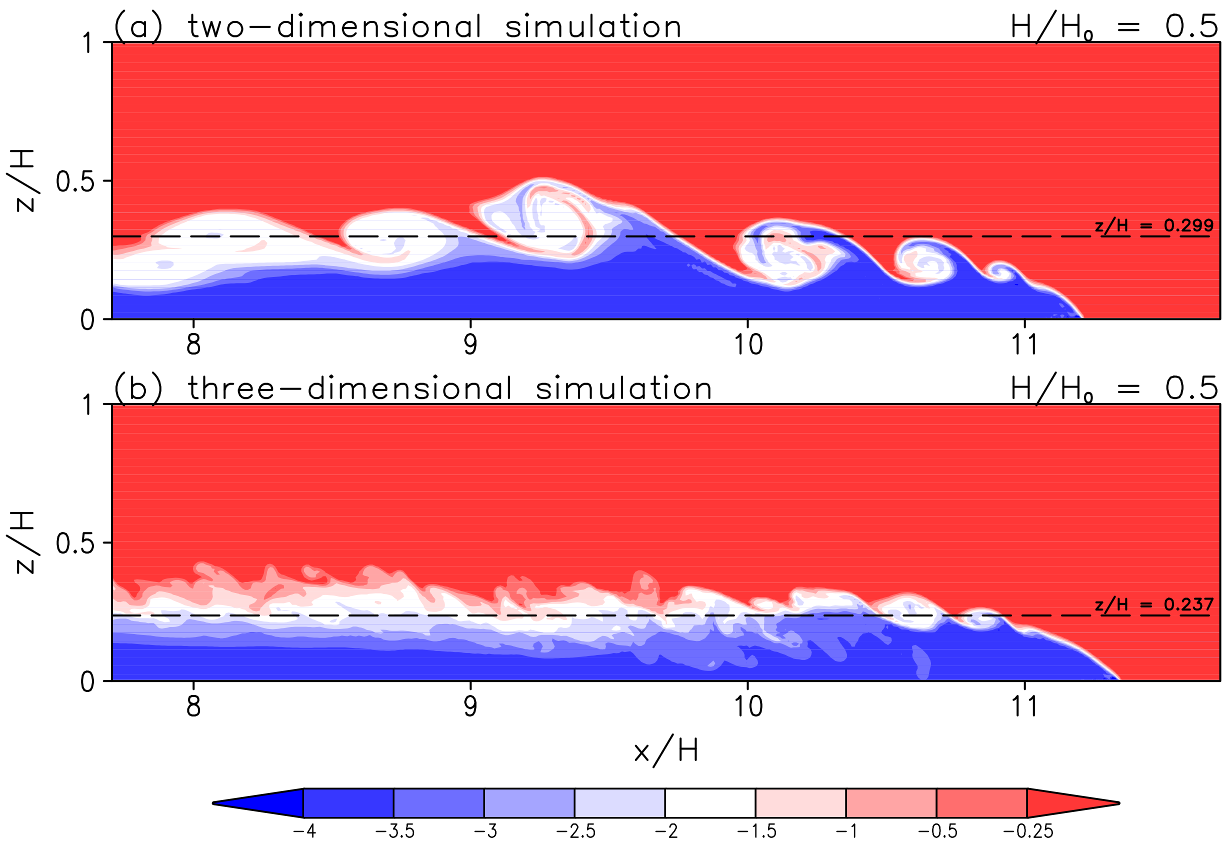

FIG. 4: The same as Fig. 3 except at $t/T = 30$ for $H/H_0 = 0.5$.

gravity currents. The analysis in section 4 strongly supports this conclusion. This issue is relevant to the severe storms community because most analytic studies of gravity currents in the atmospheric sciences literature have assumed inviscid flow (e.g., Xu and Moncrieff 1994; Liu and Moncrieff 1996; Xue et al. 1997; Xue 2000). Further studies that consider energy dissipation are thus warranted, and should focus on whether any major conclusions from these studies would be different if energy dissipation were included.

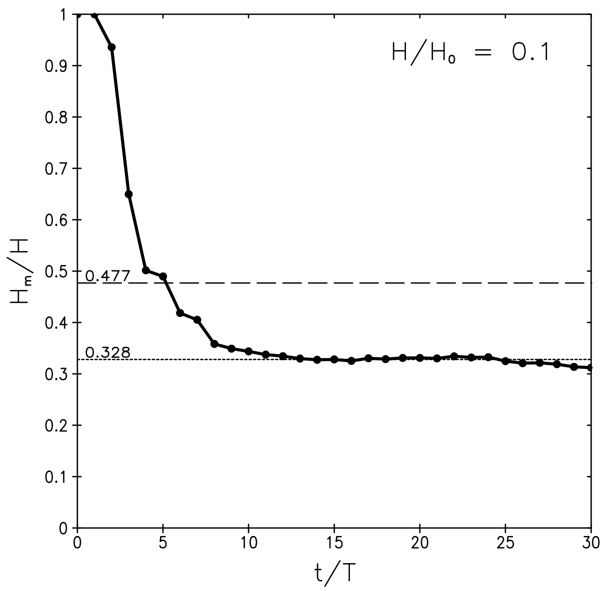

FIG. 5: Average depth of the gravity current as a function of time from the three-dimensional simulation with $H/H_0 = 0.1$. The dashed line denotes the theoretical maximum depth (for inviscid flow) and the dotted line denotes the theoretical likely maximum depth (for maximum dissipation rate).

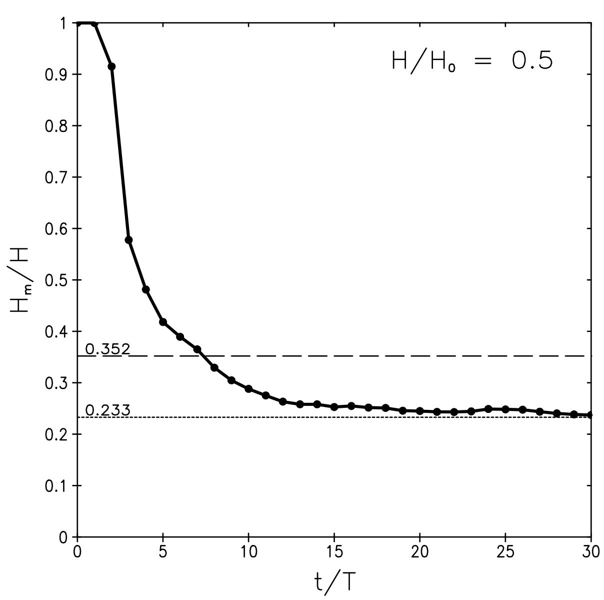

FIG. 6: As in Fig. 5 except for $H/H_0 = 0.5$.

Acknowledgements

The National Center for Atmospheric Research is sponsored by the National Science Foundation.

REFERENCES

Benjamin, T. B., 1968: Gravity currents and related phenomenon. *J. Fluid Mech.*, **31**, 209–248.

Bryan, G., D. Ahijevych, C. Davis, S. Trier, and M. Weisman, 2005: Observations of cold pool properties in mesoscale convective systems during BAMEX. Preprints, *11th Conf. on Mesoscale Processes*, Amer. Meteor. Soc., CD–ROM, JP5J.12.

Bryan, G. H., and R. Rotunno, 2008: Gravity currents in a deep anelastic atmosphere. *J. Atmos. Sci.*, **65**, 536–556.

Klemp, J. B., R. Rotunno, and W. C. Skamarock, 1994: On the dynamics of gravity currents in a channel. *J. Fluid Mech.*, **269**, 169–198.

Liu, C., and M. W. Moncrieff, 1996: An analytical study of density currents in sheared, stratified fluids including the effects of latent heating. *J. Atmos. Sci.*, **53**, 3303–3312.

Simpson, J. E., 1997: *Gravity Currents in the Environment and the Laboratory*. 2nd ed. Cambridge, 244 pp.

von Kármán, T., 1940: The engineer grapples with nonlinear problems. *Bull. Amer. Math. Soc.*, **46**, 615–683.

Xu, Q., and M. W. Moncrieff, 1994: Density current circulations in shear flows. *J. Atmos. Sci.*, **51**, 434–446.

Xue, M., 2000: Density currents in two-layer shear flows. *Quart. J. Roy. Meteor. Soc.*, **126**, 1301–1320.

Xue, M., Q. Xu, and K. K. Droegemeier, 1997: A theoretical and numerical study of density currents in nonconstant shear flows. *J. Atmos. Sci.*, **54**, 1998–2019.