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DNS ON GROWTH OF A VERTICAL VORTEX IN CONVECTION DUE TO EXTERNAL FORCES

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1. INTRODUCTION

Various types of vertical vortices, such as tornadoes and dust devils, can be seen under the unstable convective conditions. Since these vortices often cause some damage to human life activity, understanding of vortex mechanism is significant. Knowledge concerning the vortex dynamics in turbulent convection is one of the basic elements to elucidate these meteorological phenomena, which occur under more complex conditions.

According to the past studies using numerical simulations (Kanak et al., 2000; Kanak, 2005), the vertical vortices can be found in a simple flow pattern of natural convection driven by only heat. The mechanism of vortex formation is suggested as follows: Horizontal vortex which forms along an updraft region of the convective cell is tilted up by a strong local updraft at the vertex of the cell and then a pair of counter-rotating vortices is generated.

Although these studies show the vertical vortices are generated in the natural convection without mean rotation over whole domain, these vertical vortices appear to be rather weaker and larger than that in the atmosphere (Sinclair, 1964, 1973; Ryan and Carroll, 1970; Balme et al, 2003). It is one of the reasons that the convection is driven by only heat or that the extremely strong vortices in only one direction are unlikely to form owing to the absence of mean rotation over whole flow field. The formation of stronger vertical vortices as seen in the atmosphere would require other external effects, i.e. the more complex condition, for example, the topography, advection or meteorological disturbance, than the natural convection. These effects cause the vortex stretching or the concentration of the vorticity and thus the more strong vertical vortices will form.

In this study, for the sake of simplicity, we assume that the external effects appear as an updraft or a rotational flow, which causes the vortex stretching and the concentration of the vorticity. In order to analyze the vortex dynamics in turbulent convection with external effects, we performed three-dimensional direct numerical simulations for three cases of a natural convection, a convection with forced updraft, and a convection with forced updraft and rotation flow. These forced flows were imposed by adding the external force term. By detailed 3-D visualization and turbulence statics analyses, we investigated the formation mechanism and the physical structure of the strong vertical vortices in the convective boundary layers.

2. PROPLEM FORMULATIONS

2.1 *GOVERNING EQUATIONS*

In this study, the Boussinesq approximation is applied in order to consider the effect of the buoyancy by heat. The characteristic length (L), velocity (U) and temperature (θ) of the convection, respectively, are defined as the height of capping inversion base (zi), natural convection velocity (w*) and the temperature obtained by dividing the constant homogeneous heat flux at the ground surface by w*. The scales are defined as follows:

$$L = z_i = 1200\mathrm{m} \tag{1}$$

$$U = w_* = (g\beta z_i H_0)^{\frac{1}{3}} = 2.11\mathrm{m/s} \tag{2}$$

$$\Theta = \frac{H_0}{w_*} = 0.114\mathrm{K}, \tag{3}$$

where g is the gravity acceleration and β is the coefficient of volume expansion, the real scales calculated under the condition, zi=1200m and H_0=0.24 K m/s being described with them.

Using them, we define the Reynolds number Re=LU/ν and the Prandtl number Pr=ν/α as dimensionless parameters, and obtain the governing equations as follows:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{4}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla \boldsymbol{p} - \theta \boldsymbol{e_z} + \frac{1}{Re}\triangle \boldsymbol{u} \tag{5}$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = \frac{1}{PrRe}\triangle \theta. \tag{6}$$

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2.2 *COMPUTATIONAL MODEL*

To investigate the mechanism for the formation of the vertical vortices in convective boundary layer, three cases of the numerical simulations were performed. In the first case (case (A)), a pure natural convection was simulated. In the second case (case (B)), in addition to the first case (A), an updraft was forced around the center of the computational domain to examine the effect of the vortex stretching which makes vertical vortices stronger. In the third case (case (C)), rotational flow was given in the lower convective layer in addition to the model of the second case. The aim of this case (C) is to examine the mechanism of the development of strong vertical vortices when the ambient flow field has a certain amount of the positive or negative local vertical vorticity, that unbalance caused by some complicated conditions, for example, rough topography, external advection, or nonuniform heat flux etc.

Periodic boundary condition was used on the lateral surfaces and no-slip boundary condition was used on the top and bottom surfaces. On the assumption that the heat was supplied uniformly from the ground, the boundary condition for the temperature where the heat flux (H0) was constant and homogeneous was applied on the bottom surface. The initial temperature profile was set that the vertical gradient of temperature was positive in the upper domain as the inversion layer, which is set for convection not to develop above the inversion layer.

Reynolods number and Prandtl number were set at 2360, 0.71, respectively. Maker and Cell (MAC) method was applied for time marching. The computational domain was rectangular solid the size of which was 2.5 in both horizontal directions (x and y) and 1.6 in the vertical direction (z) and 256×256×128 staggered grid was used. SOR method was used as the solver of the pressure.

2.3 *EXTERNAL FORCE*

As mentioned above, an updraft and a rotational flow were imposed by external forces in case (B) and case (C) to investigate the formation mechanism and characteristics of strong vertical vortices. To impose the external force, another scheme that the velocity itself is fixed at a specified value can be also applied. However, the purpose of this study is to investigate the development of the vortex structure which has randomness under the condition that the forced flows are imposed upon the turbulent natural convection. Fixing the velocity inside the flow field directly may reduce the fluctuation and may require the more complex computational procedures as a simulation of the flow which has boundaries inside. Therefore, in this study, we adopted the method by Goldstein and Sirovich (1993), where the variable external force is given by the feedback from the difference between the specified velocity and the velocity in the computation, as follows.

In this method, the external force at the point (time t and position x) within the flow field is calculated by

$$\boldsymbol{f}(\boldsymbol{x},t) = \alpha \int_0^t (\boldsymbol{u}(\boldsymbol{x},t) - \boldsymbol{u}_0(\boldsymbol{x},t))dt + \beta(\boldsymbol{u}(\boldsymbol{x},t) - \boldsymbol{u}_0(\boldsymbol{x},t)) \tag{7}$$

where u(x,t) is the velocity at the point, u0(x,t) is a specified velocity and α, β are appropriate negative parameters. This method has an advantage in the ease in computation because only the addition of the external force term enables to give a specified velocity in the flow and thus to calculate similarly through whole computational domain. Also the velocity is not fixed strictly but oscillates around the specified value and the convergence depends on the two parameters.

The regions where the external force was set in case (C) are shown as gray blocks in Fig. 1. Updraft w=2 was imposed in the upper region and rotational flow was in the lower region. In case (B), the rotational flow was not set compared with in case (C), and no external force was set in case (A). The negative parameters were set as α=-10000 and β=-100. The results using this method are shown in the next section.

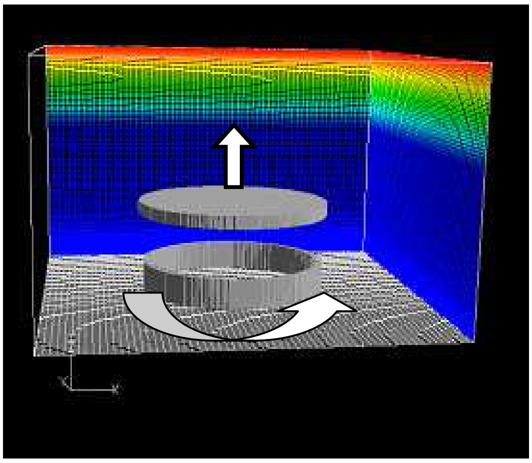

Figure 1. Illustration of the external force model used in this study. Updraft and rotational flows are given as external force, which is located within the gray regions as shown.

3 REPRODUCIBILITY OF THE VELOCITY FIELD BY THE EXTERNAL FORCE

Figure 2 shows the time series of the vertical velocity at the point (1.25, 1.25, 0.55), which is the center of the horizontal section and is the bottom of the forced updraft region if it is given. In case (A), updrafts and downdrafts occur at that point by only the heat from the ground surface. The updrafts and downdrafts occur alternately in early times, and then the mean updraft velocity is about w=1 during t=4 to 8.

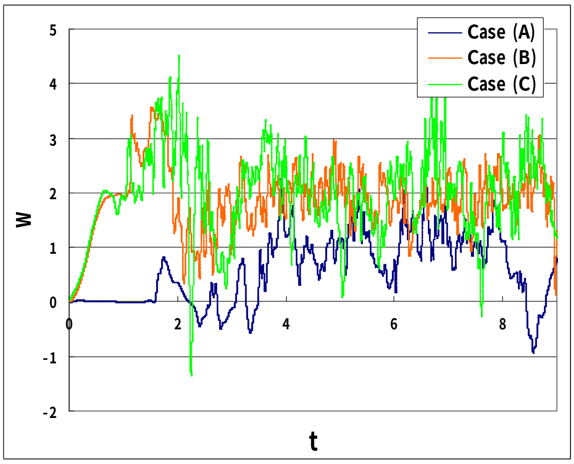

Figure 2. Time series of the vertical velocity in the three cases. This location is at the center and bottom of the updraft region (if given). The velocity is specified to be w=2 in case (B) and case (C).

Meanwhile, in case (B), it is shown that the velocity converges to the specified value w=2 averagely with oscillations, though the velocity fluctuates widely in early times. Note that w=2 is greater than that in case (A), which means that the vortex stretching will appear more clearly in case (B).

Moreover, the flow rotating and converging to the center is also added to case (B) condition in case (C), which causes the wider amplification of the velocity. Nevertheless, the mean vertical velocity is about w=2 so that the strong vortex stretching is expected to occur.

Figure 3 shows the vertical velocity at y=1.25 and the velocity vector at z=0.15. Figure 3(a), the top of which corresponds to the forced updraft region as mentioned above, illustrates that the updraft region is spreading horizontally and developing with increase in height. It is also shown by the Fig. 3(b) that the rotational flow by the external force is introduced appropriately.

4. RESULTS AND DISCUSSIONS

4.1 *STRUCTURE OF THE CONVECTIONS*

Figure 4 shows the isosurfaces of the temperature near the ground in each case, a warm color meaning an updraft and a cool color meaning a downdraft. Note that the raised surface, which means that that point has higher temperature than other points at the same height, corresponds to a strong updraft region. Figures

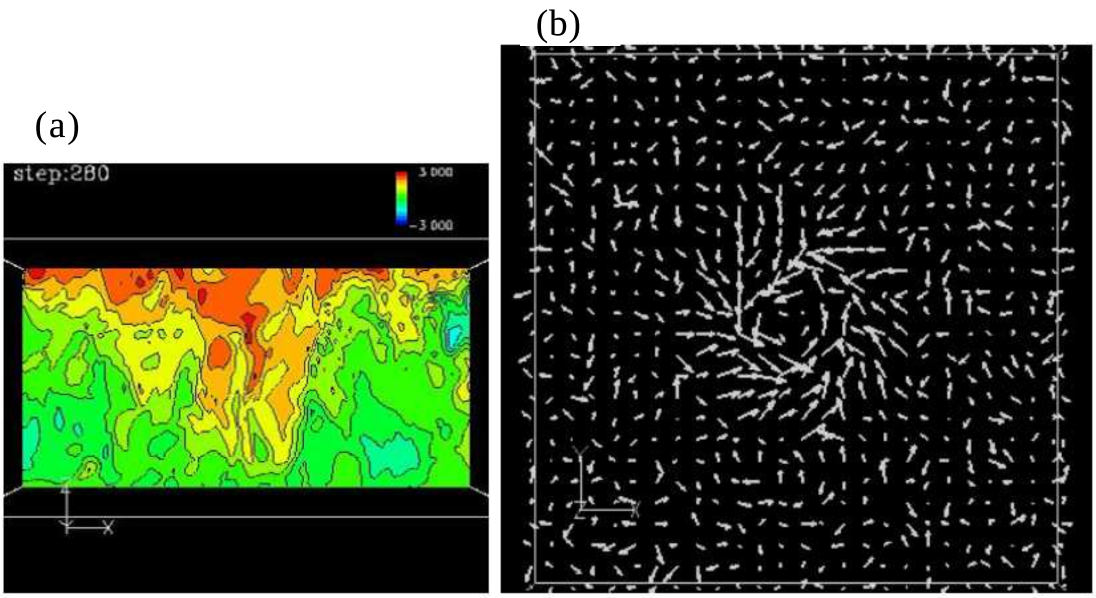

Figure 3. Flow fields under external force conditions. (a) Vertical velocity contour on the vertical cross-section at y=1.25 in case (B). The imposed updraft region locates over the shown plane. Updrafts are colored in red. (b) Horizontal velocity vectors on the horizontal cross-section at z=0.15 in case (C).

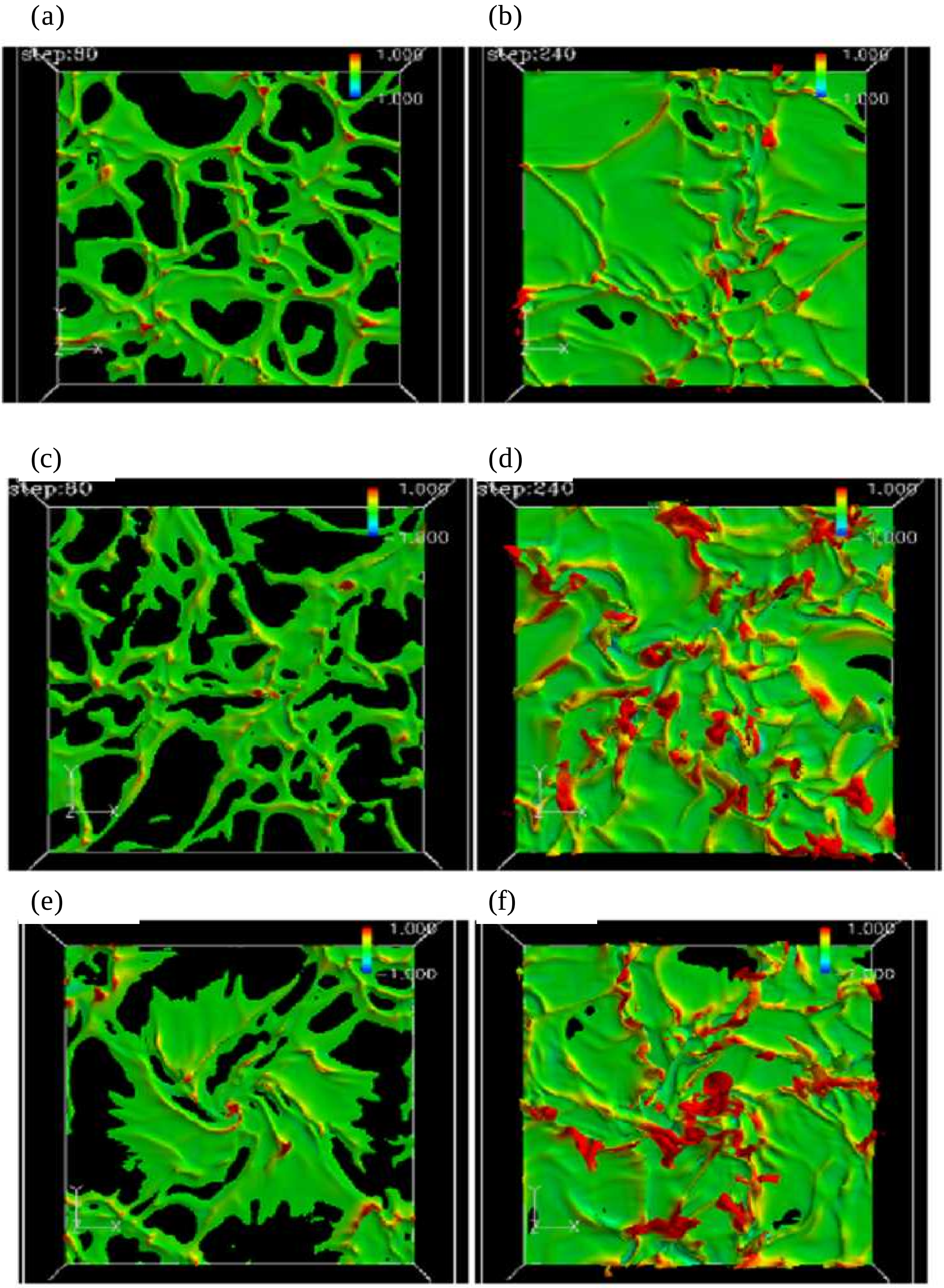

Figure 4. Pattern of convections shown by isothermal surfaces near the ground surface at: (a) t=2.4, in case (A); (b) t=7.2, in case (A); (c) t=2.4, in case (B); (d) t=7.2, in case (B); (e) t=2.4, in case (C); (f) t=7.2, in case (C). Local strong updrafts are shown as raised surface and red color.

4(a), (b) illustrate that the streaky raised surfaces form like cells which correspond to the convective cells. These convective cells get larger as time advances until the convections reach to the inversion layer and then the development of the convection is constricted. After that, the convective cells keep its size roughly about the inversion height scale, while the cells continue to appear and disappear randomly. These results about the convective cells agree with past studies and there are local updrafts at the vertices of the cell boundaries. This local updraft would be very important to form a strong vertical vortex, as described in detail below.

Figures 4(c), (d) show that the updrafts occur more actively in case (B) than in case (A), as shown prominently at t=7.2. While the size of cells in case (B) is rather smaller than that in case (A), it is also seen that it is the common structure in case (B) as in case (A) that the streaky raised surfaces form like cell and the updrafts are at the vertices of the cell boundaries.

In case (C), the flow is rotating and converging to the center with spiral updraft regions as shown in Figs. 4(e), (f). Figure (f) at t=7.2, a very strong vertical vortex is formed near the center and a strong updraft blows at the center. However, it cannot be observed that strong updrafts with vertical vortices occur at many points as it is seen in case (B).

In order to examine the relation between the vortex structure and the external flow, an additional simulation (call case (C')) where the condition is similar to case (C) except that the forced rotational flow speed is half was carried out. Figure 5 shows the horizontal cross-sections of vertical vorticity and velocity in case (C) and (C'). It shows that many vertical vortices appear

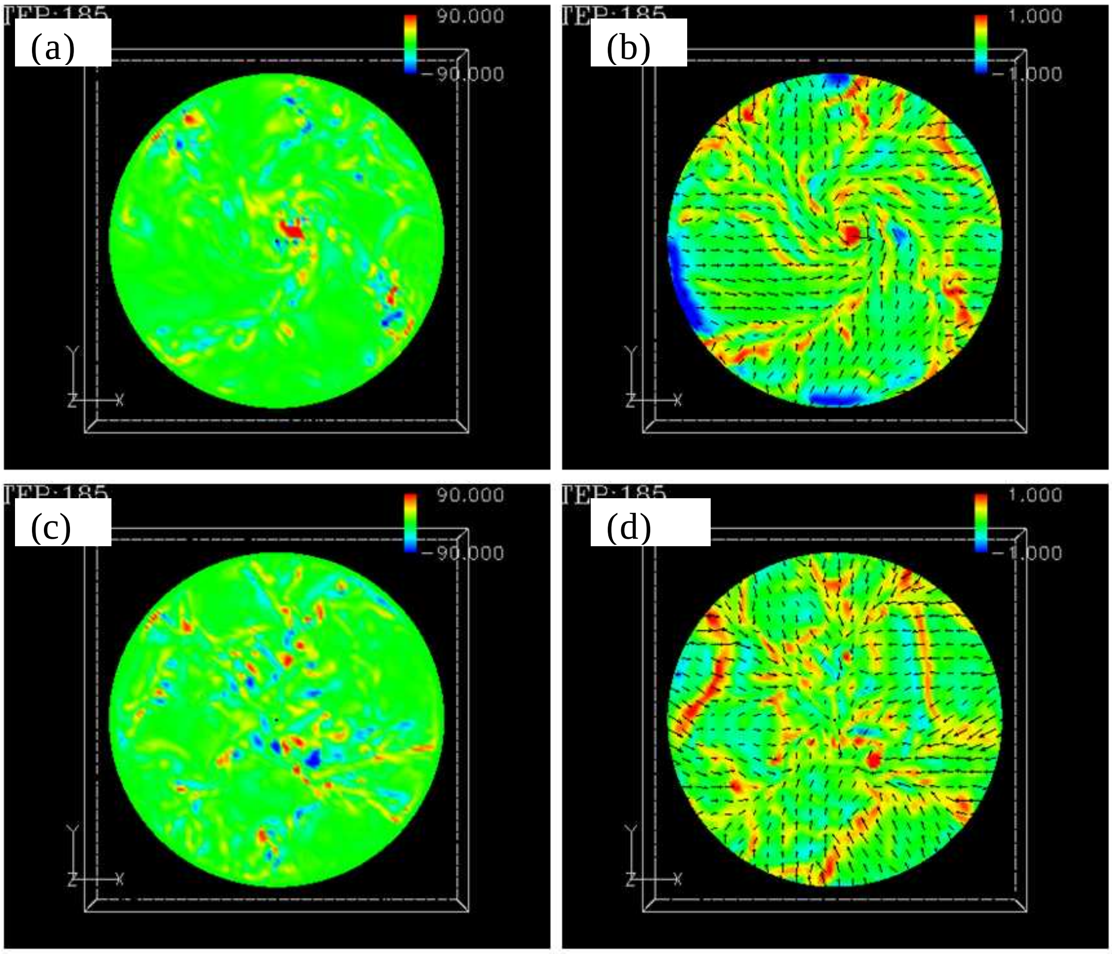

Figure 5. Structures of vertical vortices and convections: (a) vertical vorticity in case (C); (b) vertical velocity (colored) and horizontal velocity vectors in case (C); (c) vertical vorticity in case (C'); (d) vertical velocity (colored) and horizontal velocity vectors in case (C'), on the horizontal cross-section inside the rotational force region at z=0.05.

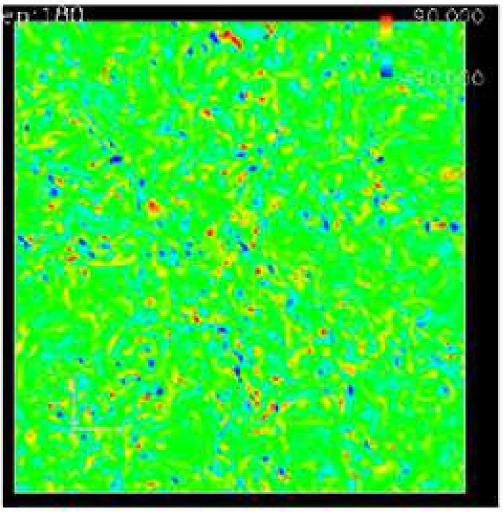

Figure 6. Vertical vorticity on the horizontal cross-section at z=0.15 in case (B). Positive vertical vorticity is shown in warm colors and negative is in cold colors.

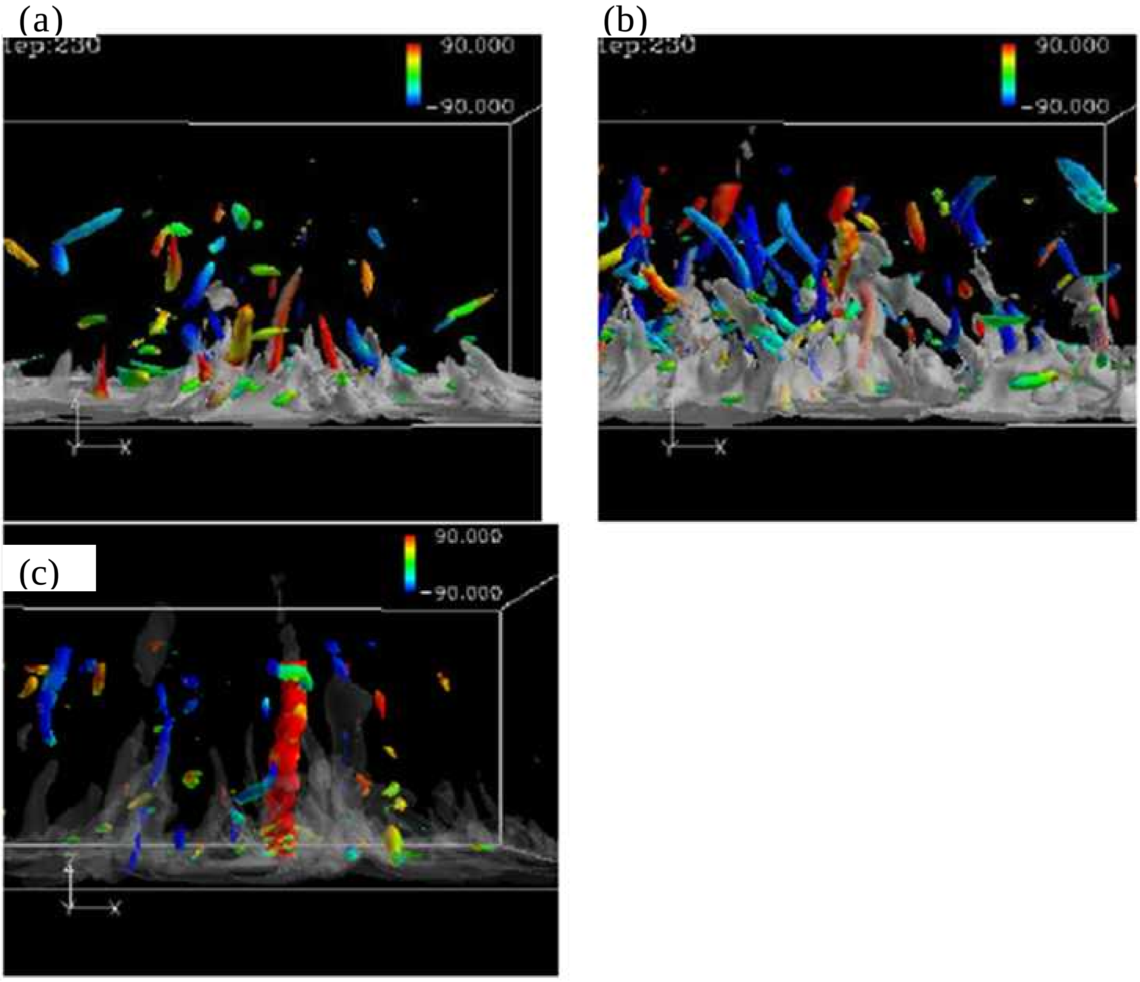

Figure 7. Strong vortex distributions defined by the second invariant of the velocity-gradient tensor and isothermal surfaces in: (a) case (A); (b) case (B); (c) case (C). Warm and cool colors, respectively, mean positive and negative vertical vorticity.

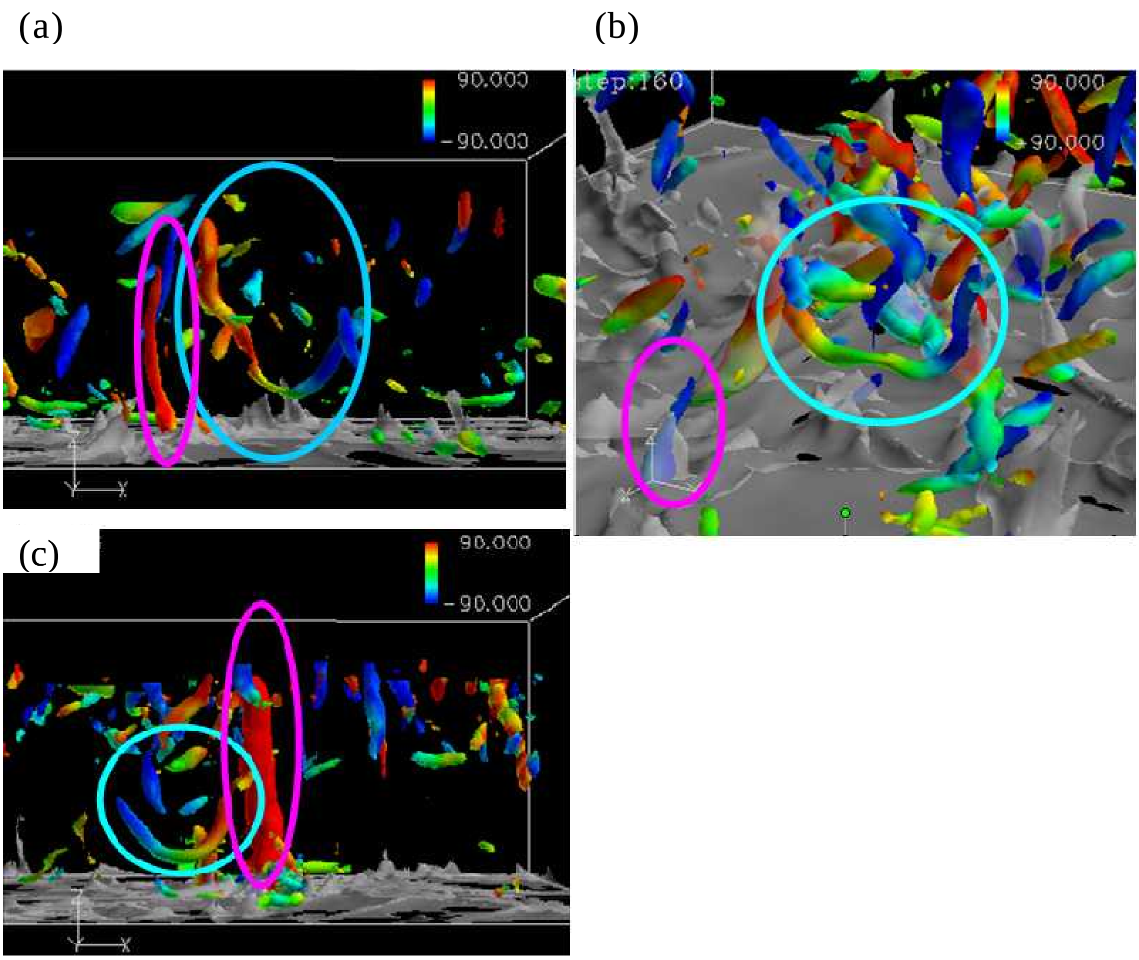

Figure 8. Two typical strong vertical vortices found in: (a) case (A); (b) case (B); (c) case (C).

in case (C'), which cannot be seen in case (C). Moreover, as the rotational flow gets weaker, the shape of the convection changes to cell, whereas, in case (C), the shape of convection is rather similar to roll, which forms parallel to the background flow and has no intersections of boundaries where the local strong updrafts occur, though the boundaries have spiral pattern. From the result, formation of many vertical vortices may occur when the vertical wind shear is smaller than a certain level. Conversely, formation of few extremely strong vertical vortices may occur when the vertical wind shear is larger than a certain level. Although that result seems to be important to reveal the mechanism of vertical vortex formation, it is still unclear that what condition is the branch point of the convective or vortex structure, or under what condition the vertical vortex forms most strongly. Further work concerning the relation between the forced flow and the vortex formation is expected to clarify them.

4.2 *THE DISTRIBUTION OF THE STRONG VORTICES*

Figure 6 shows the vertical vorticity at z=0.15 in case (B). The area where vertical vorticity has a positive value is displayed in warm colors. There are many strong vertical vortices around the center where the updraft is imposed. Moreover, the intensity and the number of the vertical vortices are greater than that in case (A), though it is not shown here.

The vortex structure in each case is shown in Fig. 7. These tubes have a strong vorticity colored by the vertical vorticity and the gray surface is the isothermal surface. It is seen that the generation of vortices in case (A) is weakest, while a number of vortices generate in case (B) where the updraft is forced. It would be caused by that the thermal plumes (shown by the gray surface) appears so frequently and intensely associated with the external updraft that the vortex stretching and concentration occur more easily. Although fewer vertical vortices appear in case (C) than that in case (B), a very strong vortex appears at about the center with a local updraft, rotating in the same direction as the forced flow. From the above, it is important thing

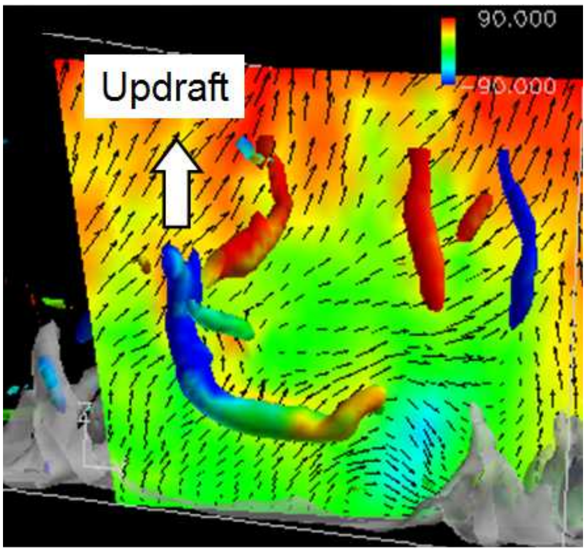

Figure 9. Vertical vortex and velocity on the cross section across that vortex. The vortex is colored by the vertical vortex and the plane is by the vertical velocity.

that the existence of the local intense updraft and the rotation of the ambient fluid contributes to form the strong vertical vortex.

4.3 *DETAILED STRUCTURE OF THE STRONG VERTICAL VORTICES*

Figure 8 shows the vortex structures at the same time in three cases. It is seen that two types of the vertical vortex occur in each case. One is the vertical vortex that the end of the horizontal vortex is bent and tilted up to it by the strong updraft, and the other is that the vertical vortex forms directly near the ground surface.

The formation mechanism of the bent vortex may be the following as shown in Fig. 9. Line-like updraft region in the boundaries of convective cells generates the horizontal vortex and then the upper updraft tilts it up and changes it to the vertical vortex. From the observation of the results, the number of these vortices is smaller and they maintain for a longer time and have stronger vorticity than the latter type. However, it may indicate how the vertical vorticity is produced from the horizontal vorticity, which the heat differences produce directly.

On the other hand, the other type of the vertical vortex has the following characteristics, which is shown by pink circles in Fig. 8. It tends to be stronger and maintain longer than the former. Almost all vortices of this type form at the place a thermal plume occur with the strong local updraft. Different from the former type, it cannot be seen that the vortex has a horizontal part and it form from near the ground surface where the ambient air flows into the center of the vortex and then grows upward by the stretching with the local updraft.

For the purpose of this study, the latter is more important because of its strength. To discuss it, Figs. 10, 11 show the detailed structure of the vortex in case (B) and (c). From Fig. 10 in case (B), the flow concentrates to the center near the ground, where the vertical vorticity is a maximum value. Figure 10 also shows that there is a large updraft region due to the external force over the vortex and the updraft stretches the vortex up and strengthens it. As shown above, the more number of strong local updrafts, that is thermal plumes, occur than in case (A) due to the forced updraft located about the half of the inversion layer height. Hence, it is thought that it leads to the increase of the number of the vertical vortex corresponding to the local updraft.

In addition to case (B), it includes the rotation flow which may make it easy to concentrate with rotation in case (C). The very strong vertical vortex appears at about the center which rotates the same direction as the forced flow. Once it forms, it exists throughout the computation due to the concentration of the ambient flow. While it is more intense than in other cases, from Fig. 11, it has a similar structure that rotating inflow comes into the center of the vortex near the ground and then it flows upward with the vortex stretching and strengthening. Case (C) is thought to show the importance of the concentration to form the strong vertical vortex.

5. CONCLUSIONS

Three-dimensional direct numerical simulations of the convective boundary layers with the forced updraft or rotational flow were carried out for examining formation of strong vertical vortices. The structures of the flow field and the turbulent vortex are compared with each other.

Convective cells the boundaries of which are updraft regions form in natural convection (case (A)). In case (B), some distortion of convective cells can be seen but similar cells still form. Also, the thermal plumes, which must be essential for vertical vortex formation, occur at the intersection of cell boundaries and are more intense in case (B). Whereas, in case (C), the shapes of convective cells change to the shapes like convective rolls which are spiral converging to the center when the forced rotational flow speed is high.

The comparison of distributions of strong

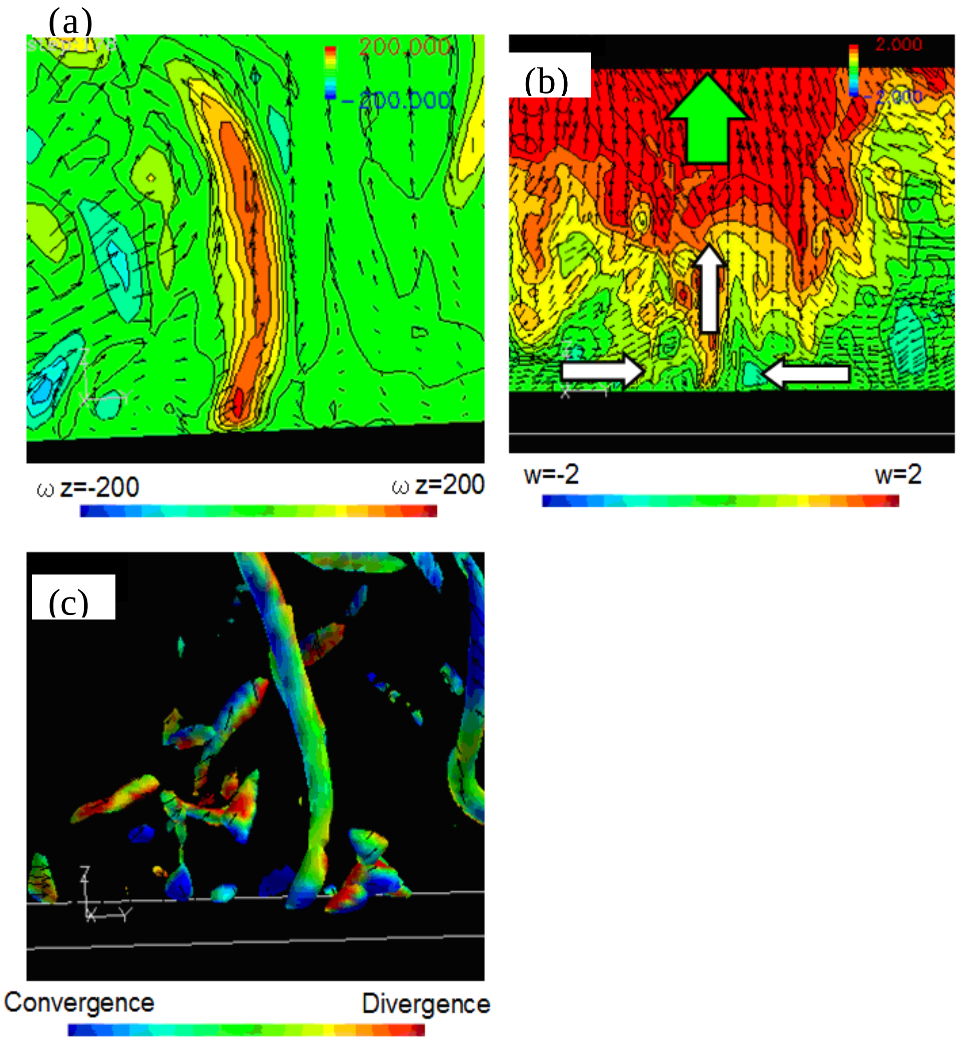

Figure 10. Structure of a strong vertical vortex in case (B). (a) Vertical vorticity contour and the velocity vectors on the cross section across the vertical vortex. (b) Flow field around the vertical vortex. Vertical velocity contour and the velocity vectors in cross section across the vortex are shown. (c) Horizontal flow convergence of the strong vertical vortex.

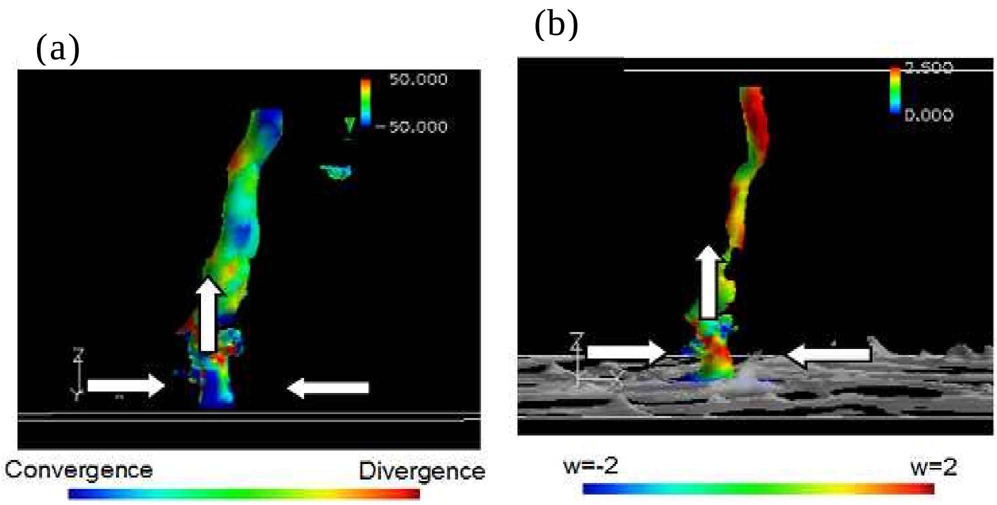

Figure 11. Structure of the strong vertical vortex, which form at about the center, in case (C). (a) Horizontal flow convergence of the srong vertical vortex. (b) Vertical velocity inside the strong vertical

vortices in each case shows the effect of the external force. In case (B), the strong vertical vortices appear most frequently of three cases because the forced updraft enhances the vortex stretching and concentration important to strengthen the vortex. Whereas, in case (C), an extremely strong vortex forms at about the center throughout the calculation because of the forced rotational flow. Moreover, additional simulation, which is similar to case (C) except that forced rotation is half, shows that many vertical vortices appear, which cannot be seen in case (C), with the shape of the convection changing from roll to cell. This indicates that formation of few but extremely strong vertical vortices like case (C) requires a certain level of rotational flow or vertical wind shear which changes the convective cells to rolls so that vertical vortices form at the fewer points with concentration.

By more detailed observation, two main types of strong vertical vortices can be found in each case. One is the vertical vortex formed by bending the end of horizontal vortex, and the other is formed directly near the ground. The latter appears to be stronger and longer-lived. What needs to be emphasized is that the strong vertical vortices are unlikely to form by the simplest mechanism where the horizontal vortex tube formed along the boundary of convective cell is tilted up and changes to vertical vortex tube.

In addition, investigation of the physical properties of the vortex clarifies the vortex structure as follows: at first, the strong vertical vortex forms from the bottom at which the ambient flow horizontally convergent to the center of the vortex and then grows upward with stretching. The vertical vortex has a local updraft within itself, which means that the vertical vortices are likely to relate the thermal plumes, which occur in convective cells.

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