# **P2.1** CROSSBEAM WIND MEASUREMENTS USING SPACED-ANTENNA AND DOPPLER BEAM SWINGIING BASED ON MONOPULSE CONFIGURATIONS WITH THE NATIONAL WEATHER RADAR TESTBED

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## 1. INTRODUCTION

The Spaced-Antenna Interferometry (SAI) technique has been used to measure crossbeam wind components as well as radial wind. Briggs et al. [1950] developed the Full Correlation Analysis (FCA) method, in which both the auto-correlation and cross-correlation are used. Doviak et al. [1996] developed general equations that link the auto-correlation and cross-correlation functions to the statistical characteristics of the wind and refractive index fields. Methods other than the FCA have been used to measure crossbeam wind. Intersection (INT) method [Holloway et al., 1997] shows that the intersection of auto-correlation and cross-correlation functions are related to baseline wind. Lataitis et al. [1995] used a method called Slope-at-Zero-Lag (SZL); it shows baseline wind is proportional to the slope of the magnitude of cross-correlation function at zero lag. A Cross-Correlation Ratio (CCR) method was recently proposed by Zhang et al. [2003]; it shows that the logarithm of the CCR is linearly proportional to the crossbeam wind along the baseline direction.

Another common technique used to measure baseline wind is called the Doppler Beam Swinging (DBS) method. In this technique, crossbeam wind components are derived from measurements of the Doppler velocities along two beams sequentially pointed in the plane of the wind component.

The phased array radar of the National Weather Radar Testbed (NWRT) located in Norman, Oklahoma is a monopulse radar, having two receiving channels: a sum channel and a difference channel. Crossbeam wind measurements typically use the cross-correlation and auto-correlation of two spaced receiving antennas. The left and right halves of NWRT array can be formed and used to measure the crossbeam wind parallel to the baseline that linked two phase centers of the two halves. However, signals from each half of the array cannot be directly obtained. Nevertheless, instead of using two signals of two spaced antennas, the sum and different signals can be used to estimate crossbeam wind directly. In addition, the DBS method can be also applied to NWRT's sum channel. In this work, we examine the relative performance of wind measurements using SAI and DBS.



**Fig. 1** The NWRT antenna is being installed at Norman, Oklahoma

## 2. SAI FOR MONOPULSE CONFIGURATION

First, we need to express the auto-correlations,  $C_{ss}(\tau)$  and  $C_{ss}(\tau)$ , of the sum and difference signal in terms of cross-correlation and auto-correlation of the virtual signals from the two halves of the NWRT array. The expressions are [Doviak et al. 2006]

$$C_{dd}(\tau) = C_{11}(\tau) + C_{22}(\tau) - C_{12}(\tau) - C_{21}(\tau) \quad (1)$$

$$C_{ss}(\tau) = C_{11}(\tau) + C_{12}(\tau) + C_{21}(\tau) + C_{22}(\tau) \quad (2)$$

where [Zhang and Doviak, 2007]

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$$C_{11}(\tau) = C_{22}(\tau) = \exp(-2jkv_{x'}(0)\tau - 2k^{2}\sigma_{tx'}^{2}\tau^{2} - 2k^{2}\sigma_{e\phi}^{2}\tau^{2}v_{y'}^{2}(0) - 2k^{2}\sigma_{e\phi}^{2}\tau^{2}v_{z'}^{2}(0))$$
(3)  

$$C_{12}(\tau) = \exp(-2jkv_{x'}(0)\tau - 2k^{2}\sigma_{tx'}^{2}\tau^{2} - 2k^{2}\sigma_{e\phi}^{2}(v_{y'}(0)\tau - \Delta y_{12}'/2)^{2}$$
(4)  

$$-2k^{2}\sigma_{e\phi}^{2}(v_{z'}(0)\tau - \Delta z_{12}'/2)^{2}$$
(5)  

$$-2k^{2}\sigma_{e\phi}^{2}(v_{y'}(0)\tau + \Delta y_{12}'/2)^{2}$$
(5)

If the two halves of the array are overlapped, the auto-correlation of sum signal cannot be directly obtained with equation (2). According to Zhang and Doviak [2007], we can derive the auto-correlation coefficient of  $C_{ss}(\tau)$  using a similar method (wind shear is not considered):

$$C_{ss}(\tau) = \exp[-2jkv_{x'}(0)\tau - 2k^{2}\sigma_{tx'}^{2}\tau^{2} - 2k^{2}\sigma_{e\phi s}^{2}\tau^{2}v_{y'}^{2}(0) - 2k^{2}\sigma_{e\phi s}^{2}\tau^{2}v_{z'}^{2}(0)]$$
(6)

 $v_{x'}(0)$  is the mean wind at the center of  $V_6$  and parallel to the beam axis.  $\sigma_{tx}^2$  is the variance of turbulence parallel to the beam axis.  $\sigma_{e\phi s}$  and  $\sigma_{e\theta s}$  are the effective two-way beamwidths in azimuth and zenith for sum mode.  $v_{y'}(0)$  is the velocity of mean baseline wind component.  $v_{z'}(0)$  is the velocity of cross-baseline mean wind component.  $\sigma_{e\phi}$  and  $\sigma_{e\theta}$  are the effective two-way beamwidths in the azimuth and zenith direction for the left and right halves of the array.  $\Delta y'_{12}$  and  $\Delta z'_{12}$  are the receiving antennas separations along y' and z' direction.

Taking the Fourier Transform of (1) and (6), we have:

$$S_{ss}(v) = \frac{2}{\lambda} \int C_{ss}(\tau) \exp(j2kv\tau) d\tau$$
(7)

$$S_{dd}(v) = \frac{2}{\lambda} \int C_{dd}(\tau) \exp(j2kv\tau) d\tau$$
(8)

By performing the integration, we obtain (considering only two receivers are separated in the y' direction):

$$S_{ss}(v) = \frac{1}{\pi} \sqrt{\frac{\pi}{2\sigma_{tx}^{2} + 2\sigma_{e\phi s}^{2}v_{y'}^{2}(0)}}$$

$$\cdot \exp(-\frac{(v - v_{x}(0))^{2}}{2\sigma_{tx}^{2} + 2\sigma_{e\phi s}^{2}v_{y'}^{2}(0)})$$
(9)

$$S_{dd}(v) = \frac{2}{\pi} \sqrt{\frac{\pi}{2\sigma_{tx'}^2 + 2\sigma_{e\phi}^2 v_{y'}^2(0)}}$$
  
•exp $\left(-\frac{(v - v_{x'}(0))^2}{2\sigma_{tx'}^2 + 2\sigma_{e\phi}^2 v_{y'}^2(0)}\right)$  (10)

•
$$(1 - \exp(-\frac{k^2 \sigma_{e\phi}^2 \Delta y_{12}^{-2} \sigma_{tx}^2}{2\sigma_{tx}^2 + 2\sigma_{e\phi}^2 v_{y'}^2(0)})\cos(\alpha))$$

$$\alpha = \frac{k\sigma_{e\phi}^2 v_{y'}(0)\Delta y_{12}}{\sigma_{tx}^2 + \sigma_{e\phi}^2 v_{y'}^2(0)} (v - v_{x'}(0))$$
(11)

By evaluating  $S_{ss}(v)$  and  $S_{dd}(v)$  at  $v = v_{x'}(0)$ , equations (9) and (10) will become more concise. They can be written as:

$$S_{ss}(v_{x'}(0)) = \frac{1}{\pi} \sqrt{\frac{\pi}{2\sigma_{tx'}^2 + 2\sigma_{e\phi s}^2 v_{y'}^2(0)}}$$
(12)

$$S_{dd}(v_{x'}(0)) = \frac{2}{\pi} \sqrt{\frac{\pi}{2\sigma_{tx'}^{2} + 2\sigma_{e\phi}^{2}v_{y'}^{2}(0)}}$$
  
•[1 - exp( $-\frac{k^{2}\sigma_{e\phi}^{2}\Delta y_{12}^{'}\sigma_{tx'}^{2}}{2\sigma_{tx'}^{2} + 2\sigma_{e\phi}^{2}v_{y'}^{2}(0)})$ ] (13)

Equation (12) and (13) can be solved for two unknown variables:  $\sigma_{tx}$  and  $v_{y'}(0)$ . By solving these two equations, we can derive  $v_{y'}(0)$ . For the NWRT, the transmitting 3-dB beamwidth is  $\sigma_{\theta T} = \sigma_{\varphi T} = 1.53$ degrees, receiving 3-dB beamwidth is 1.72 degrees. The left and right half receiving 3-dB beamwidths are 2.50 degrees. Thus,  $\sigma_{e\phi s}$  is equal to 0.0085,  $\sigma_{e\phi}$  is equal to 0.0097.

### **3. DBS BASED ON THE NWRT**

Other than SAI, we also can apply the DBS method using the NWRT. The DBS is a very common approach to obtain the horizontal wind component. Assuming wind is uniform, we use two radial velocities from two beam pointing positions to retrieve crossbeam wind which is in the same plane of two beam pointing positions. Assume  $v_1$  and  $v_2$  are two radial velocities measured by Doppler radar,  $v_1$  is measured from boresight beam and  $v_2$  is measured from azimuth tilted beam. The horizontal wind velocity is obtained by:

$$v_h = v_1 \tan \theta_z + \frac{v_2 - \frac{v_1}{\cos \theta_z}}{\sin \theta_z}$$
(14)

According to Doviak et al.[2004], if  $\theta_z$  is very small, which is always true when we need to compare SA and DBS performances for wind measurements, equation (14) can be written as:

$$v_h = \frac{v_2 - v_1}{\theta_z} \tag{15}$$

Unlike a mechanically rotated dish antenna, phase array radar allows the steering of the PAR beam on a pulse-to-pulse basis. This capability provides diversity in receiving signals from two beams. For example, we can receive a group of signals on one beam pointing position, and then receive the other group of signals on the other beam pointing position. In addition, we can alternately receive signals from two beam pointing positions, this can be done only by phase array radar. By receiving signals alternately, we can decrease the variance of horizontal wind velocity. We find that the variance of horizontal wind is related to the variance of two radial velocities, the covariance of two radial velocities and  $\theta_z$ , which can be written as:

$$\operatorname{var}[v_{h}] = \frac{1}{\theta_{z}^{2}} \operatorname{var}[v_{1}] + \frac{1}{\theta_{z}^{2}} \operatorname{var}[v_{2}] - \frac{2}{\theta_{z}^{2}} \operatorname{cov}[v_{1}, v_{2}] (16)$$

### 4. Results

We ran Monte Carlo simulations to compare the performance of wind estimators using DBS and SAI with the NWRT/PAR. In order to let SAI and DBS have matched resolution volume, according to Doviak et al.[2004a],  $\theta_z$  is obtained by:

$$\theta_{z} = \frac{\theta_{6}^{(2)}(SA) - \theta_{6}^{(2)}(DBS)}{\sqrt{2}}$$
(17)

For the NWRT,  $\theta_6^{(2)}(SA)$  is equal to 1.85 degree,  $\theta_6^{(2)}(DBS)$  is equal to 1.62 degree. Thus  $\theta_z$  is equal to 0.16 degree. In the DBS method, we use two different receiving strategies: 1. Receiving a group of signals from first beam position and then receiving the other group of signals from the second beam position; 2. Receiving signals alternately from two beam positions. We will see the standard deviation of the crossbeam wind estimates is much smaller if we apply the second strategy. This is because the estimation errors are highly correlated and tend to cancel each other. For the SA method, T<sub>d</sub> is equal to 2s, T<sub>s</sub> equal to 1ms, and the number of pulses is equal to 2000. For DBS strategy 1, T<sub>d</sub> is equal to 1s for each beam position, T<sub>s</sub> is equal to 1ms, and the number of pulses is 1000 per beam position. Under DBS strategy 2, T<sub>d</sub> is equal to 2s for each beam position,  $T_s$  is equal to 2ms, and the number of pulses is 1000 per beam position. We have calculated the standard deviations and biases of crossbeam wind using the different methods for varies intensities of turbulence(figures 2 and 3)



**Fig. 2** Normalized standard deviation of estimates  $v_{0y'}$  versus  $\sigma_{tx'}$ . NWRT parameters:  $\lambda = 0.0938m$ ,  $\Delta y'_{12} = 1.22m$ , Meteorological parameters:  $v_{0y'}$ = 20m/s,  $v_{0x'} = 0m/s$ ,  $v_{0z'} = 0m/s$ .



Fig. 3 Relative bias of estimates  $v_{0y}$  versus  $\sigma_{tx}$ .

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## 5. Conclusions

It is shown that if wind across the beam is uniform, the NWRT is capable of measuring crossbeam wind velocity as well as the radial wind velocity. We find that directly using auto-correlations of the sum and difference signals, we can produce satisfactory crossbeam wind measurements. In future work, cross-correlation of the sum signal and difference signals will be investigated and shear will be included.

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