P2.2 **Range Overlaid Staggered PRT**

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ABSTRACT

Limitations in resolving both range and velocity unambiguously arise from uniform sampling as is typically done in Doppler weather radars. If the sampling period or pulse repetition time (PRT) is made large for extended range coverage, no overlaid conditions occur, but Doppler velocity measurements become ambiguously aliased. Conversely, if the PRT is made small to unambiguously resolve velocities, range-overlaid signals become more likely. Staggered Pulse Repetition Time (SPRT) has been shown (Sirmans 1976, Sachidananda and Zrnić 2003, Torres et al. 2004) to mitigate range and velocity ambiguities by decreasing velocity aliasing while extending the radar coverage. In other words, with the proper choice of PRTs, SPRT can eliminate the occurrence of overlaid echoes without sacrificing velocity aliasing. However, the performance of the SPRT algorithm to accurately dealias Doppler velocities deteriorates as the spectrum width of the weather signal increases in relation to the Nyquist interval. By allowing some overlaid signals to occur, shorter PRTs can be used to increase the SPRT dealiasing performance.

In this paper, we explore the use of range-overlaid SPRT as a means to improve the quality of Doppler velocity estimates. Additionally, we present a method to recover velocities beyond the unambiguous range of the standard SPRT algorithm.

1. INTRODUCTION

In Doppler weather radar systems like the WSR-88D, it is desirable to unambiguously track weather at long distances while estimating the velocity unambiguously. However, when uniform sampling is utilized, unambiguous resolution in both range and velocity is generally not realized. That is, the mitigation of range (velocity) usually results in the ambiguity of the weather velocity (range). The relationship is such that the product of unambiguous range (r_a) and unambiguous velocity (v_a) is constant for a specified radar wavelength:

$$r_a v_a = c\lambda / 8 \qquad (1)$$

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where c is the speed of light and λ is the wavelength of the transmitted pulse.

Because of the large range extent and significant velocities associated with weather conditions, SPRT has been suggested (Sirmans 1976) as a viable means to reduce the ambiguity that arises in both range and velocity. Zrnić and Mahapatra (1985) showed that when SPRT is used, the range and velocity product can be extended:

$$r_a v_a = \left(\frac{c\lambda}{8}\right)\left(\frac{1+\kappa}{1-\kappa}\right) \qquad (2)$$

where κ is the constant of proportionality between T_1 and T_2 (i.e. T_1 / T_2) with $T_2 > T_1$. Thus, with the proper choice of T_1 and T_2, both range and velocity can be unambiguously determined.

Practical limits exist for any system. For the WSR-88D, Torres et al. (2004) evaluated the system limitations and provided a general SPRT algorithm description that realizes the theoretical maximum extent in velocity utilizing velocity difference transfer functions for any choice of the constant of proportionality κ. The SPRT algorithm described assumes that the weather extent does not exceed the unambiguous range of the longest PRT (T_2) guaranteeing that reflectivity (power) can be estimated without any overlaid echoes. Additionally, the algorithm allows recovery of Doppler moments (velocity and spectrum width) within the shortest PRT (T_1) but allows overlaid echoes to occur. This algorithm is being implemented into the WSR-88D system (Torres et al. 2009) with a κ of 2/3.

Since weather signals are allowed to overlay, an overlay determination is needed to recover Doppler moments in regions where overlays could occur. If weather signals overlay, the stronger signal dominates the Doppler estimates. These Doppler estimates can be recovered if the power ratio (stronger-to-weaker) exceeds a level such that the bias and standard deviation are reasonably low. Sirmans studied the Doppler moment bias associated with overlaid occurrences in a uniformly sampled environment (Sirmans 1990 and Sirmans 1998). He found that biases in velocity estimates can be maintained below 1 m/s for the stronger weather signal when the power ratio exceeds 5 dB; whereas, the weaker weather signal is unrecoverable. Later, he found that spectrum width overlaid power ratio needed to be about 20 dB to provide useful spectrum width estimates for the stronger weather signal. The focus of this paper is to provide overlaid threshold values such that the Doppler moments can be reliably estimated to some accuracy for the SPRT algorithm.

2. OVERLAID ECHOES IN SPRT

The SPRT uses alternate pulses transmitted at two PRTs (T_1 and T_2). For this paper, the algorithm described by Torres et al. (2004) with κ = 2/3 ($T_1 = \kappa T_2$) is used. With the proper choice of T_1 and T_2, overlaid weather echoes can be avoided as shown in figure 1a. The algorithm allows overlaid echoes to extend beyond the receive time of T_1 (figure 1b), but not beyond the receive time of T_2 (figure 1c). The receive times can be broken into regions that are half of the receive time of T_1. Thus, two regions are created for T_1 (I and II), and three regions are created for T_2 (I, II, and III). Note in figure 1b that during receive time for region II in both T_1 and T_2 there are no overlaid echoes. Additionally, in figure 1b, there are no overlaid echoes in region I for T_1 and region III for T_2. As mentioned before, this allows the reflectivity (power) to be recovered unambiguously in all three regions. That is, the reflectivity estimate is recoverable to the maximum extent of the unambiguous range of T_2 ($r_{a2} = cT_2 / 2$) without overlaid echoes biasing the estimates. Thus, the reflectivity (power) becomes a useful means of determining the amount of overlay that occurs in region I and region III.

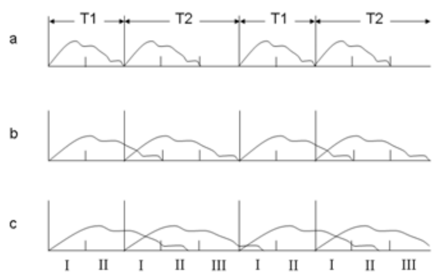

Figure 1. Overlaid Weather in SPRT.

In the algorithm, Doppler moments for region I are calculated from the lag-one autocorrelation between pulses T_1 and T_2. In fact, the difference of two lag-one autocorrelations is used for the final velocity estimate. The two autocorrelations are constructed as:

$$R_{12}(n,1) = \frac{2}{M}\sum_{m=0}^{M/2-1} V^*(n,2m)V(n,2m+1) \qquad (3)$$

$$R_{21}(n,1) = \frac{2}{M-2}\sum_{m=0}^{M/2-2} V^*(n,2m+1)V(n,2m+2) \qquad (4)$$

Where the xx in R_{xx} is the lag-one autocorrelation from pulse to pulse (i.e., equation 3 is from T_1 to T_2 and equation 4 is from T_2 to T_1), n is the range gate index within T_1, M is the number of pulse samples, V is the voltage measured, and m is the pulse index. For the sake of simplicity, we assume that the sequence of pulses starts with T_1 and M is even. For a generalized approach, the reader is referred to Torres et al. (2004).

Region III Doppler moments are not recovered using the algorithm described by Torres. However, another set of lag-one autocorrelations can be constructed in range from only the T_2 pulses:

$$R_{22}(n+N_1,1) = \frac{2}{M-2}\sum_{0}^{M/2-2} V^*(n,2m+1)V(n+N_1,2m+1) \qquad (5)$$

$$R_{22}(n+N_1,1) = \frac{2}{M-4}\sum_{0}^{M/2-3} V^*(n+N_1,2m+1)V(n,2m+3) \qquad (6)$$

Here, all variables are as before, except that n is restricted to range gates occurring in the first half of T_1 and N_1 is the number of range gates in T_1.

From figure 1b, it is seen that only the region I pulses from T_2 can be overlaid. In equations 3 and 4, the odd pulses (2m and 2m+2 with index starting at 0) are from region I of T_1 and the even pulses are from region I of T_2. In equations 5 and 6, all the pulses are from T_2. Thus, overlaid echoes can only occur in the range gates for n and not for $n+N_1$.

At this point, it becomes necessary to introduce the SPRT kernel [10100]. The SPRT kernel represents the smallest periodic sampling of the sequences of voltages from T_1 and T_2 when placed in their appropriate time slots in the uniform sampling $T_u = T_2 - T_1$ (e.g. Sachidananda and Zrnić 2003). In the kernel, a 1 represents the presence of a pulse and a 0 represents the absence of a pulse. In this way, the SPRT sequence can be seen as the product of the periodic extension of the SPRT kernel with a sequence of uniformly sampled voltages at a PRT equal to T_u.

Following the previous discussion, let's introduce the overlaid kernels. The overlaid kernel from equations 3 and 5 is [00100] and the overlaid kernel from equation 4 and 6 is [10000]. The overlaid kernels result from the occurrence of overlaid echoes from region I of T_2 in the above lag-one autocorrelations. A 1 in the overlaid kernels represents the presence of an overlaid echo; whereas, a 0 represents no overlaid echo or the absence of a pulse.

Comparing the overlaid kernels to the SPRT kernel it is easy to see that only half the power of the overlaid echoes contaminates the Doppler estimates (i.e. every other pulse is contaminated by overlaid echoes). There is an additional benefit that is not readily seen, but is very apparent when comparing the power spectra of the three kernels. That is, both overlaid spectra are evenly distributed across the SPRT spectrum. In figure 2, the SPRT kernel spectrum (blue) is shown against the overlaid kernel spectra (red). Note that both overlaid cases have the same power spectrum and that the power in the central coefficient of the SPRT kernel is 6 dB greater than any of the overlaid kernel coefficients.

Finally, consider the correlation between the overlaid signal and the true signal. Zrnić and Mahapatra (1985) convincingly argue that overlaid echoes will not coherently contribute to the bias of the Doppler estimates, but they warn that the overlaid echoes will increase the standard error of estimates.

The lack of correlation between the overlaid and true signal along with the preceding overlaid kernel discussion suggest that SPRT can tolerate overlaid echoes better than with uniform sampling. Specifically, the power ratio for SPRT can be at least 6 dB lower than the power ratio for uniform sampling as formulated by Sirmans (1990, 1998).

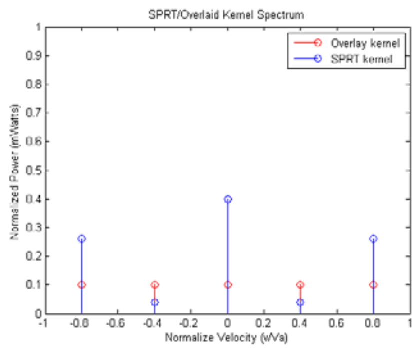

Figure 2. Comparison of kernel spectra.

3. EVALUATION METHOD

The standard model for weather simulations is a Gaussian spectrum (e.g. Doviak and Zrnić 1993). To construct the simulation of overlaid echoes in region I and III of SPRT, two uncorrelated uniform time series (V_a and V_b) are created at a PRT (T_u = $T_1/2$) as described by Sachidananda and Zrnić (2003). A third time series (V_c) is created as the composite of delayed $V_a(t+2)$ and $V_b(t)$. For region I, $V(n,m)$ ($n \le N_1/2$, m=0:2:M-2) is created from every fifth pulse of V_a starting with the first pulse. Then, using every fifth pulse of V_c starting with the first pulse, fill in $V(n,m)$ ($n \le N_1/2$, m=1:2:M-1). For region III, $V(n+N_1,m)$ ($n \le N_1/2$, m=1:2:M-1) is created from every fifth pulse of V_b starting with the third pulse. For region II, V_a is used to create $V(n,m)$ ($N_1/2 < n \le N_1$, m=0:M-1) by taking only those samples that match the ones in the periodic extension of the SPRT kernel (κ=2/3), all other pulses are dropped. In figure 3, each region is shown along with the time series that is used in the region. To summarize, V_a is used in regions I and II (not necessarily the same V_a), V_b is used in region III, and V_c (delayed V_a + V_b) is used in region I.

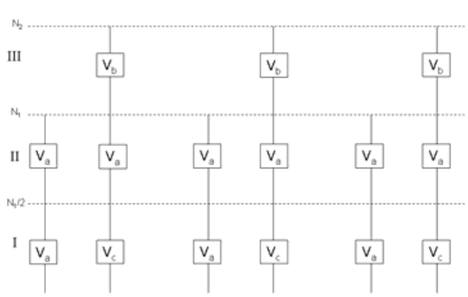

Figure 3. SPRT time series construction.

With this construction, a set of time series simulations were created and passed to the algorithm described by Torres et al. (2004) with the addition of the lag-one autocorrelations (equations 5 and 6) to recover Doppler moments in region III. The parameters for the simulation are listed in table 1. The true velocity was varied over the range from -100 m/s to 100 m/s across regions I and II; while, the true velocity was varied from 50 m/s to -50 m/s in region III. Since the unambiguous velocity is only 50 m/s, the true velocity in regions I and II aliases. The signal-to-noise (SNR) was held constant at 20 dB in regions I and II. In region III, the SNR was varied from 0 dB to 40 dB.

Table 1. Simulation Parameters

Parameter	Value
λ	~10 cm
T_1	1002 µs
T_2	1503 µs
N_1	600 bins
N_2	900 bins
r_{a1} (for T_1)	150 km
r_{a2} (for T_2)	225 km
v_a	50 m/s
v_{true}	-100 m/s to 100 m/s (bins 1 to 600) 50 m/s to -50 m/s (bins 601 to 900)
SNR_1	20 dB
SNR_2	20 dB
SNR_3	0 dB to 40 dB
σ_v	4 m/s
κ	2/3

Figures 4 through 9 show three plots: SNR (top), velocity (middle), and spectrum width (bottom). The solid green lines in regions I and II are the true values; whereas, the solid red line in region III represents the true values. The overlaid signals are shown with dashed lines. The dashed green line in region III is the overlaid signal from region I, and the dashed red line in region I is the overlaid signal from region III. There are no overlaid signals in region II. Note that the dashed green line for velocity shows the true velocity as being aliased. The blue line represents the estimates of the moments.

In figure 4, the power ratio (S_1/S_2) is 20 dB. As mentioned, the power estimate is never overlaid and should provide quality estimates regardless of the power ratio. However, the Doppler moments are influenced severely by the overlaid echoes with a power ratio of 20 dB. The velocity and spectrum widths are recoverable in region I but not in region III. Note that the velocity is aliased in region I and II. This behavior is expected and will occur if the true velocity exceeds the unambiguous velocity (50 m/s).

The power ratio is reduced to 10 dB in figure 5. Note that the velocity is very good (low variance) in region I; whereas the spectrum width is starting to deteriorate. At a power ratio of 0 dB in figure 6, the velocity is completely recoverable in all regions, but the spectrum width has high variance in both regions I and III. By decreasing the power ratio to -5 dB in figure 7 (i.e. region III has the stronger signal), the velocity exhibits low variance and is still recoverable in all regions. However, the spectrum width has a high variance in regions I and III, but is starting to improve in region III. As the region III signal becomes more dominant in figures 8 and 9 with power ratios of -10 dB and -20 dB respectively, the region III Doppler moments experience less influence from the region I overlaid signal. Consequently, the Doppler moments exhibit less variance in region III.

4. SUMMARY

An improvement to the SPRT algorithm is shown that allows recovery of Doppler moments beyond the unambiguous range of the shortest PRT. For a given dwell time, moment estimates are better when using a shorter PRT because more pulses are collected. This is true in SPRT as well. With the implementation of the new lag-one autocorrelations, the range coverage for Doppler moments is increased by 50 percent allowing the use of a shorter PRT for the same Doppler coverage. In turn, the use of a shorter PRT provides improved estimates.

Although the analysis is preliminary, it is evident that acceptable recovery of Doppler estimates (velocity and spectrum width) in the presence of overlaid echoes is possible. In fact, the simulations indicate that velocity recovery in both regions I and III is possible if the strongest signal is not more than 5 dB stronger than the weakest signal. This suggests that there is a 10 dB overlap region where the power ratio (strongest signal to weakest signal) is between 0 dB and 5 dB allowing recoverable velocities for both regions. In the uniform sampling reported by Sirmans (1990) the stronger signal velocity is unrecoverable when the power ratio is below 5 dB. Additionally, spectrum width is shown to be recoverable if the power ratio (strongest signal to weakest signal) is above 10 dB indicating about a 10 dB improvement over uniform samples at 20 dB as reported by Sirmans (1998).

Part of the reason for the better performance of SPRT is because only half of the overlaid signal is contaminating the Doppler spectrum. Other factors that improve the recovery of overlaid echoes are the incoherency and even distribution of the overlaid signal in the SPRT spectrum. It was shown in figure 2 that the SPRT spectrum is 6 dB above the overlaid spectrum. Nevertheless, the simulations show that about 10 dB of improvement over uniform sampling is achieved.

5. ACKNOWLEDGEMENTS

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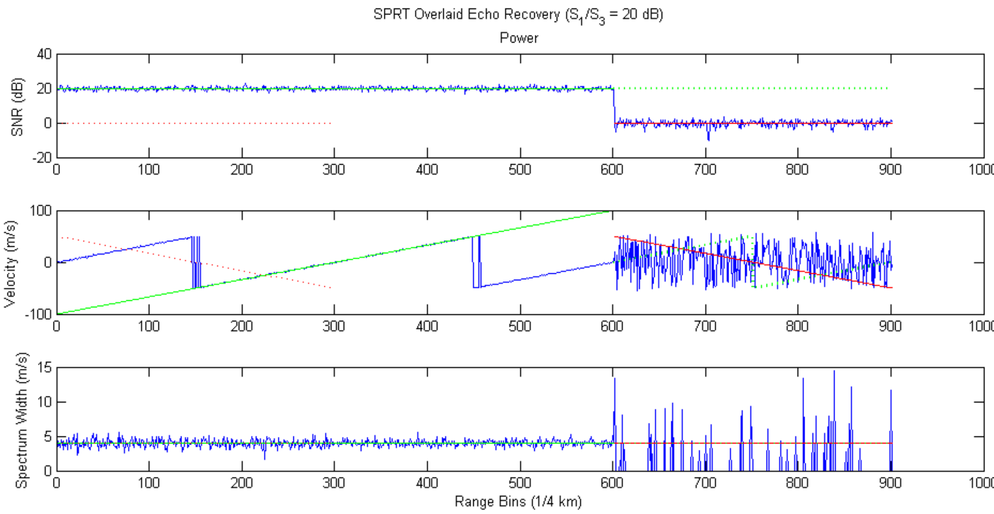

Figure 4. SPRT overlaid echo recovery with a power ratio of 20 dB.

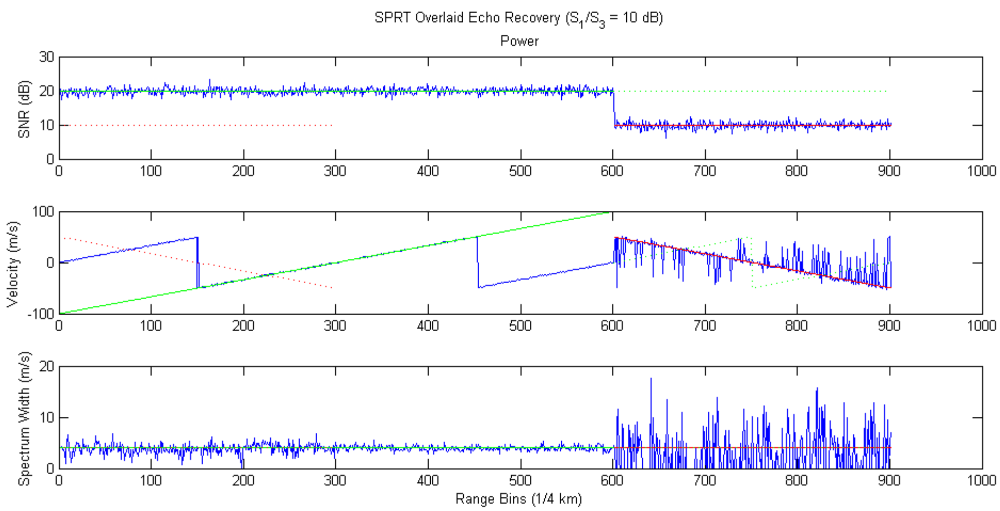

Figure 5. SPRT overlaid echo recovery with a power ratio of 10 dB.

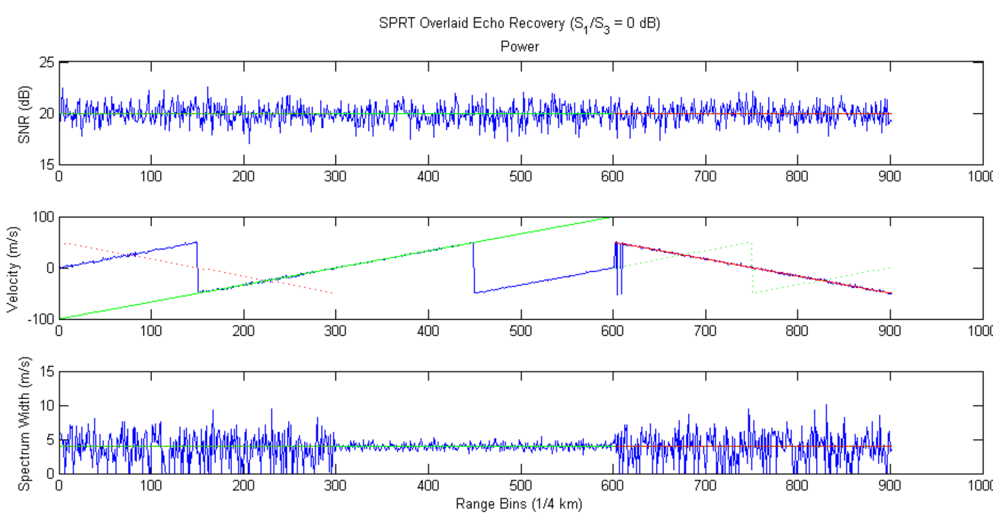

Figure 6. SPRT overlaid echo recovery with a power ratio of 0 dB.

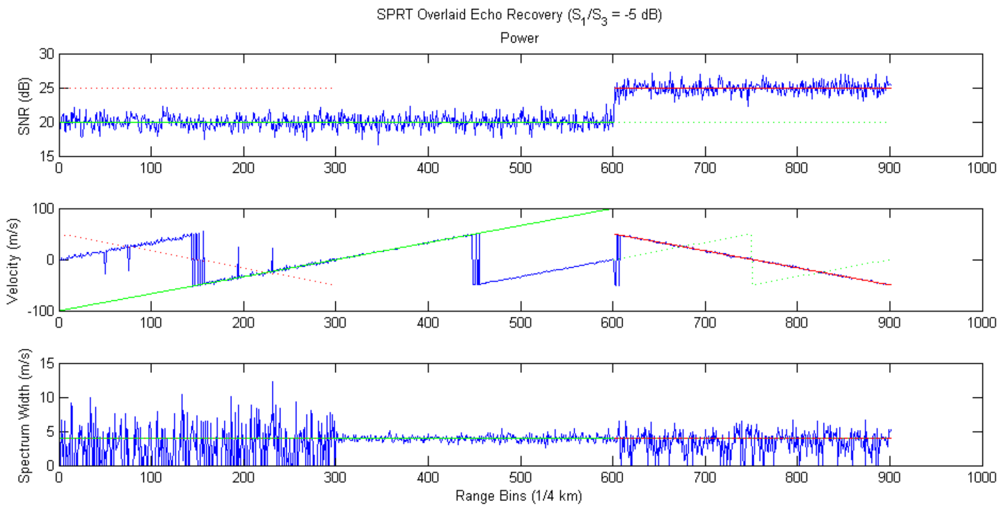

Figure 7. SPRT overlaid echo recovery with a power ratio of -5 dB.

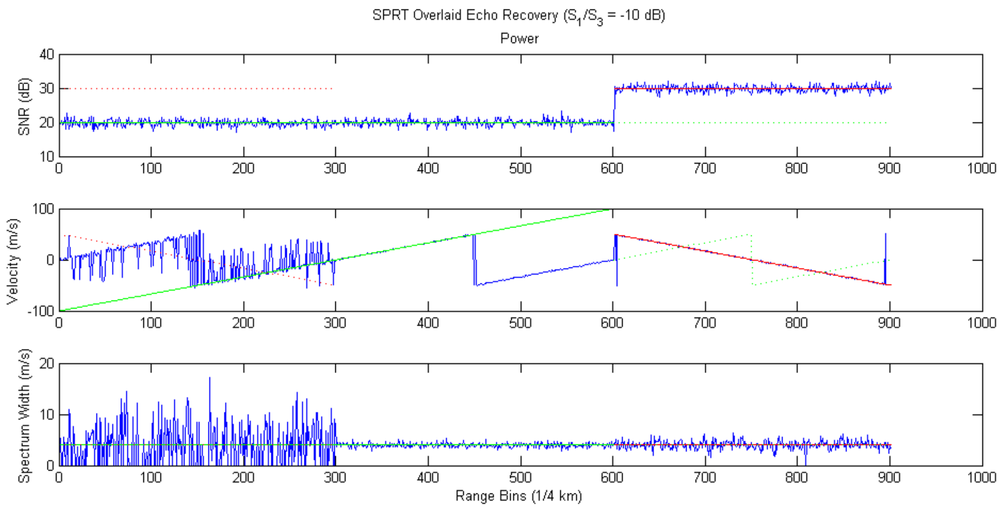

Figure 8. SPRT overlaid echo recovery with a power ratio of -10 dB.

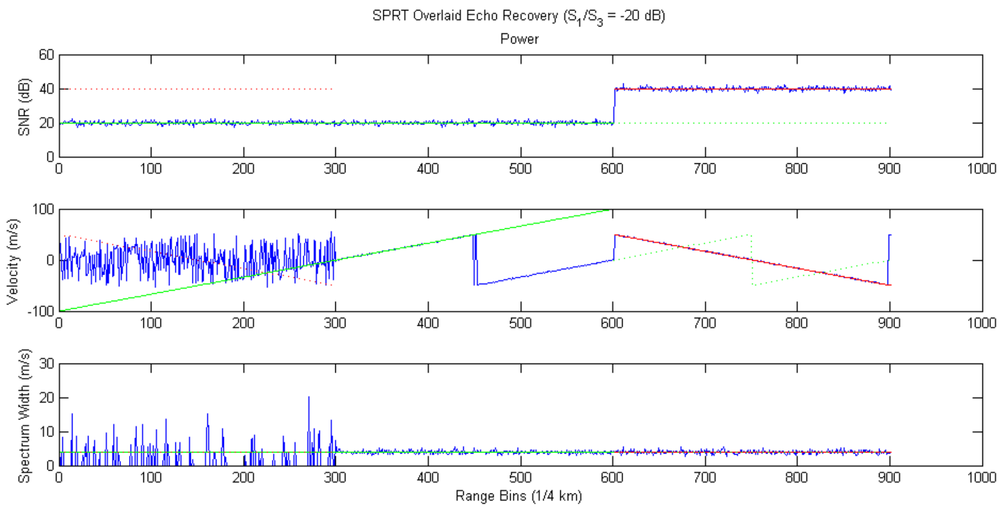

Figure 9. SPRT overlaid echo recovery with a power ratio of -20 dB.