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Calibration of Polarimetric Phased Array Radar for Improved Measurement Accuracy

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INTRODUCTION

The requirement to satisfy a Z_{dr} antenna induced bias of less than 0.1 to 0.2 dB presents significant design challenges to the phased array radar system design. Three of the key parameters that affect the Z_{dr} bias imbalances in the transmitter and receiver channels and depolarization introduced by the antenna radiating structure. Zrnic⁽¹⁾ has shown that -30 dB cross-polarization of an antenna alone at an arbitrary phase can introduce a bias error of ± 2 dB.

In most current applications with reflector antennas, errors in the antenna transmit and receive channels are calibrated to minimize the Z_{dr} measurement error and the antenna is designed with a very low cross-polarization. This problem is somewhat more challenging in the phased array due to variations in the antenna characteristics over the scan coverage region. Urkowitz⁽¹⁾ has shown that errors in polarization characteristics of phased array antennas with ideally orthogonal radiating elements introduced in transmission may be corrected in the receive channels of the antenna where compensation is more practical. For phased arrays with non-orthogonal radiating elements, Zhang et. al.⁽²⁾ has shown that a matrix can be used to correct the measured scattering matrix of the reflected weather data for sufficiently narrow beam antennas. This correction matrix approach is recast, herein, to facilitate the analysis of all of the array measurement parameters requirements

necessary to achieve improved Z_{dr} accuracy. These include signal amplitude, phase imbalances in the transmitted, and receive signal, radiating element cross-polarization characteristics and antenna array tilt.

The correction matrix will be obtained by measurement of the polarization characteristics of the phased array over all scan angles. A description of the algorithm is included in section 1 of this report. Section 2 of this report includes a brief discussion of the required measurement accuracies to satisfy challenging Z_{dr} bias requirements.

1. POLARIZATION COMPENSATION

Figure 1 is a schematic showing salient features of the phased array. The horizontal and vertically polarized channels at each radiating element include a separate phase shifter, high power amplifier (HPA) and duplexed low noise amplifier (LNA) and radiating element. An ideal directional coupler shown between each circulator and antenna element provides a measurement of the incident (alternate sampling approaches can be used) transmitted power, which will be used to measure the coupled signal amplitude and phase for calibration of the array. The antenna radiates vertical and horizontal polarization simultaneously producing a slant polarization and receives both polarizations in two separate channels. The modules for each polarization are separately combined in the receive mode to provide a low sidelobe antenna patterns.

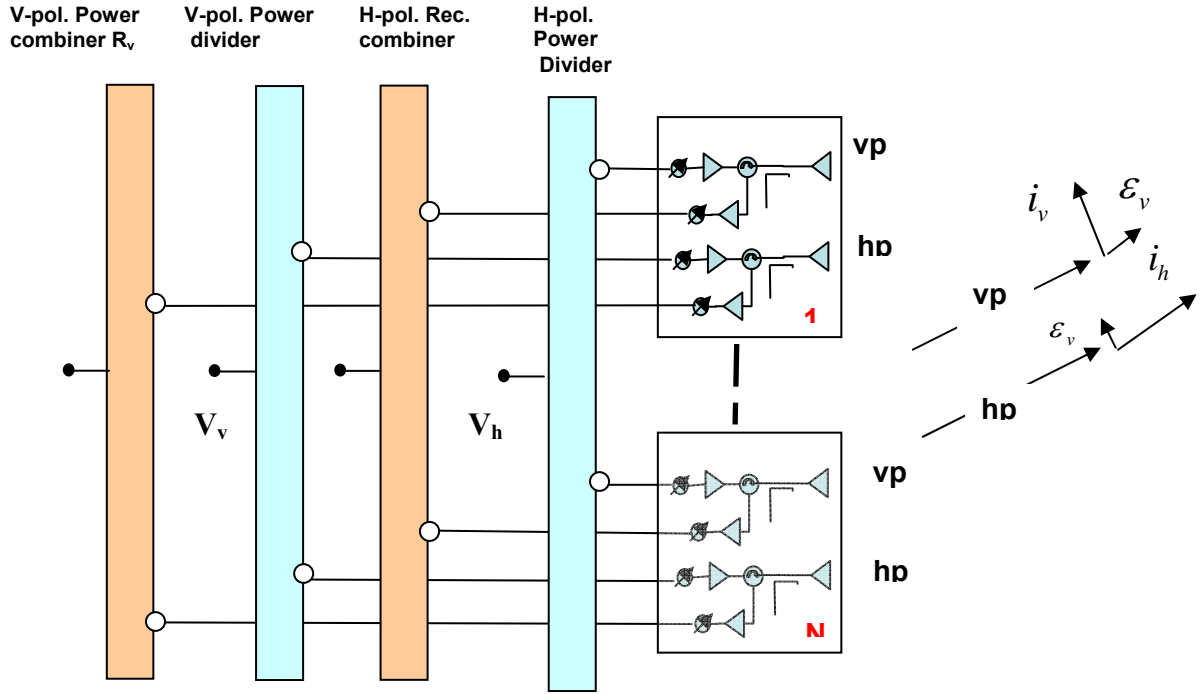


Figure 1. Block diagram of phased array

In accordance with the coordinate system in Figure 2, when only the h-port is excited, the unit vector of the radiated field is

$$\hat{\mathbf{e}}_h = i_h \hat{\mathbf{h}}_i + \epsilon_h \hat{\mathbf{v}}_i \quad (1)$$

and when only the v-port is excited, the unit vector of the radiated field is

$$\hat{\mathbf{e}}_v = \epsilon_v \hat{\mathbf{h}}_i + i_v \hat{\mathbf{v}}_i \quad (2)$$

i_h and i_v are the magnitude of the co-polar components and ϵ_h and ϵ_v are cross-polarized components of the horizontal and vertical ports, respectively. The radiated field is subscripted i to denote signal incident on scattering objects.

Consider an array with $(2M+1)(2N+1)$ elements located in a parallelogram lattice as shown in Figure 2. The element locations are represented by the vector

$$\bar{\mathbf{r}}_{mn} = m\bar{\mathbf{d}}_1 + n\bar{\mathbf{d}}_2 \text{ where } |m| \leq M \text{ and } |n| \leq N$$

where $\bar{\mathbf{d}}_1$ and $\bar{\mathbf{d}}_2$ are shown in Figure 3.

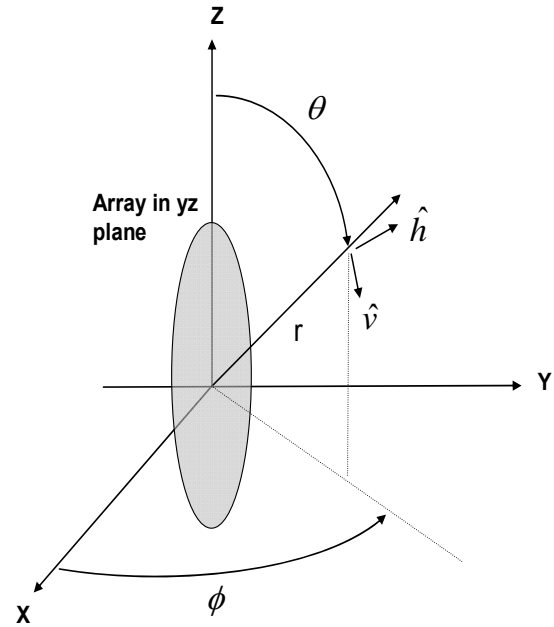


Figure 2. Coordinate System

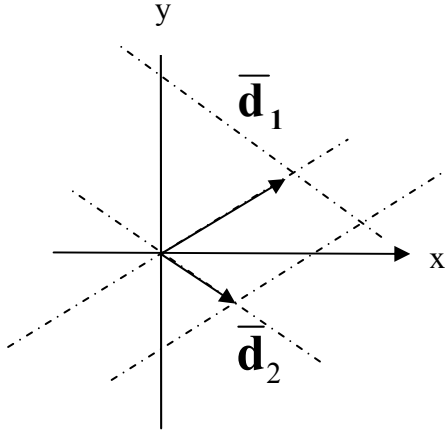


Figure 3. Radiating Element Lattice

Adapting notation similar to Bringi and Chandrasekhar⁽³⁾ extended to the he phased array antenna, the radiated field at a specific point in space r, θ, ϕ is:

$$\bar{E}_{ht}^i = \sqrt{\frac{Z_0}{2\pi}} M_h \sqrt{g_h} e^{j\phi_{ht}} (\hat{i}_h \hat{h}_i + \epsilon_h \hat{v}_i) \frac{e^{-jk_0 r}}{r}$$

when $M_v = 0$ (3)

$$\bar{E}_{vt}^i = \sqrt{\frac{Z_0}{2\pi}} M_v \sqrt{g_v} e^{j\phi_{vt}} (\epsilon_v \hat{h}_i + \hat{i}_v \hat{v}_i) \frac{e^{-jk_0 r}}{r} \text{ when } M_h = 0$$

(4)

where

$$M_h = \sum_{m,n}^{M,N} M_{hmn} e^{j(\bar{k}_T - \bar{k}_{T0}) \times \bar{r}_{mn}} \text{ and } M_v = \sum_{m,n}^{M,N} M_{vmn} e^{-j(\bar{k}_T - \bar{k}_{T0}) \times \bar{r}_{mn}}$$

$$\text{and } P_h = \sum_{m,n}^{M,N} |M_{hmn}|^2, \text{ and } P_v = \sum_{m,n}^{M,N} |M_{vmn}|^2 \text{ are}$$

the h and v polarized transmit powers, respectively.

g_h, ϕ_h and g_v, ϕ_v are the h and v element gain and phase

$$\bar{k}_T = \frac{2\pi}{\lambda} (u\hat{i}_x + v\hat{i}_y), \bar{k}_{T0} = \frac{2\pi}{\lambda_0} (u_0\hat{i}_x + v_0\hat{i}_y)$$

$$u = \sin\theta \cos\phi, v = \sin\theta \sin\phi$$

$$u_0 = \sin\theta_0 \cos\phi_0, v_0 = \sin\theta_0 \sin\phi_0$$

θ_0, ϕ_0 is the array pointing angle

Z_0 is the free space impedance = 377ohms

λ =free space wavelength at frequency f

r is distance from the center of array to the field point.

M_h and M_v are the transmit horizontal and vertical polarized array factors and the coefficients in these respective summations determine the antenna sidelobes and beamwidth.

Hence

$$E_{ht}^i \hat{h}_i = \sqrt{\frac{Z_0}{2\pi}} (M_h \sqrt{g_h} e^{j\phi_{ht}} \hat{i}_v + M_v \sqrt{g_v} e^{j\phi_{vt}} \epsilon_v) \hat{h}_i$$

(5)

$$E_{vt}^i \hat{v}_i = \sqrt{\frac{Z_0}{2\pi}} (M_h \sqrt{g_h} e^{j\phi_{ht}} \epsilon_h + M_v \sqrt{g_v} e^{j\phi_{vt}} \hat{i}_v) \hat{v}_i$$

(6)

In matrix form,

$$\begin{pmatrix} E_{ht}^i \\ E_{vt}^i \end{pmatrix} = \sqrt{\frac{Z_0}{2\pi}} \frac{e^{-jk_0 r}}{r} \begin{pmatrix} \sqrt{g_h} e^{j\phi_{ht}} i_h & \sqrt{g_v} e^{j\phi_{vt}} \epsilon_v \\ \sqrt{g_h} e^{j\phi_{ht}} \epsilon_h & \sqrt{g_v} e^{j\phi_{vt}} i_v \end{pmatrix} \begin{pmatrix} M_h \\ M_v \end{pmatrix} \quad (7)$$

or

$$\begin{pmatrix} E_{ht}^i \\ E_{vt}^i \end{pmatrix} = \sqrt{\frac{Z_0}{2\pi}} \frac{e^{-jk_0 r}}{r} \begin{pmatrix} i_h & \epsilon_v \\ \epsilon_h & i_v \end{pmatrix} \begin{pmatrix} \sqrt{g_h} e^{j\phi_{ht}} & 0 \\ 0 & \sqrt{g_v} e^{j\phi_{vt}} \end{pmatrix} \begin{pmatrix} M_h \\ M_v \end{pmatrix} \quad (8)$$

$$\text{Let } \beta^2 = \sqrt{\frac{g_h}{g_v}} e^{j(\phi_h - \phi_v)}$$

β is identical in transmit and receive since the same radiating element is used in both modes.

Then

$$\begin{pmatrix} E_{ht}^i \\ E_{vt}^i \end{pmatrix} = \sqrt{\frac{Z_0}{2\pi}} \frac{e^{-jk_0 r}}{r} (g_h g_v)^{\frac{1}{4}} e^{j\frac{(\phi_h + \phi_v)}{2}} \begin{pmatrix} i_h & \epsilon_v \\ \epsilon_h & i_v \end{pmatrix} \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \begin{pmatrix} M_h \\ M_v \end{pmatrix} \quad (9)$$

In a similar manner to the analysis for transmit, it can be shown that the receive voltage collected by the entire array after beamforming is given by

$$\begin{pmatrix} V_{hr} \\ V_{vr} \end{pmatrix} = \frac{\lambda_0}{\sqrt{8\pi Z_0}} \frac{e^{-jk_0 r}}{r} (g_h g_v)^{\frac{1}{4}} e^{j\frac{\phi_{hr} + \phi_{vr}}{2}} \begin{pmatrix} R_h \beta & 0 \\ 0 & R_v \beta^{-1} \end{pmatrix} \begin{pmatrix} i_h & \epsilon_h \\ \epsilon_v & i_v \end{pmatrix} \begin{pmatrix} E_{hr}^i \\ E_{vr}^i \end{pmatrix} \quad (10)$$

where

$$R_h = \sum_{m,n}^{M,N} R_{hmn} e^{j(\bar{k}_T - \bar{k}_{T0}) \cdot \bar{r}_{mn}} \text{ and } M_v = \sum_{m,n}^{M,N} R_{vmn} e^{j(\bar{k}_T - \bar{k}_{T0}) \cdot \bar{r}_{mn}}$$

R_h and R_v are the horizontally and vertically receive polarized antenna array factors physically incorporated in the receive manifolds. The coefficients of R_h and R_v are the aperture weighting factors that determine the antenna receive sidelobes and beamwidth. E_{hr} and E_{vr} are the reflected electric field components incident on the antenna. The reflected field of a single scattering object is given by the scattering matrix

$$\begin{pmatrix} E_{hr} \\ E_{vr} \end{pmatrix} = \begin{pmatrix} S_{hh} & S_{vh} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} E_{ht}^i \\ E_{vt}^i \end{pmatrix} \quad (11)$$

For the case of simultaneous transmission and simultaneous reception, the scattering matrix is assumed diagonal.

Combining 7, 10 and 11 we get

$$\begin{pmatrix} V_{hr} \\ V_{vr} \end{pmatrix} = \frac{\lambda}{4\pi r^2} e^{-j2k_0 r} \sqrt{g_h g_v} e^{j\frac{\phi_h + \phi_v}{2}} \begin{pmatrix} R_h \beta & 0 \\ 0 & R_v \beta^{-1} \end{pmatrix} \begin{pmatrix} i_h & \epsilon_h \\ \epsilon_v & i_v \end{pmatrix} \begin{pmatrix} S_{hh} & 0 \\ 0 & S_{vv} \end{pmatrix} \begin{pmatrix} i_h & \epsilon_v \\ \epsilon_h & i_v \end{pmatrix} \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \begin{pmatrix} M_h \\ M_v \end{pmatrix} \quad (12)$$

Evaluating (12), we get

$$\begin{aligned}\frac{V_h}{C_0} &= s_{hh} \left[(i_h^2 \beta^2 M_h + i_h \varepsilon_v M_v) + s_{vv} (\varepsilon_h^2 \beta^2 M_h + \varepsilon_h i_v M_v) \right] R_h \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}} \\ \frac{V_v}{C_0} &= s_{hh} \left[(\varepsilon_v i_h M_h + \varepsilon_v^2 \beta^{-2} M_v) + s_{vv} (i_v \varepsilon_h M_h + i_v^2 \beta^{-2} M_v) \right] R_v \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}}\end{aligned}\quad (13)$$

where $C_0 = \frac{\lambda}{4\pi} \frac{e^{-j2kr}}{r^2}$

Integrating the right hand side of (13) over the beamwidth of the antenna, we get

$$\begin{pmatrix} V_h \\ V_v \end{pmatrix} = C_0 \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} s_{hh} \\ s_{vv} \end{pmatrix} \quad (14) \quad \text{determinant} = \Delta = AD - BC$$

where

$$A = \iint (i_h^2 \beta^2 M_h + i_h \varepsilon_v M_v) R_h \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}} d\Omega \quad (15)$$

$$B = \iint (\varepsilon_h^2 \beta^2 M_h + \varepsilon_h i_v M_v) R_h \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}} d\Omega \quad (16)$$

$$C = \iint (\varepsilon_v i_h M_h + \varepsilon_v^2 \beta^{-2} M_v) R_v \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}} d\Omega \quad (17)$$

$$D = \iint (i_v \varepsilon_h M_h + i_v^2 \beta^{-2} M_v) R_v \sqrt{g_h g_v} e^{j \frac{\phi_h + \phi_v}{2}} d\Omega \quad (18)$$

$$\frac{s_{hh}}{s_{vv}} = \frac{DV_h - BV_v}{-CV_h + AV_v} \quad (20)$$

$$\left| \frac{s_{hh}}{s_{vv}} \right|^2 = \frac{(DV_h - BV_v)(D^* V_h^* - B^* V_v^*)}{(-CV_h + AV_v)(-C^* V_h^* + A^* V_v^*)}$$

$$\left| \frac{s_{hh}}{s_{vv}} \right|^2 = \frac{|DV_h|^2 + (BV_v)^2 - 2 \operatorname{Re}(DV_h B^* V_v^*)}{|CV_h|^2 + (AV_v)^2 - 2 \operatorname{Re}(CV_h A^* V_v^*)} \quad (21)$$

Equation (14) is solved to obtain s_{hh} and s_{vv} . For the purposes of analysis of the antenna parameter deviations on the estimate of Z_{dr} , the scattering parameters are assumed to have a uniform density and fill the entire beam-width.

For small matched h and v beamwidths, where variations of the antenna parameters are negligible over the beam, Z_{dr} for a specific beam-pointing angle is given by:

Combining (14) and (15), we get

$$Z_{dr} = \left[\begin{pmatrix} V_h \\ V_v \end{pmatrix} \begin{pmatrix} M_v \\ M_h \end{pmatrix} \begin{pmatrix} R_v \\ R_h \end{pmatrix} \begin{pmatrix} i_v \\ i_h \end{pmatrix} \right]^2 \beta^{-4} \left[1 + \frac{\varepsilon_h}{i_v} \beta^2 \left(\frac{M_h}{M_v} - \frac{R_h V_v}{R_v V_h} \right) - \frac{\varepsilon_v}{i_h} \beta^{-2} \left(\frac{M_v}{M_h} - \frac{R_v V_h}{R_h V_v} \right) \right]^2 \quad (22)$$

$\frac{i_v}{i_h}$ is the ratio of the antenna co-polar unit

vectors characteristics and $\frac{\varepsilon_v}{i_h}$ and $\frac{\varepsilon_h}{i_v}$ are the antenna cross-pol. characteristics.

The measured values of V_h and V_v are the

$$Z_{dr} = \left| \frac{S_{hh}}{S_{vv}} \right|^2 = \left| \frac{V_h}{V_v} \right|^2 \frac{P_v G_{tv} G_{rv} |i_v|^4}{P_h G_{th} G_{th} |i_h|^4} \left| 1 + \frac{\varepsilon_h}{i_v} e^{-j\phi_\Delta} \left(\sqrt{\frac{P_h G_{th}}{P_v G_{tv}}} - \sqrt{\frac{G_{th} V_h}{G_{rv} V_v}} \right) - \frac{\varepsilon_v}{i_h} e^{j\phi_\Delta} \left(\sqrt{\frac{P_v G_{tv}}{P_h G_{th}}} - \sqrt{\frac{G_{rv} V_v}{G_{th} V_h}} \right) \right|^2$$

where $\phi_\Delta = \phi_h - \phi_v$

(23)

2. MEASUREMENT TOLERANCE REQUIRED TO ACHIEVE Z_{dr}

This section considers the required measurement accuracies required to achieve a specified Z_{dr} within a tolerance range for the parameters in equation (22) for Z_{dr} . The following equations summarize the analysis of the allowable worst-case (maximum allowable amplitude variation at all phase angles) variation of the parameters in equation (22) for a specified

variations in $\Delta Z_{dr}(dB)$ when the ratios $\frac{M_v}{M_h}$ and

$\frac{R_v}{R_h}$ are assumed to have a nominal value of unity and the absolute magnitude of the cross pol parameters are set equal

to each other, i.e. $\left| \frac{\varepsilon_h}{i_v} \right| = \left| \frac{\varepsilon_v}{i_h} \right| = \left| \frac{\varepsilon}{i} \right|$

$$\Delta \left| \frac{M_v}{M_h} \right| (dB) < \frac{\Delta Z_{dr}(dB)}{\left[1 + \left| \frac{\varepsilon}{i} \right| \left[\left| \beta^2 \right| \left(2 + \left| \frac{V_v}{V_h} \right| \right) + \left| \beta^{-2} \right| \left(2 + \left| \frac{V_h}{V_v} \right| \right) \right] \right]} \quad (24)$$

$$\Delta \left| \frac{V_h}{V_v} \right| (dB) < \frac{\Delta Z_{dr}(dB)}{\left[1 + \left| \frac{\varepsilon}{i} \right| \left[\left| \beta^2 \right| \left| \frac{V_v}{V_h} \right| \left(2 + \left| \frac{V_h}{V_v} \right| \right) + \left| \beta^{-2} \right| \left| \frac{V_h}{V_v} \right| \left(2 + \left| \frac{V_v}{V_h} \right| \right) \right] \right]} \quad (25)$$

$$\Delta \left| \frac{R_v}{R_h} \right| (dB) < \frac{\Delta Z_{dr}(dB)}{\left[1 + \left| \frac{\varepsilon}{i} \right| \left[\left| \beta^2 \right| \left(1 + \left| \frac{V_h}{V_v} \right| + \left| \frac{V_v}{V_h} \right| \right) + \left| \beta^{-2} \right| \left(1 + 2 \left| \frac{V_h}{V_v} \right| \right) \right] \right]} \quad (26)$$

$$\Delta \left| \frac{i_v}{i_h} \right| (dB) < \frac{\Delta Z_{dr}(dB)}{2} \quad (27)$$

$$\Delta \left| \frac{\epsilon_v}{i_h} \right| (dB) < \frac{\Delta Z_{dr} (dB)}{\left[\left| \frac{\epsilon_v}{i_h} \right| \beta^2 \left(1 + \left| \frac{V_h}{V_v} \right| \right) \right]} \quad (28)$$

$$\Delta \left| \frac{\epsilon_h}{i_v} \right| (dB) < \frac{\Delta Z_{dr} (dB)}{\left[\left| \frac{\epsilon_h}{i_v} \right| \beta^{-2} \left(1 + \left| \frac{V_v}{V_h} \right| \right) \right]} \quad (29)$$

$$|\Delta \beta| (dB) < \frac{\Delta Z_{dr} (dB)}{4 \left[1 + \frac{3}{2} \left| \frac{\epsilon}{i} \right| \left(\left| \beta^2 \right| \left(1 + \left| \frac{V_v}{V_h} \right| \right) + \left| \beta^{-2} \right| \left(1 + \left| \frac{V_h}{V_v} \right| \right) \right) \right]} \quad (30)$$

At a specific frequency and beam pointing angle errors in the parameters in equation (19) contributes an error in Zdr.

A typical example is illustrated here for the case when the bias errors for each of the measured parameters are nominally assumed equal to zero and the horizontal and vertically polarized beams have matched beamwidths. A preliminary budget of allowable component tolerances to achieve $\Delta Z_{dr} = 0.2$ dBpk due to uncertainties in the parameter measurements of the variables is shown in Table 1 for nominal values of $\text{abs}(V_h/V_v) = 1$ (corresponding to a Zdr

approximately equal to 0 dB).

The first six columns list the nominal values of the antenna parameters. The eighth column of the table is a nominal allocation of the standard deviation of ΔZ_{dr} in dBrms to each of the polarization compensation variables listed in column 7. The sum of the entries in this column is equal to 0.2 dBpk. Column 9, the component tolerance is obtained by calculating corresponding component tolerances from equations (24) to (30). In the case of small errors, $(\text{abs}(V_h/V_v))^2$ is approximately equal to Z_{dr} .

M_v/M_h (dB)	$\text{abs}(V_h/V_v)$	i_v/i_h	β	ϵ_{vh} (dB)	ϵ_{hv} (dB)		Allocated ΔZ_{dr} budget dBpk	Component dBrms
						ΔZ_{dr} dBpk	0.20	
1	1	1	1	27	27	$\Delta M_v/M_h$	0.0185	0.0049
1	1	1	1	27	27	$\text{abs}(V_h/V_v)$	0.0200	0.0053
1	1	1	1	27	27	$\text{abs}(R_v/R_h)$	0.0200	0.0054
1	1	1	1	27	27	i_v/i_h	0.0600	0.0100
1	1	1	1	27	27	β	0.0600	0.0158
1	1	1	1	27	27	ϵ_{vh}	0.0120	0.0448
1	1	1	1	27	27	ϵ_{hv}	0.0120	0.0448

Table 1. Budget of Component Standard Deviations for $\Delta Z_{dr} = 0.2$ dBpk

The accuracy requirements in equations (24) to (30) are for the total antenna. Variations in, M_v/M_h and R_v/R_h occur as a consequence of amplitude and phase errors in each of the radiating element paths. The element level errors change at every beam position in a random fashion and, therefore, there is a corresponding change in the value of Zdr. Minimizing the allowable spread in Zdr requires that the measured element amplitude and phase errors be controlled within corresponding bounds. Similar considerations apply to the errors in the received voltage ratio given by R_h/R_v .

The allowable element level amplitude error in the measurement of the signal coupled from each element has been analyzed using a Monte Carlo approach and the results showing the allowable element level voltage and phase are shown in Figure 5 for phased arrays with 1224, 4896 and 9792 elements for various values of delta Zdr, for cross polarization values of 27 dB when the h and v beams are matched. Measurement requirements are inversely proportional to the number of elements in the array and are more stringent for antennas with low cross-polarization characteristics.

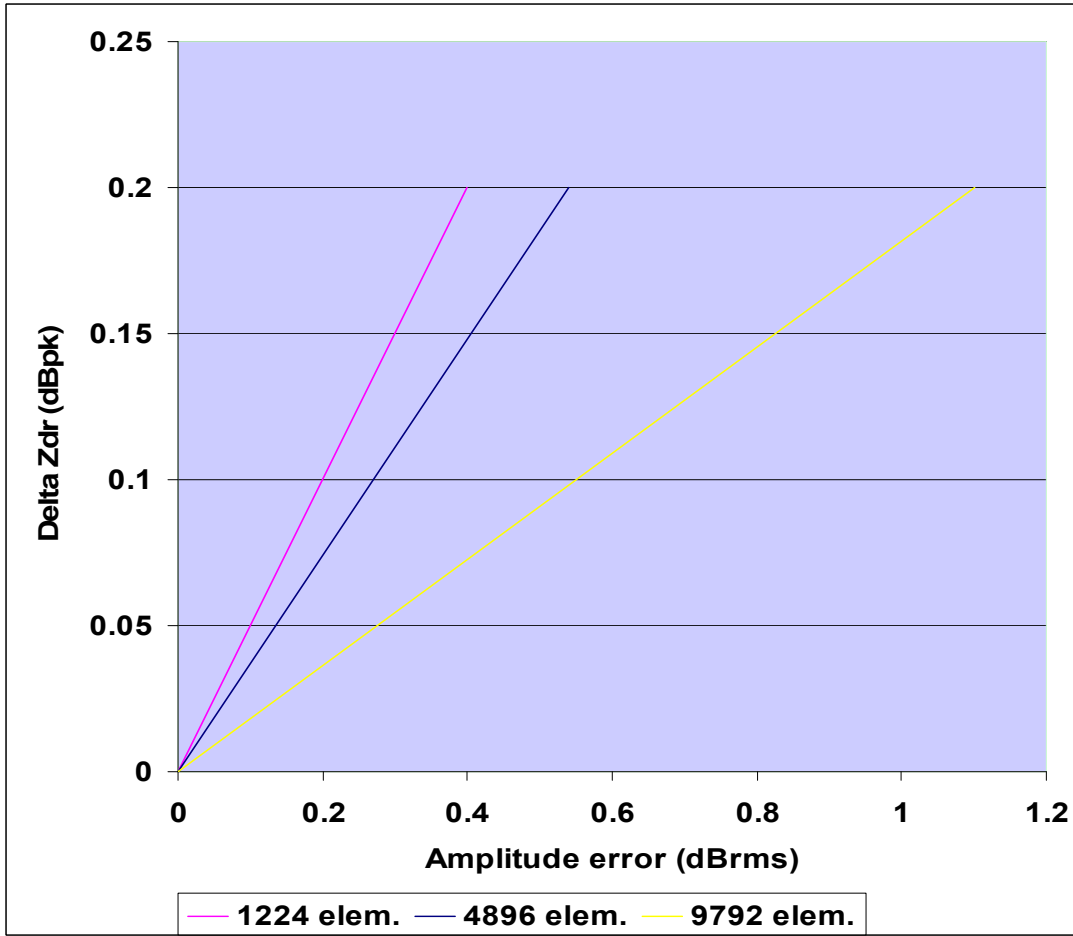


Figure 5 Allowable element amplitude to achieve delta Zdr (cross-pol=27 dB, $M_h/M_v=R_v/R_h=1$, $I_v/I_h, \beta=1$)

CONCLUSIONS

The requirement to achieve accurate measurements of Zdr presents new challenges to the design of phased arrays. Unlike dish antennas whose properties are generally invariant with respect to scan angle, transmitter and receiver

imbalance and antenna cross polarization change over the scan coverage of the antenna. A polarization compensation approach utilizing antenna calibration and real time measurement can be used to correct the measured scattering matrix and provide improved measurement accuracy for Zdr.

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