# MEAN SEA LEVEL PRESSURE REDUCTION IN CANADA AND THE CORRECTION FOR PLATEAU EFFECT AND LOCAL LAPSE RATE ANOMALY: FERREL VERSUS BIGELOW

Christopher A. Hampel\*,

Test and Evaluation, Meteorological Services of Canada, Environment Canada

### 1. INTRODUCTION

Before novel techniques for defining and deducing mean sea-level pressure were introduced in recent years<sup>1</sup>, the standard problem in MSL pressure reduction was understood essentially as the challenge of representing the virtual temperature profile of a fictitious column of free air that would otherwise have existed in the absence of a mountain or a plateau<sup>2</sup>. This conceptualization of the problem focused on improving mean sea-level extrapolations based on measurements obtained at the mountain or plateau station in question.

In 1886, Ferrel attributed the cause of what he identified as spurious amplifications of the annual variation in reduced MSL pressure to seasonal amplifications of temperature differences between measurements based on land as opposed to free air. Assuming a constant atmospheric lapse rate in his analysis, he introduced what is now referred to as the plateau correction to account for this effect.<sup>3</sup>

The plateau correction is meant to be applied only to North American stations to compensate for the large gradient in elevation associated with the western plateau. Ferrel's empirical relation, including the constant which he deduced over a century ago that defines the magnitude of the correction, is still in use in North American networks today and is considered requisite by the Smithsonian Meteorological Tables<sup>4</sup>.

In 1905, Bigelow improved upon Ferrel's work and included in his analysis an attempt to relate local variations in the lapse rate to the temperature<sup>5</sup>. measured station This improvement supposedly allowed for more accurate local representations of the temperature profile of the fictitious air column. Like Ferrel, Bigelow expressed his variable lapse rate-version of the correction in units of pressure to be added algebraically to the reduced pressure. The correction was also applied both above and below the 305 gpm height limit, although different assumptions were used for the two regimes<sup>2</sup>. Despite the greater detail promised by Bigelow's approach, his methods for deducing variations in local lapse rate above 305 gpm are notoriously

difficult to reproduce and may offer only small improvements to Ferrel's much simpler formulation.

In 1963, the WBAN introduced a modified version of Bigelow's correction by expressing it in units of temperature to be added directly to the mean virtual temperature argument in the dry-air hypsometric equation. This version of Bigelow's correction is currently in use in the surface weather and climate networks of the Meteorological Services of Canada (MSC). In this paper we present results of an examination of the differences between Ferrel's and Bigelow's formulations of the Plateau Correction using Canada's national weather and climate network as a test case.

### 2. BIGELOW: THE PLATEAU CORRECTION IN USE BY THE METEOROLOGICAL SERVICES OF CANADA (MSC)

The dry-air form of the hypsometric equation used in MSL pressure reduction by MSC is the familiar,

$$P_{MSL} = P_s \exp\left(\frac{kH}{T_{mv}}\right) \tag{1}$$

where  $P_{MSL}$  is the reduced sea level pressure in hPa,  $P_s$  is the station pressure in hPa, k is the dry air constant and is assigned a value of 0.0341636 °K/gpm, H is the geopotential height in gpm, and  $T_{\rm mv}$  is the corrected mean (vertical) virtual temperature of the air column expressed in °K :

$$T_{mv} = T_{mv}^* + F(t_s)$$
 (2)

Here, *F* is Bigelow's correction for plateau effect and local lapse rate anomaly and is uniquely a function of the station temperature,  $t_s$ . Note also that *F* is expressed in units of temperature as it is applied as a correction to the mean virtual temperature of the fictitious air column. MSC represents *F* by a quadratic:

$$F(t_s) = At_s^2 + Bt_s + C \tag{3}$$

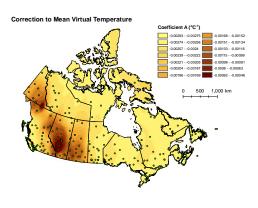
where the coefficients A, B and C are uniquely determined for each station according either to the methods originally identified by Bigelow or by the weighted averaging of the correction functions of nearby plateau stations.

<sup>\*</sup> *Corresponding author address*: Chris A. Hampel, Environment Canada, 4905 Dufferin Street, Toronto, ON, M3H 5T4; e-mail: <u>christopher.hampel@ec.gc.ca</u>

 $T_{mv}^{*}$  is the uncorrected vertical mean virtual temperature of the air column and is given by

$$T_{mv}^{*} = T_{0} + t_{s} + \frac{\Gamma H}{2} + e_{s}C_{h}$$
(4)

where  $T_0 = 273.15$  °K,  $t_s$  is the station 12 hour mean temperature in °C;  $\Gamma$  is the atmospheric lapse rate and has a value of 0.0065°C/gpm;  $e_s$  is the station vapor pressure and is a function of the station temperature;  $C_h$  is a function of elevation.



Correction to Mean Virtual Temperature

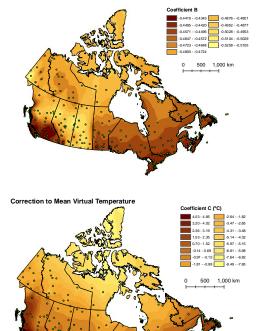


Figure 1: National Distribution of Plateau Coefficients, A, B and C, of the function,  $F(t_s)$ 

Intuitively, we can express Bigelow's correction,  $J_B$  in pressure coordinates as the difference in MSL pressures obtained with and without the correction, F applied to the mean virtual temperature argument, as follows:

$$J_{B} = P_{s} \left[ \exp \left( \frac{kH}{T_{mv}^{*} + F} \right) - \exp \left( \frac{kH}{T_{mv}^{*}} \right) \right]$$
(5)

## 3. FERREL: THE CORRECTION IDENTIFIED IN THE SMITHSONIAN TABLES

Ferrel's original formulation of the plateau correction is:

$$J_F = \sigma H \left( T_{mv}^* - T_{mvn}^* \right) \tag{6}$$

where  $\sigma$  is Ferrel's original plateau constant and is given by 0.000210 °K<sup>-1</sup> gpm<sup>-1</sup> and  $T_{mvn}^*$  is the annual mean of the vertical mean virtual temperature and can be estimated by substituting the station annual mean temperature for the station temperature argument in equation 4. The MSL pressure reduced using this form of the correction is:

$$P_{MSL}^{*} = P_{s} \exp\left(\frac{kH}{T_{mv}^{*}}\right) + \sigma H\left(T_{mv}^{*} - T_{mvn}^{*}\right)$$
(7)

The difference in MSL reduced pressure between Ferrel's and Bigelow's methods can be expressed in general as:

$$\Delta P = P_{MSL} - P_{MSL}^* \tag{8}$$

$$= \sigma H \left( T_{mv}^* - T_{mvn}^* \right) - P_s \left[ \exp \left( \frac{kH}{T_{mv}^* + F} \right) - \exp \left( \frac{kH}{T_{mv}^*} \right) \right]$$

$$=J_{F}-J_{B}=\Delta J \tag{9}$$

Here we see that the difference in the reduced pressures is simply the difference between the two formulations of the plateau correction.

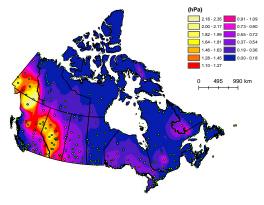
### 4. THE NATIONAL DISTRIBUTION OF DIFFERENCES: $\Delta J$

For a given station, we can use the values of the station elevation and the minimum and maximum station temperatures from climatology to calculate a realistic range of mean virtual temperatures over which to integrate  $\Delta J$ . The rms integral acts as a worst-case measure of the impact of differences that would arise between the two methods over a range of temperatures expected for a given station.

$$\varepsilon_{rms} = \sqrt{\frac{1}{\left(T_{mv,max}^{*} - T_{mv,min}^{*}\right)_{T_{mv,max}^{*}}} \Delta J^{2} dT_{mv}^{*}}$$
(10)

Results of the analysis for 350 Canadian stations are shown in figure 2 and indicate that significant differences occur in Alberta in the vicinity of the lee-trough of the Rocky Mountains and in the mountainous regions straddling the British Columbia-Yukon border.

**RMS Pressure Difference** 



**Figure 2:** National distribution of rms differences in MSL pressure between MSC's application of Bigelow's correction for the plateau effect which accounts for variations in local lapse-rate, and Ferrel's correction for the plateau effect which assumes a constant lapse rate.

### 5. DISCUSSION AND CONCLUSION

MSC does not use station climate data when calculating the coefficients A,B and C that define the plateau correction for new stations. We extrapolate coefficients for new stations using nearest neighbours or 'point of departure' stations<sup>6</sup>. The method used is a Cressman weighted average and, as such, represents an empirical estimation of the coefficients for a given station. When checked against known climate data for the station, the extrapolation may prove to be inaccurate.

We can readily test the accuracy of the extrapolation since it follows from Bigelow's analysis that above and below 305 gpm, the value of the correction should equal zero when the station temperature argument,  $t_s$  is equal to the annual mean. The roots of the quadratic representing F for a given station should thus yield the climatic mean if the extrapolation that generated the coefficients was accurate.

Results of analysis indicate significant departures between the roots of F and the actual annual mean temperature for Canadian stations.

If we correct for these departures by translating the function F horizontally along the temperature axis by an amount equal to this difference for each station, the average value of  $\varepsilon_{ms}$  for all stations is reduced from 0.32 hPa to 0.16 hPa, indicating that 50% of the observed differences are likely due to errors in the Cressman estimation of the plateau function, F, for each station. The remaining differences and their correlations with physical variables such as orography and elevation require further scrutiny and investigation. However, in the interest of simplifying the national practice of MSL pressure reduction, the above results quantify the worst case impact on MSL pressure fields that would result should MSC decide to adopt the methods of Ferrel in its application of the Plateau Correction.

#### References

1. Mesinger, F. and Treadon, R.E., 1995, "Horizontal" Reduction of Pressure to Sea Level: Comparison against the NMC's Shuell Method, Monthly Weather Review, **123**, p 59-68

2. Reichelderfer, F.W., et al, 1963, Manual of Barometry (WBAN), Appendix 7.2

3. Ferrel, W., 1886, "Report on Reduction of Barometric Pressure to Sea Level and Standard Gravity," Appendix 23 of Annual Report of the Chief Signal Officer, U.S. War Department, Washington, D.C.

*4. List, R.J.*, 1984, Smithsonian Meteorological Tables, Sixth Revised Edition,

5. *Bigelow, F.H.*, 1901 "Report on the Barometry of the United States, Canada, and the West Indies,"volume II, Report of the Chief of the Weather Bureau,

6. Savdie, I., 1982, AES Barometry Program, Network Planning and Standards Division, Environment Canada, TR9,

#### Acknowledgements

Thanks to Mark Torgerson for obtaining station climate data and to Bruce Persaud for his time and expertise in generating the maps.