**JP1.11 TEMPERATURE CORRECTION FOR THE GEONOR WEIGHING GAUGE**

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## 1. INTRODUCTION

In October of 2004, a poster paper by Lamb (2005) was written for the AMS Annual Meeting in 2005. In the summer of 2005, Surface Weather and Climate Networks purchased a Thermal Products Solutions (Lunaire/BlueM) ETCU-09 environmental chamber. Over the summers of 2005, 2007 and 2008, about 270 Geonor vibrating wire transducers were tested, independently of the gauge.

The transducers show consistancy of performance and averages of the results are presented here. Additionally, the temperature coefficient demonstarted in the tests is of opposite sign to that used in the paper cited above. This paper begins by correcting an error made in the earlier paper.

## 2. VIBRATING WIRE TRANSDUCER

Lamb (2005) contained two primary sections which discussed the science and engineering; “Vibrating Wire Transducer” and “Datalogger Measurement”. The corrections relate only to the vibrating wire. There are no corrections to “Datalogger Measurement”.

“Vibrating Wire Transducer” contains three sub-sections, ‘Fundamentals’, ‘Temperature Sensitivity’ and ‘Zero Drift’. The corrections relate only to the first two. There are no corrections to ‘Zero Drift’.

In Lamb (2005), there is an error in Equation 6 which affects Equation 12 which is the basis for the graphs of Figures 1, 5 and 6. Equation 6 should not have been solved for weight since it is not an actual weight that is being measured. Also, beginning with Eq. 6 and continuing through Eq. 12, when the text referred to an equation, the equation number was one less than it should have been.

The corrected first two sub-sections of “Vibrating Wire Transducer”, essential to this paper, are presented here.

### 2.1 *Fundamentals*

The transducer consists of a wire with magnetic properties under tension and two magnets placed nearby. One magnet “plucks” the wire to cause oscillation and the other magnet detects the vibration. Because the output is continuous, the two magnets must be part of an oscillating circuit providing positive feedback to the plucking magnet. For the oscillation to be the natural frequency of the wire, care is taken that the other components of the circuitry cause no forcing. This is the method presented in DiBiagio (2003) of the Norwegian Geotechnical Institute and is likely the method used in the Geonor.

The resonant frequencies of a wire under tension are

$$f^2 = n^2F/4L^2\mu, \quad n = 1, 2, - - - \qquad (1)$$

where L is length of the wire, μ is the mass/unit length and n = 1 for the fundamental resonant frequency.

For a weighing gauge, the parameter being measured is the increase in weight beyond a reference weight comprising the empty bucket and the mechanism for holding said bucket. From Bakkehoi (1985), the relationship between strain and frequency is

$$f^2 – f_0^2 = \varepsilon Eg/4L^2\rho C \qquad (2)$$

where $\varepsilon$ is strain, E is Young’s modulus of elasticity, g is gravity, $\rho$ is density of the wire and C is a constant of the device. In this case, $f_0$ is stated to be the zero strain frequency. However, that would also mean the force, F, and the frequency, f, are both zero. Equation (2) is therefore modified to be

$$f^2 – f_0^2 = (\varepsilon - \varepsilon_0)Eg/4L^2\rho C \qquad (3)$$

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Young's modulus is

$$E = \text{stress/strain} = \sigma/\varepsilon = (F/A)/(\Delta L/L) \quad (4)$$

where A is the cross sectional area of the wire and stress has units of pressure and strain is dimensionless.

Now using $\varepsilon E = F/A$ and $\rho A = \mu$,

$$f^2 - f_0^2 = (g/C)(F - F_0)/4L^2\mu \quad (5)$$

which is consistant with Equation (1) and where C has dimensions of acceleration.

By taking the difference in frequencies, the gauge is measuring the increase in force beyond the reference force. Then $F - F_0$ can be replaced by F = Wg, where W is the weight that has been added to the system. Equation (5) becomes

$$f^2 - f_0^2 = (g^2/4L^2\mu C)W \quad (6)$$

where the constant has dimensions of $(Kg*sec^2)^{-1}$ and C must have dimensions of $m/sec^2$.

Tunbridge (1988) state that the following two equations are in general use for the gauge

$$M_i = K(f^2 - f_0^2) \quad (7)$$

$$M_i = A(f - f_0) + B(f - f_0)^2 \quad (8)$$

Equation (8), which has become more commonly used, may be put into the form

$$M_i = B(f^2 - f_0^2)[1 + (A - 2Bf_0)/B(f + f_0)] \quad (9)$$

where $M_i$ is the measurand and $|(A - 2Bf_0)/B(f + f_0)| < 0.15$ for $f \geq f_0$. The constants, B and K, have dimensions of $cm*sec^2$.

### 2.2 *Temperature Sensitivity*

The wire will have a temperature sensitivity given by

$$L = L_a[1 + c(T - T_a)] \quad (10)$$

where c is the linear coefficient of expansion and $T_a$ is the ambient temperature. Geonor uses the same bright polished music wire as DiBiagio (2003), a steel alloy wire of propietary composition. For steel and iron, the coefficient of linear expansion is 0.000012/C°. The mass per unit length, μ, will change inversely as the change in length of the wire

$$\mu = \mu_a/[1 + c(T - T_a)] \quad (11)$$

Since $c(T - T_a) << 1$, $[1 + c(T - T_a)]^{-1} \cong [1 - c(T - T_a)]$ and Equation (6), expressed as precip, becomes

$$P = K(g^2/4L_a^2\mu_a C)[1 - c(T - T_a)]W \quad .$$

$$= K[1 - c(T - T_a)](f^2 - f_0^2) \quad (12)$$

The change in precip reading, $\Delta P = P - P_a$, over the range from -50°C to 50°C is shown in Figure 1. Note that the total change is 0.12% of the reading which is reasonably consistant with the magnitude of 0.001%FS(reading)/°C specified by Geonor but is of opposite slope. Also note that Geonor's specification is for the gauge, not just the transducer.

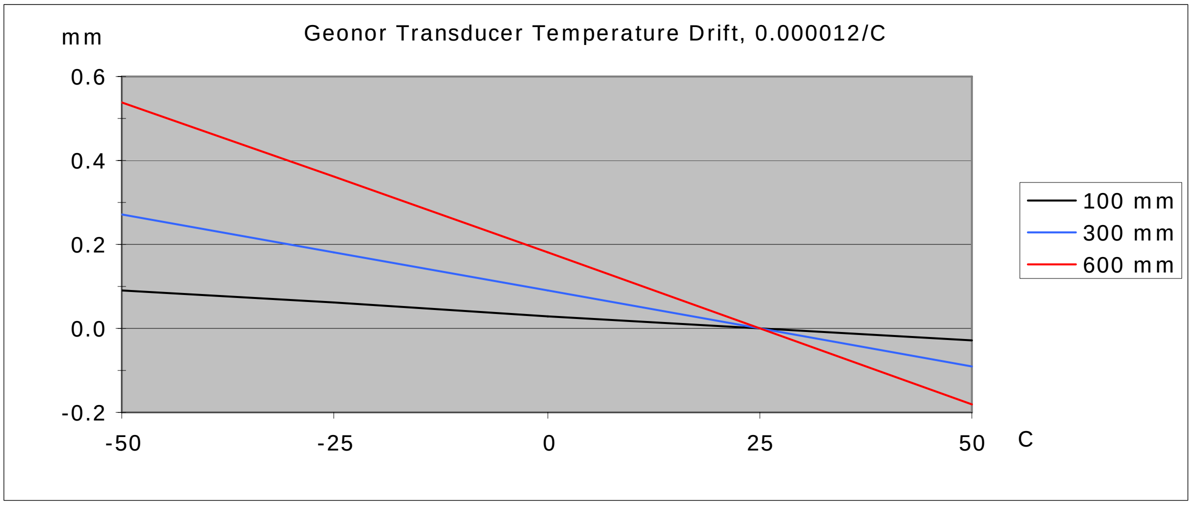


Figure 1

## 3. CHAMBER TESTS

### 3.1 *Transducers*

Seventeen sets of 16 transducers were aged in the environmental chamber.

A platform was constructed for the chamber from which 16 transducers could be hung with weights attached. The transducers were arranged in rows (sub-sets), 1 through 4 front to back, each row containing four transducers, A through D left to right. Brass weights, accurate to < 1%, were hung from the transducers; 0.5 kg on row 1, 1.0 kg on row 2, 2.0 kg on row 3 and 3.0 kg on row 4.

With a complete Geonor weighing gauge, the zero frequency, $f_0$, occurs at a zero weight of 1/3 of the empty bucket and supporting cradle. The bucket holds 12 L (12 kg) of water, representing 600 mm of precipitation or 50 mm/kg.

With the weights stated above, the expected depths would be 3 times 25, 50, 100, 150 mm. However the reported depths at 22°C are 15 (14.7), 90 (90.5), 240 (240.4) and 390 (390.5) mm. The zero weight is then 60 mm (1.2 kg) or 20 mm (0.4 kg)/transducer and the measured weights are then 3 times 0.1, 0.6, 1.6 and 2.6 kg.

### 3.2 *Chamber Program*

The chamber program begins at 22°C and goes through the following sequence: 22, -40, -30, -20, -10, 0, 10, 20, 30, 40, 50, 27 and 21 °C. The program ramps between temperatures at 1°C/minute except the last one which is at 0.2°C/minute. The program holds at each temperature for 1 hour except for the first one which is for 2 minutes and the last one where the program ends and shuts down the chamber.

### 3.3 *Procedure*

The data logger program, running in an idle state, is started by manually setting the start flag about 10 minutes before the chamber is started. A temperature sensor is placed in the chamber for the logger to measure. Minutely averages are obtained of frequency and temperature. Minutely precip levels are calculated and ten minute averages of precip and temperature are calculated and stored. At least one data point is collected before the chamber is started. The chamber, at a 23/24 °C room temperature, is then started.

The end flag is at zero. If end = 0 and the temperature < 23°C, the minutely averages of the measured frequencies of sub-set 1 are stored. When the chamber exceeds 45°C, the end flag is set to one. If end = 1 and the temperature < 23°, the program calculates a new $f_0$ and 'A' constant for the transducers in sub-set 1 if the cal flag is zero (default). Also, the end flag is set to two.

Beginning one minute later, if end = 2 and time is at a multiple of ten minutes in an hour, the start and end flags are set to zero and the logger goes to an idle state.

Note that the weight in sub-set 1 is close enough to the gauge zero weight to justify calculating the new constants. Also note that logger time and chamber time are not synchronized.

## 4. RESULTS

Two transducers in sub-sets 1 failed. The locking thumbscrews on two transducers in sub-sets 2 were not released and those units were retested. The locking thumbscrew on one transducer in sub-sets 3 was not released and that unit was set aside for retesting. One transducer in sub-sets 4 showed a sudden 7 mm shift and it was set aside for retesting.

The following results are based on 66 units each in sub-sets 1 & 2 and 67 units each in sub-sets 3 & 4 for a total of 266 units. The results of the aging runs are shown in Figures 2 – 5, each figure being at a given weight and each curve being the average of the four transducers in a sub-set.

For those transducers in sub-sets 1 for which a new $f_0$ and 'A' constant was calculated, not using the new values would result in errors ranging from -0.030 to 0.039 mm for gauges at full capacity. As a result, the new values were not issued to field personnel.

In Figure 6, all 17 sub-sets of each weight are averaged together and referenced to the 22°C value. Because an actual weight is being measured, the error contributed by the zero weight would have been included. This was corrected in Figure 6 by multiplying by ratio of the measured weight to the total weight.

Note that the values in Figure 6, resulting from measurement, are roughly times greater than the values in Figure 1, resulting from theory.

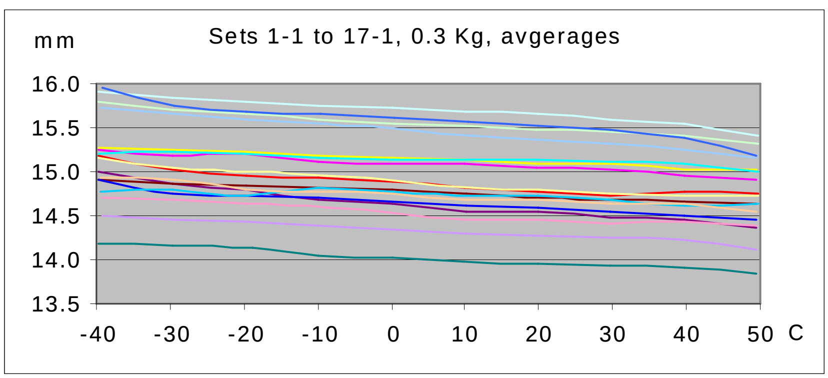


Figure 2

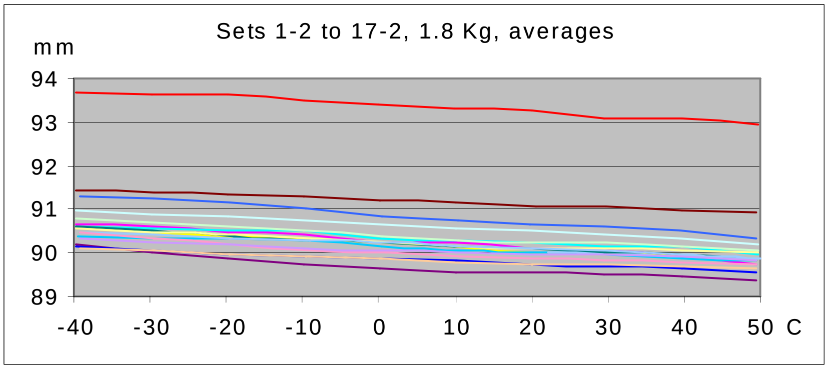


Figure 3

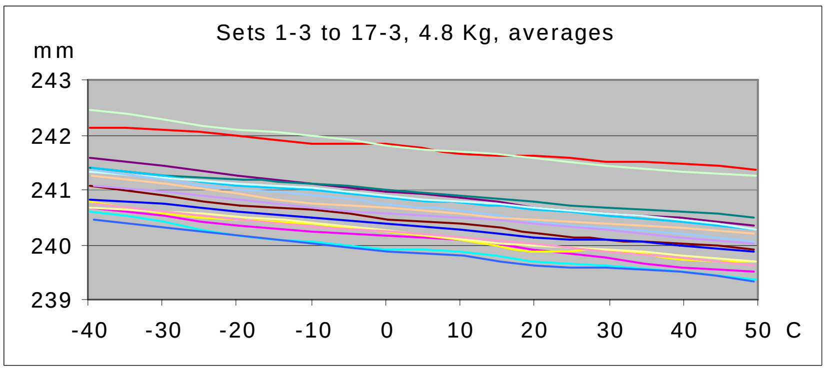


Figure 4

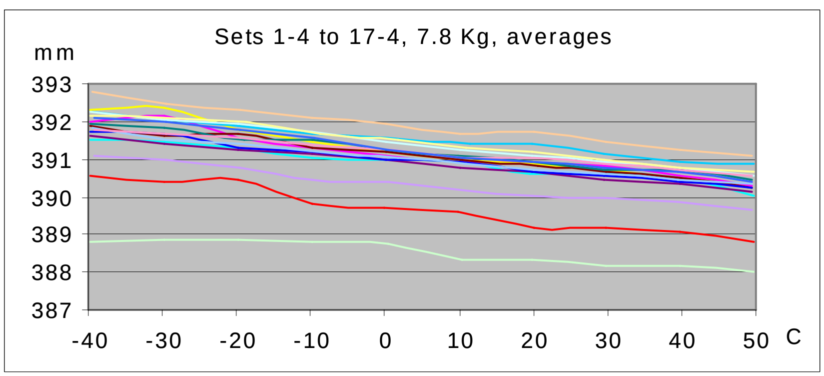


Figure 5

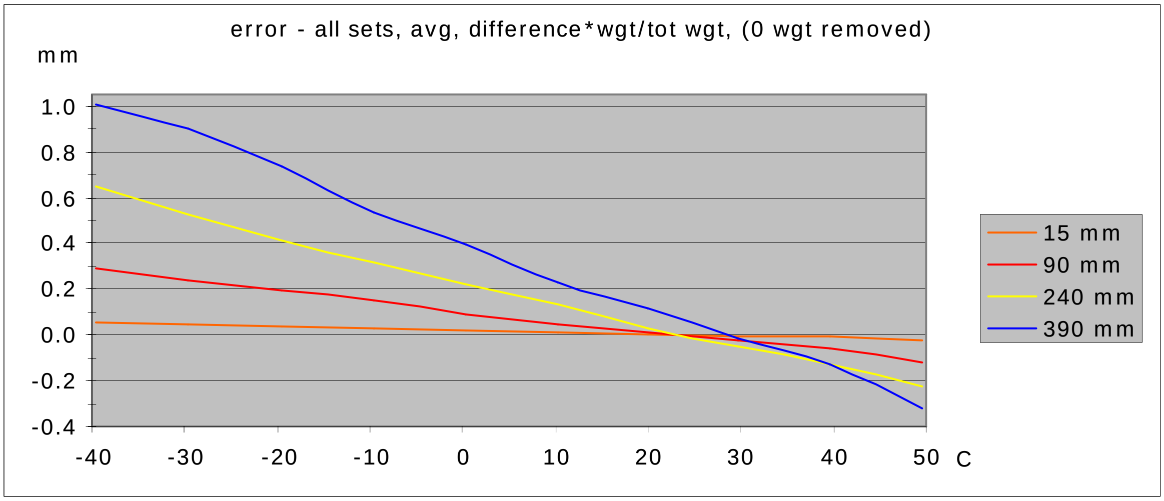


Figure 6

## 5. RATIONALIZING THEORY and MEASUREMENT

It was pointed out by Lamb (2005) that the composition of the wire in the Geonor transducer was propietary. Geonor declined to provide any information on the wire. The discrepency between theory and measurement is in two parts: the algebraic sign and the magnitude of the temperature coefficient of expansion.

### 5.1 *Magnitude of Coefficient of Expansion*

To rationalize the magnitude, the authors devised a basic measurement procedure to determine the the coefficient of expansion with tools and methods readily available to Surface Weather and Climate Networks.

A single transducer was used. The outer cover, magnets and the locking thumbscrew were removed but the wire remained intact within the body of the transducer. A digital caliper with a resolution of 0.01 mm or 0.0005 inch was used to measure the length of the wire.

One end of the wire is attached to the upper body of the transducer and the other end is attached to a solid metal cylinder 40 mm long which is free to slide through a hole 20 mm long in the bottom of the transducer. The caliper measured the distance between the upper body and the cylinder. There is enough play between the cylinder and the hole to cause successive measurements to vary considerably. A thin shim was made to insert in the hole to reduce the play.

The unit under test was maintained at 0° C or 100° C with freezing or boiling water and three tests were performed. The nominal length of the wire at 22° C is 51.75 mm and the results of the tests are shown in Table 1.

For Test 1, numerous measurements were taken and an approximate center value was used and there was no shim in place. For Test 2, a shim was in place but it covered too much of the circumference of the cylinder which caused binding and these measurements were not used. For Test 3, the shim covered about 40% of the circumference which produced good results.

Notice that the resolution for these tests is about 1 in 5100 and that small errors in measurement will create large errors in the calculation of the temperature coefficient. The purpose here was to obtain a temperature coefficient of sufficient validity to justify the claim that it is the reason for the difference between the theory and the chamber tests. It is left to others to obtain a more accurate measure of the temperature coefficient of the wire.

| | length @ 0°C | length @ 100°C | coefficient, c |
|---|---|---|---|
| Test 1 | 51.59 mm | 51.84 mm | (0.25/100)/51.75 = 0.000048 |
| Test 2 | 51.27 mm | 51.28 mm | see discussion |
| Test 3 | 51.72 mm | 51.86 mm | (0.14/100)/51.75 = 0.000027 |

Table 1

Figure 7 is the graph of Figure 1 with the precip levels set to 90, 240 and 390 mm, the temperature scale set to -40ºC to 50ºC, reference temperature set to 22ºC and the temperature coefficient set to 0.000044/ºC, which is in the range defined by Test 1 and Test 3 in Table 1. This coefficient gives good agreement Figure 6.

### 5.2 *Sign of Coefficient of Expansion*

The Geonor manual presents a temperature coefficient for the gauge with a single transducer, but not for the transducer itself. The other two supports consist of chain which would likely have a temperature coefficient close to 0.000012/°C as Lamb (2005) had assumed for the transducer wire. Then, as the wire changes in length faster than the chains, there will be a very small change in the level of the bucket, changing the temperature coefficient of the gauge relative to the transducer. As shown in the section on Gauge Level Correction, if the weight in the bucket is liquid the temperature coefficient of the gauge will be positive.

If three transducers are used, the expected temperature coefficient of the gauge would be the same as for the transducer.

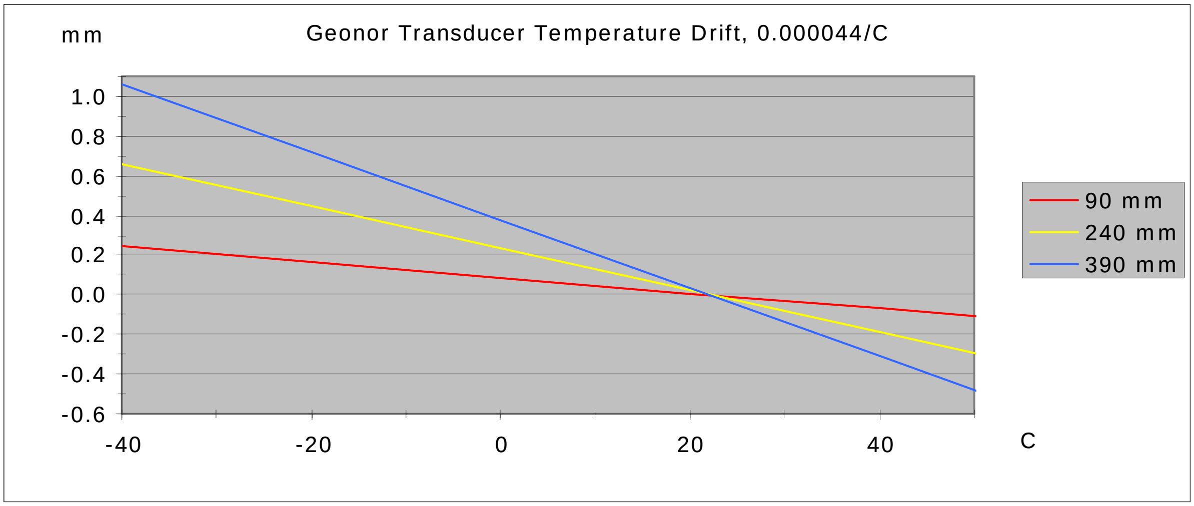


Figure 7

## 6. TEMPERATURE CORRECTION

To develop a temperature correction equation, the error due to zero weight must also be included. Figure 8 shows the total error due to temperature shift.

It can be seen with a close look at Figures 6, 7 and 8 that the measured temperature drift varies somewhat with tension on the wire whereas in the theory, it does not. While using a constant drift rate will reduce the error curves of Figure 8, to obtain the maximum reduction in the error of Figure 8, it is necessary to use the an error curve that matches the drift rate curve.

Figure 9 shows the error component due to the zero weight. Linear trend lines are shown for 15 mm and 90 mm which show an average drift rate somewhat less than that obtained by using the end points of the curves. This is more true of 15 mm than for 90 mm. For 240 mm and 390 mm, the trend lines become unnecessary.

The drift rates for Figures 6, 8 and 9 are shown in Table 2.

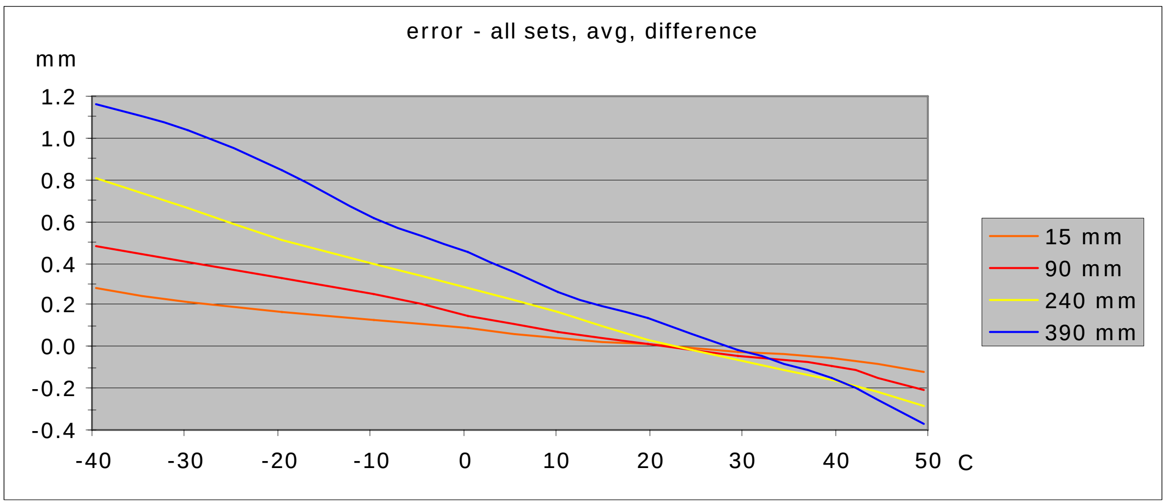


Figure 8

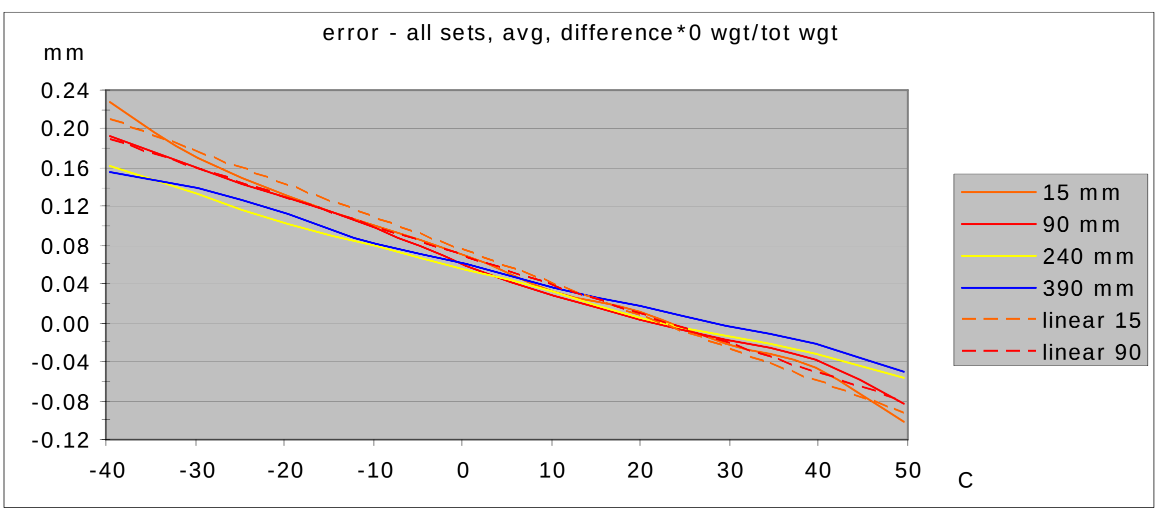


Figure 9

| | 15 | 90 | 240 | 390 | 540(est) | mm |
|---|---|---|---|---|---|---|
| Fig.6 | -0.0000609 | -0.0000510 | -0.0000404 | -0.0000379 | | mm/°C |
| Fig.9 | -0.0000556 | -0.0000505 | -0.0000404 | -0.0000379 | | mm/°C |
| Fig.8 | -0.0000566 | -0.0000508 | -0.0000404 | -0.0000379 | -0.0000370 | mm/ºC |

Table 2

The scale of Figures 6 and 8 is such that trend lines would give the same results as the curves. All the Figures would have the same drift rate if the trend lines in Figure 9 were not used. As it is, the drift rates in Figure 8 are a weighted average of the drift rates of Figures 6 and 9.

Figure 10 shows the variation of drift rate for the total error (Figure 8) and also an approximation to the drift rate curve. The approximation is determined by Equation (13) where P is the measured weight.

$$d\_rate(P) = -.000058+.000022(1-e^{-P/180}) \quad (13)$$

A correction function can now be presented as given in Equation (14).

$$corr(T,P) = (T - T_a)(P + 60)d\_rate \quad (14)$$

Subtracting corr(T,P) from the error curves of Figure 8 gives the resultant error shown in Figure 11.

The data logger will output temperature corrected values every 15 minutes for each of the three measurements. However, the correction calculation is performed minute by minute. A less satisfactory temperature correction could be performed off logger on the 15 minute data.

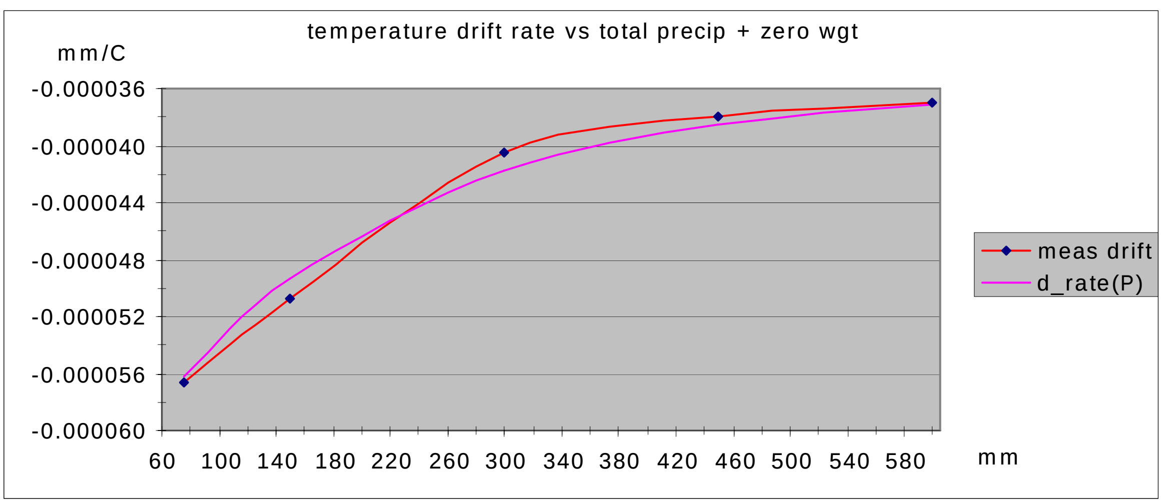


Figure 10

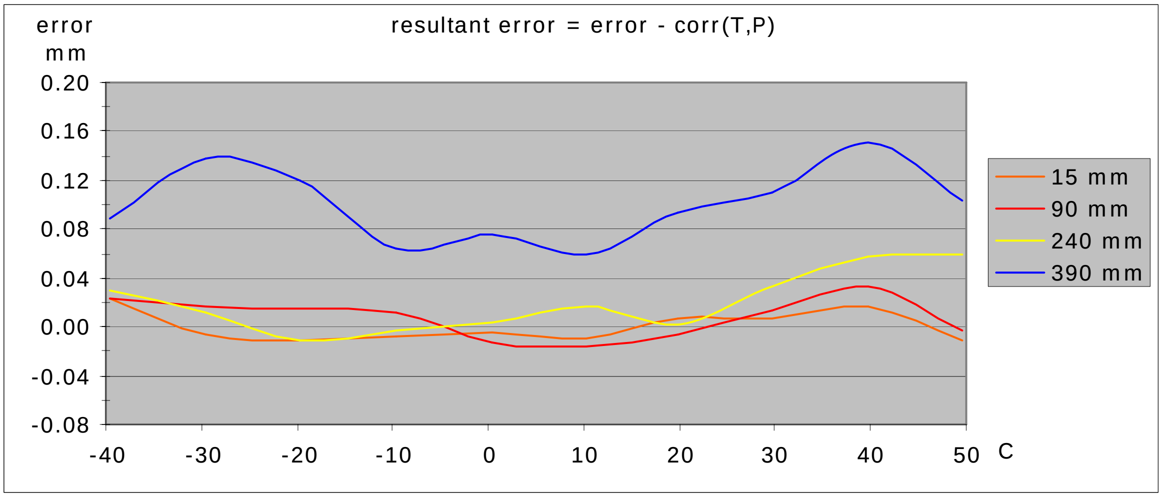


Figure 11

## 7. GAUGE LEVEL CORRECTION

An off logger algorithm will apply a voting scheme to determine if all three measurements can be used and then provide a single averaged official value. It may occur that only two of the measurements will be used to determine the official value.

If the gauge becomes off level, the measurement will increase for one transducer, decrease for one transducer and may increase or decrease for the third transducer. A simple qualitative test shows that when the bucket swings toward a transducer, its measurement will increase if the weight is liquid and will decrease if the weight is solid. When weight moves away from a transducer, that wire would normally see an increase in tension but if the weight is liquid it will shift downhill decreasing the weight on the uphill transducer and increasing the weight on the downhill transducer. The two opposing effects make the change in measurement much weaker for liquid than for solid.

When three transducers are used, the level correction will occur in the off logger calculation of the single averaged official value. If only two transducers are used, the level correction may or may not function properly, depending on the direction the gauge goes off level. The level correction will also function with changes in temperature since the temperature correction produces a small reasonably constant resultant error curve.

A more complete discussion of the level correction can be included in a discussion of the off logger algorithm for triple transducers.

## 8. CONCLUSION

An error in the paper by Lamb (2005) was corrected and the theory and practice of the vibrating wire sensor rationalized. For gauges with triple transducers, a useful temperature correction equation was developed, producing a resultant error curve which is reasonably flat over temperature. This then permits the off logger triple transducer algorithm to produce a single official value which is also level corrected.

## 9. REFERENCES

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