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MULTIPLE EQUILIBRIA IN A CUMULUS ENSEMBLE MODEL EMPLOYING THE WEAK TEMPERATURE GRADIENT APPROXIMATION

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1. INTRODUCTION

In the tropical atmosphere, there exist regions devoid of active convection even in conditions which may support vigorous convection. Generally, global circulation models have difficulty in accurately modeling such regions. Understanding this phenomena using small scale numerical models may provide physical insight for improving parameterizations in large-scale models.

In recent work, Sobel et. al. (2007) demonstrated that the weak temperature gradient (WTG) approximation employed in a single column model of the atmosphere may result in multiple stable equilibria. The stable equilibria correspond to either a state with persistent deep convection or one that is extremely dry. In both cases, boundary conditions support deep convection, but the initial moisture profile dictates which equilibrium state is ultimately realized. We explore this phenomenon using a cloud resolving model (CRM), also in the context of the WTG approximation.

In the following sections, we briefly review the WTG approximation and the gross moist stability (GMS). The GMS is a diagnostic that proves useful in interpreting our numerical results, which are presented in section 4.

2. WEAK TEMPERATURE GRADIENT APPROXIMATION

The WTG approximation is a parameterization of large-scale dynamics that can be used in single-column or limited-area models (e.g., Sobel and Bretherton 2000). The WTG approximation is based on the observation that in the tropics, the vertical profile of potential temperature is nearly homogeneous in the horizontal. Physically, surface heat fluxes and radiation may locally perturb the profile of potential temperature, resulting in buoyancy anomalies. Gravity waves then redistribute these anomalies throughout the tropics, thus approximately maintaining horizontal homogeneity. In the model, the vertical profile of potential temperature for the large scale environment is specified, and convection evolves so as to maintain this profile within the modeled domain. This is accomplished by generating a hypothetical vertical velocity which is just sufficient to counteract the buoyancy anomalies. This WTG vertical velocity significantly impacts the evolution of convection (Raymond and Zeng, 2005).

In our implementation of the WTG approximation, we specify a time scale for locally perturbed potential temperature profiles to relax to the large scale environmental profile. The environmental profile is taken to be the radiative convective equilibrium (RCE) profile for a 2D domain with an SST of 303 K and a mean horizontal wind speed of 5 m/s. The wind speed and SST essentially govern the surface fluxes in the model. The relaxation time scale corresponds to the time it takes gravity waves to cross the modeled domain. An infinite relaxation time is equivalent to turning off WTG mode, while an instantaneous relaxation time ($t_{relax} = 0$) amounts to a strict enforcement of the WTG approximation (corresponding to infinite gravity wave speed). As we will show below, the existence of the dry equilibrium is dependent on this relaxation time scale; the model is more likely to remain dry if the relaxation time is small. In other words, the closer the atmosphere is to obeying the WTG approximation, the more likely it is to exhibit multiple equilibria.

3. NORMALIZED GROSS MOIST STABILITY

The gross moist stability can be thought of as a measure of precipitation efficiency. It was first introduced in 1987 by Neelin and Held as a means to model tropical convergence based on the moist static energy budget (which is approximately conserved in moist processes). In this work, we choose to define the normalized gross moist stability (NGMS) in terms of moist entropy (which is also approximately conserved in moist convective pro-

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cesses). Specifically, we define Γ to be the NGMS:

$$\Gamma = \frac{T_R[\nabla \cdot (s\mathbf{v})]}{-L[\nabla \cdot (r\mathbf{v})]} \quad , \tag{1}$$

where s is the moist entropy, r is the total cloud water mixing ratio, $\mathbf{v}$ is the horizontal wind. The square brackets indicate a vertical pressure integral and ∇ is the horizontal divergence operator. T_R and L are a constant reference temperature and the latent heat of condensation, and are included so that Γ is dimensionless. We thus define NGMS as the ratio of moist entropy export to moisture import. In a majority of cases, NGMS is a positive quantity, but can become negative if moist entropy and moisture are both imported (or exported) into a convecting region. Observations suggest that the sign of NGMS is related to the shape of the profile of vertical motion (Back and Bretherton 2006; López and Raymond 2005). A thorough discussion on the role of NGMS in tropical dynamics can be found in Raymond et. al. (2009).

4. RESULTS

We used a 2-dimensional cloud-resolving model similar to that of Raymond (2000). The model was run in non-WTG mode to RCE as described in section 2. The domain-averaged vertical profiles of potential temperature and total cloud water mixing ratio resulting from these runs were then used to represent the mean state of the large scale atmosphere used in the WTG runs.

The WTG experiments varied horizontal domain size (50, 100, or 200 km), surface wind speed (v_y, 0-20 m/s), and the time for the vertical potential temperature profile to relax to that of the surrounding environment (t_{relax}, 0.01-27 hours, though not all of these values are realistic according to the gravity wave interpretation of this time scale). In this series of experiments, the initial moisture in the model domain was either set to that of the surrounding environment or to zero, corresponding to a completely dry domain. Most experiments were run for 4 months, and steady state values were taken as the average over the last 30 days.

In all domain sizes, we found a range of horizontal wind speeds and relaxation times which sustained multiple equilibria. Figure 1 shows the precipitation rate dependence on wind speed for a 50 km domain with a relaxation time of 1.85 hours. For these parameters, the model exhibits multiple equilibrium for horizontal wind speeds in the range 3-12 m/s. Below 3 m/s, only the dry equilibrium exists. Here, surface fluxes are insufficient to balance radiative cooling during the time of the simulation and the troposphere remains dry. For wind speeds greater than 12 m/s, surface fluxes overcome convective inhibition, which allows shallow convection to moisten the free troposphere and effectively destroy the dry equilibrium. The range of wind speeds permitting both equilibria is significant.

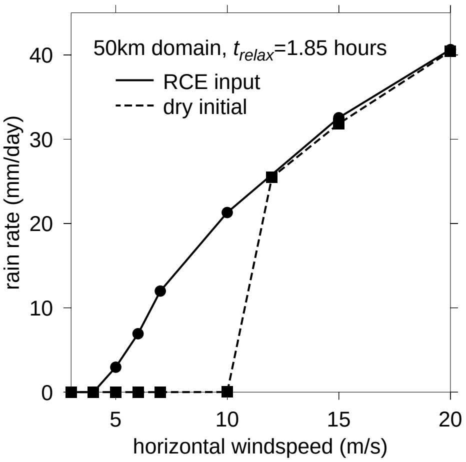

Figure 1: Precipitation as a function of surface wind speed. Solid line represents runs initialized with RCE moisture profile, dotted line shows experiments initially dry. Multiple equilibria exist for wind speeds between 3 and 12 m/s.

The existence of multiple equilibria also depends on the size of the modeled domain as well as on the WTG relaxation time. This dependence is shown in figure 2. All symbols in figure 2 represent numerical experiments in initially dry conditions with a horizontal wind speed of 10 m/s. Runs which eventually produced precipitation are shown with solid blue circles, while those which remained dry are shown by open red squares. For given surface flux conditions (as determined by wind speed and SST), we find that the likelihood of having multiple equilibrium decreases with increasing domain size and relaxation time. This tendency can be rationalized if we consider the probability per unit area per unit time that the system will initiate convection. The probability that convection happens somewhere will increase as the area or time increases. Thus, it is less likely to maintain a dry equilibrium for large domain sizes, or for longer relaxation times. This is the reason that the WTG approximation is essential for the existence of multiple equilibria in models: larger relaxation times are weaker realizations of WTG. Stronger enforcement of the WTG approx-

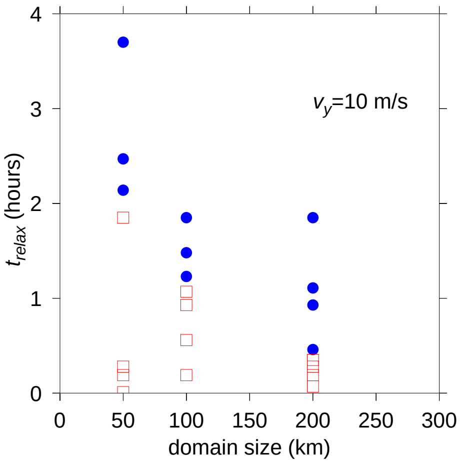

Figure 2: Dependence of multiple equilibrium on relaxation time scale and domain size. All experiments shown were initiated with zero moisture and wind speeds of 10 m/s. Blue circles correspond to simulations which eventually reached the precipitating equilibrium; experiments which remained dry are represented by red squares.

imation corresponds to smaller values of t_{relax}, and an increased likelihood for maintaining the dry equilibrium.

Sobel et. al. (2007) suggested that the persistence of the dry equilibrium in the presence of positive CAPE requires that the free troposphere be able to remain completely dry, even over a moist boundary layer. Though not shown, all experiments which remained dry observed this condition.

The dry equilibrium state can exist even in conditions which are favorable to vigorous convection. Thus, in order to better understand the existence of multiple equilibria, it is important to characterize the corresponding environment. The NGMS provides a useful prognostic tool for this purpose (Raymond et. al. 2009). Of particular interest is the possibility of negative values of NGMS, which can occur either in a transient state where convection is developing, or in the steady state of a non-precipitating environment.

Observations suggest that the shape of the profile of vertical motion dictates the sign of NGMS (Back and Bretherton, 2006; López and Raymond, 2005). Bottom heavy convective profiles, as occur in developing systems, tend to import both moist entropy and moisture, resulting in a negative NGMS. An example of this in our simulations is shown in figure 3. In this experiment, a 200 km domain with a surface wind speed of 10 m/s and WTG relaxation time of 0.93 hours was initialized with a dry atmosphere. The conditions were insufficient to maintain the dry state, and after about 23 days, the system rapidly transitioned to the precipitating equilibrium state. Prior to this transition, the system imports moist entropy and exports moisture ($\Gamma > 0$). Near the beginning of the transition, the system begins to import moisture, which increases the saturation fraction (defined as the precipitable water divided by the saturated precipitable water). Since the moisture convergence goes from negative to positive, and is in the denominator of NGMS, the NGMS goes from very large and positive to large and negative. As the saturation fraction reaches a maximum value, the system begins to export moist entropy while importing moisture, resulting in a positive NGMS in the steady state.

In the steady state, NGMS can only be negative in the dry equilibrium case. In regions of descent, convection is suppressed and both moisture and entropy are exported. Figure 4 shows the time evolution of NGMS, moisture convergence, and entropy divergence for a numerical experiment which remained dry. In this example, we modeled a 200 km domain with 10 m/s surface winds and a relaxation time of 0.19 hours.

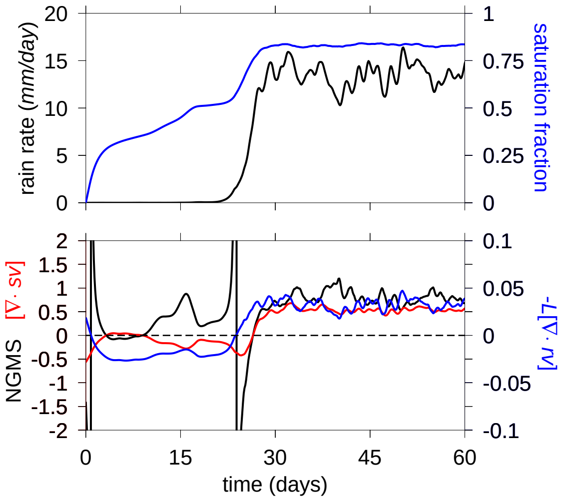

Figure 3: Time evolution of (top) rain rate (black), saturation fraction (blue); (bottom) NGMS (black), moist entropy divergence (red) and moisture convergence (blue) for initially dry 200 km domain. As convection develops, system evolves toward the precipitating equilibrium. For a period of about 3 days, the domain is importing both moist entropy and moisture, resulting in a negative NGMS.

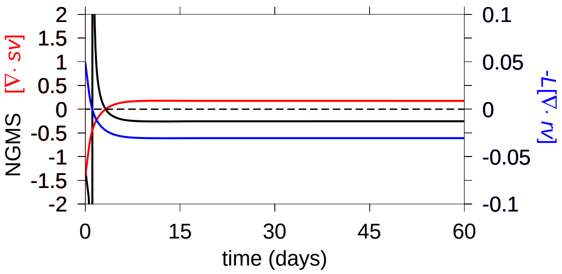

Figure 4: Time evolution of NGMS for an experiment which remained dry under conditions which would support vigorous convection. In this case, domain is exporting both moist entropy (red) and moisture (blue), resulting in a negative NGMS (black).

5. CONCLUSIONS

We present results from a series of numerical experiments using a 2-dimensional cloud resolving model in the context of the weak temperature gradient approximation. Our results compliment the discovery of multiple equilibria in a single column model also employing the WTG approximation (Sobel et. al. 2007). In addition to the initial moisture profile of the modeled domain, we found that the existence of multiple equilibria depends on domain size, the strength of surface fluxes (modulated by surface wind speeds in these experiments), and the time scale for the vertical profile of potential temperature to relax to the large scale mean. This time scale is related to the enforcement of the WTG approximation, indicating that WTG is essential for the existence of multiple equilibria.

The NGMS was introduced as a quantity useful for characterizing the environment which controls precipitation. In particular, negative values of NGMS, which were previous thought to be nonphysical, can manifest either during the evolution of convection from an unstable dry state to a stable convecting one, or in a stable dry state.

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