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POSSIBLE STRETCHING MECHANISMS PRODUCING THE TORNADO VORTEX IN THE MID-LEVEL

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1. INTRODUCTION

The estimations of maximum wind speed of tornado are discussed on some works, e.g., Lilly (1969), Dergarabedian and Fendell (1970), and Fiedler and Rotunno (1986). They tried to estimate it using the energy conservation law, but resulted in rather small values. Nowadays, some parameters (e.g., EHI, VGP, STPC, etc.) are used for tornado forecasting (Rasmussen and Blanchard, 1998; Thompson et al., 2003). In these parameters, baseline climatologies are important as pointed out by Rasmussen and Blanchard (1998). These works suggest that the parameters exhibit relatively good correlation with tornado intensity. We believe that we could estimate well-calibrated tangential velocity by more physical considerations.

For this purpose, we developed a new tornado forecast parameter named as tornado velocity parameter (TVP). The purpose of TVP is to estimate a maximum tangential velocity that can be generated under a specified atmospheric condition. In addition, we can estimate the tornado diameter, rotation period, ground wind speed, and damaged width.

For the estimation of TVP, the stretching of tornadic vortex in the mid-level should be calculated using a 1D vertical convection model. The purpose of this research is, 1) to verify the model by comparing with some published materials, and 2) to clarify some features on the tornadic vortex.

In section 2 and 3, the principle and an example of TVP calculation will be presented. Some features of tornadic vortex in the mid-level will be derived in section 4. Some case studies are performed in section 5.

2. PHYSICAL ASPECTS

According to some observations, a tornadic vortex can approximately be thought as a rigid body rotation (e.g., Wurman and Gill, 2000). So, hereafter we think it as a rigid body rotation. A collaboration of the vortex stretching and the law of angular momentum conservation is thought as a mechanism of acceleration of wind speed larger than 100ms^{-1}. Adequate energy to stretch the vortex is necessary. Previous works considered the main source of energy as the convective available potential energy (CAPE), and estimated the maximum tangential velocity of tornado. For example, Fiedler and Rotunno (1986) give 62ms^{-1} when CAPE=3800Jkg^{-1}. This is approximately a half of observed maximum velocity.

In order to gain insight of the acceleration due to stretching, we shall consider a simple mechanical system that consists of a weight and a string penetrating the weight as shown in Fig.1. We shall hold the ends of the string by both hands and give some angular velocity. Pulling the ends of the string up and down will gain more angular velocity. The rotation of the weight makes stress in the string due to centrifugal force. Both hands give some work against this stress to the weight through the string. This work is the source of energy for acceleration. We notice that the force for acceleration acts to the weight not in the direction of its movement, but in the direction of radius of the rotation against the centrifugal force. This is the principle of acceleration due to stretching.

In the case of tornadic vortex, it is crucial how long the low-level mesocyclone as an initial vortex is stretched. In addition to CAPE, the vertical wind shear is an important source of energy for stretching. The vertical wind shear horizontally pulls and transforms the vortex against the stress inside the vortex, thus gives some work to the vortex, resulting in acceleration of rotation. The energy conservation law can be written approximately in the next form,

$$\frac{1}{2}m\overline{v_{LLM}^2} + m(CAPE + CIN) + MS = \frac{1}{2}m\overline{v_t^2} + \frac{1}{2}m\overline{w^2}, \quad (1)$$

where m is the mass of rigid vortex, v_{LLM} is the rotation velocity of the initial vortex, CIN is the covective inhibition, MS is the work given by the vertical wind shear in the mid-level, and v_t and w are the rotation and vertical velocity of the tornadic vortex, respectively. MS is too large to neglect. Moreover, the estimation of MS is difficult. Therefore we cannot estimate v_t from the equation (1). Alternately, employing the law of angular momentum and mass conservation enables

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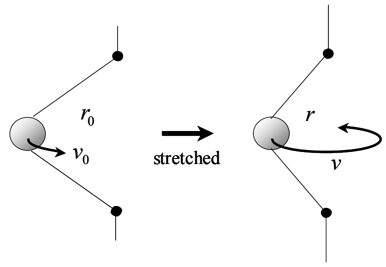

Fig. 1 principle of acceleration by stretching.

us to estimate v_t without quantification of *MS*. In this method, the length of stretching should be estimated. This point is the difference between the work of Fiedler and Rotunno (1986) and ours.

The diameter and the rotation velocity of the vortex should be known for application of the law of angular momentum conservation. In addition, these two physical quantities should be related to some environmental parameters. For further consideration, the initial vortex should be discussed. The tilting of the horizontal vortex is considered as a source of tornadic vortex. Assuming a 10ms^{-1}-tangential-velocity vortex as an initial vortex of a 100m-diameter and 50ms^{-1}-tangential-velocity tornado, its diameter will be 500m from the law of angular momentum conservation. In order to reduce the vortex diameter to 100m by stretching, the vortex should be stretched 25 times in its length. Therefore, when the vortex is vertically stretched to the height of 12.5km, the height of the initial vortex should also be 500m. Davies (2004) reported that the level of free convection (LFC) is low at tornado occurrence. Considering that the horizontal vortex below LFC is a main source of tornadic vortex, the diameter of the initial vortex should be approximately equal to the height of LFC (z_{LFC}), and the rotation velocity should be approximately equal to a half of the integral of the vertical wind shear from the ground to the height of LFC. We can imagine that tornado is difficult to occur in the case of high LFC because the length of stretching should be short and the diameter of the initial vortex is too large. The idea is summarized in Fig.2. The genesis mechanism of the initial vertical vortex is in discussion even today (e.g., Markowski et al., 2003). However, our consideration does not depend on the genesis mechanism if the angular momentum is conserved through the genesis. Simultaneous observation of the diameter and the rotation velocity of initial vortex and comparison with LFC are not conducted. In section 3, we assume that the initial diameter is equal to the height of LFC, and the initial tangential velocity is a half of the difference between the velocity at LFC and that at ground level. We verify this assumption by comparing the results with many results from damage surveys.

3. DEFINITIONS AND SOME EXAMPLES OF TORNADO PARAMETERS

We apply conservation laws for the initial vortex with diameter z_{LFC} and the height z_{LFC}, and the final tornadic vortex. This does not mean that the initial vertical vortex is cut at the height of z_{LFC}. The height actually stretched is important, but the condition of the upper end of the vortex is not. The laws of mass conservation and angular momentum conservation can be written in the next form,

$$\rho_{LNB}\pi R^2 L_{LNB} = \rho_{LFC}\pi r_0^2 z_{LFC} \equiv M \, , \tag{2}$$

$$RMV = r_0 M v_0 \, , \tag{3}$$

where ρ_{LNB} (ρ_{LFC}) is the mean air density from the ground to LNB (LFC), R (r_0) is the radius of the tornadic vortex (the initial vertical vortex), L_{LNB} (z_{LFC}) is the length of the tornadic vortex (the initial vertical vortex), and V (v_0) is the tangential velocity of the tornadic vortex (the initial vertical vortex). We refer to the parameter η obtained from the equation (2) as a stretching factor.

$$\eta \equiv \frac{r_0}{R} = \left(\frac{\rho_{LNB} L_{LNB}}{\rho_{LFC} z_{LFC}} \right)^{1/2} \tag{4}$$

The tangential velocity of the initial vertical vortex is equal to a half of the integral of $\partial v / \partial z$ between the ground and LFC.

$$v_0 = \frac{1}{2} \int_0^{z_{LFC}} \frac{\partial v(z)}{\partial z} dz = \frac{1}{2} \left(v(z_{LFC}) - v(0) \right) \equiv \frac{\Delta V}{2} \tag{5}$$

The tornado velocity parameter (TVP) is defined as the next equation.

$$TVP \equiv V = \frac{r_0}{R} v_0 = \eta \frac{\Delta V}{2} \tag{6}$$

The diameter of the initial vertical vortex is assumed to be z_{LFC}. Then, the diameter D, the angular velocity Ω, and the rotation period T of the tornado vortex can also be obtained.

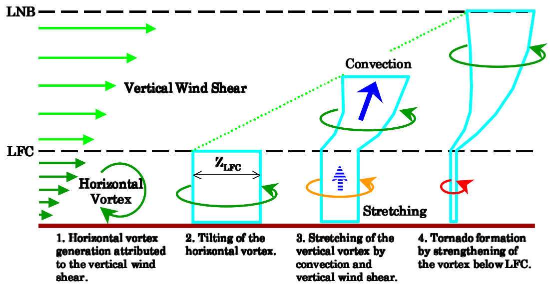

Fig.2 Physical process assumed for TVP calculation.

$$D = 2R = \frac{2r_0}{\eta} = \frac{z_{LFC}}{\eta} \quad (7)$$

$$\Omega = \frac{V}{R} = \frac{2\eta}{z_{LFC}} \cdot \eta \frac{\Delta V}{2} = \eta^2 \frac{\Delta V}{z_{LFC}} \quad (8)$$

$$T = \frac{2\pi}{\Omega} = \frac{2\pi z_{LFC}}{\eta^2 \Delta V} \quad (9)$$

The tornadic vortex is stretched by the vertical wind shear in the mid-level as described in section 2. Assuming that the vortex is simply advected by the environmental wind, L_{LNB} can be calculated by the next equation.

$$L_{LNB} = z_{LFC} + \int_{z_{LFC}}^{z_{EL}} \left[1+\left\{\left(\frac{\partial V}{\partial z}\right)\int_{z}^{z_{EL}}\left(\frac{d\zeta}{w}\right)\right\}^2\right]^{1/2} dz \quad (10)$$

$$w(z) \equiv \left(2g\int_{z_{LFC}}^{z} \frac{T_{vp}-T_v(z')}{T_v(z')}dz'\right)^{1/2} \quad (11)$$

Fig.3 depicts TVP and related parameter fields (TVP, TVP+surface wind, TVP+LFC wind, diameter, rotation period, and LCL) for 17 September 2006 tornado outbreaks in Kyushu, Japan. F-scales are well reproduced as wind speed represented by TVP+LFC wind, where LFC wind is assumed as the velocity of tornado movement. A statistical survey suggests that F-scales were reproduced for 76% of all tornadoes in 2001-2007 in Japan. TVP does not need any baseline climatology as in the cases of EHI and STPC, and produces better estimate of tornado intensity. Recently, additional tests are performed by some researchers in Japan. Almost the same results are obtained.

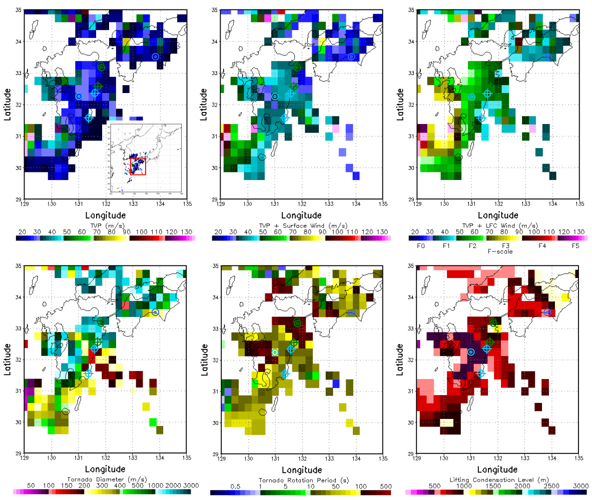

Fig.3 TVP and related parameter fields for 17 September 2006 tornado outbreaks in Kyushu, Japan.

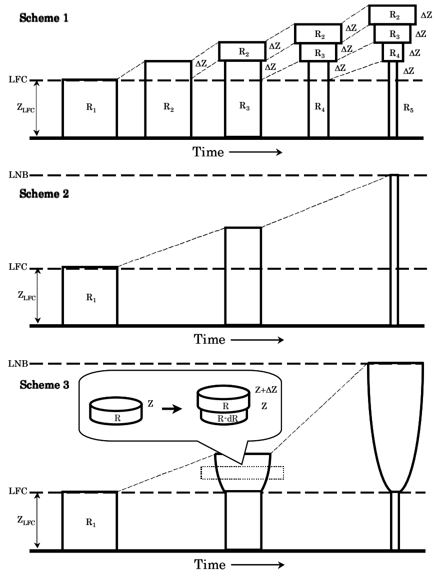

Fig.4 Three typical schemes of vortex stretching.

4. VORTEX STRETCHING

In section 3, we derived equations (2) and (3) regarding the vortex shape after stretching as cylindrical. This is acceptable if the diameter at LFC is the same as that described by equation (7). Here we shall investigate the details of the vortex stretching in the mid-level. Our interests are 1) the shape of the stretched vortex in the mid-level, 2) the vortex diameter at LFC, 3) temporal evolution of the length and the diameter of the vortex, and 4) the correspondence to the calculation with observed data (e.g., photos and radar data). Similar discussions can be found in Fiedler and Rotunno (1986).

We shall consider three typical schemes in Fig.4 for the vortex stretching in the mid-level. In scheme 1, no stress works in the vortex. This scheme corresponds to such that many thermal is produced continually at LFC. In scheme 2 and 3, some stress works in the vortex. In scheme 2, the effect of the stretching can propagate upward and downward while in scheme 3 the effect can only propagate downward. In scheme 3, the diameter at the top of the vortex does not vary because the effect of the stretching propagates upward. The diameter profile *D(z)* and the diameter at LFC, $D_{LFC}(z_{top})$, where z_{top} is the top of the vortex, can be obtained with some computation.

Scheme 1

$$D(z) = z_{LFC}\left(\frac{\rho_{LFC}}{\rho_{LNB}}\right)^{\frac{1}{2}} \exp\left(\frac{z - z_{LNB}}{2z_{LFC}}\right) \quad (12)$$

$$D_{LFC}\left(z_{top}\right) = z_{LFC}\left(\frac{\rho_{LFC}}{\rho_{top}}\right)^{\frac{1}{2}} \exp\left(-\frac{z_{top} - z_{LFC}}{2 z_{LFC}}\right) \quad (13)$$

Scheme 2

$$D(z) = z_{LFC}\left(\frac{\rho_{LFC}\, z_{LFC}}{\rho_{LNB}\, z_{LNB}}\right)^{\frac{1}{2}} \quad (14)$$

$$D_{LFC}\left(z_{top}\right) = z_{LFC}\left(\frac{\rho_{LFC} z_{LFC}}{\rho\left(z_{top}\right) z_{top}}\right)^{\frac{1}{2}} \quad (15)$$

Scheme 3

$$D(z) = z_{LFC}\left(\frac{\rho(z) z}{\rho_{LNB}\, z_{LNB}}\right)^{\frac{1}{2}} \quad (16)$$

$$D_{LFC}\left(z_{top}\right) = z_{LFC}\left(\frac{\rho_{LFC}\, z_{LFC}}{\rho\left(z_{top}\right) z_{top}}\right)^{\frac{1}{2}} \quad (17)$$

Actually the vortex stretching would not be conducted as quasi-static process assumed here. In addition, we neglect the vortex breakdown and the volume expansion by pressure reduction in the mid-level because these processes do not affect the diameter at LFC. In reality, the diameter profile would become exponential even in the case of scheme 2 due to these effects. Our 1D convection model employs scheme 2. We note that even in the case of scheme 3 the vortex diameter at LFC becomes the same as that in scheme 2.

The 3D shape of the vortex can be obtained as a set of 3D coordinates in the stretching calculation. The vortex volume and the estimated axis of radar reflectivity field taking the drop velocity into account are defined as follows.

$$V\left(z_{top}\right) = \pi R_{LFC}\left(z_{top}\right)^2 \cdot z_{LFC} + \int_{z_{LFC}}^{z_{top}} \pi R(z)^2 dz \quad (18)$$

$$\left(\int_{z_{LFC}}^{z} \frac{u(z)}{w(z)} dz + u(z)\int_{z_{LFC}}^{z} \frac{dz}{w(z)} + \int_{z}^{z_{LFC}} \frac{u(z)}{w_0} dz, \int_{z_{LFC}}^{z} \frac{v(z)}{w(z)} dz + v(z)\int_{z_{LFC}}^{z} \frac{dz}{w(z)} + \int_{z}^{z_{LFC}} \frac{u(z)}{w_0} dz\right) \quad (19)$$

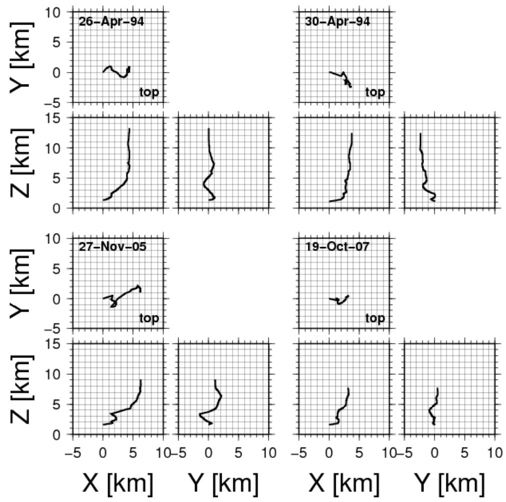

Fig.5 Three dimensional shape of the vortex in the mid-level for four tornadic events listed in Table 1.

5. CASE STUDY

For four tornadic cases in the United States, we conducted the stretching calculation using proximity-sounding data. These are listed in Table 4 with TVP+LFC wind and some parameters. The criteria of proximity are within 200km in distance and 70 minutes in time difference for the first two examples, and 100 km and 1 hour for the next two.

The scheme 1 gives unrealistic small diameter at LFC (less than one meter). On the other hand, the schemes 2 and 3 give some hundred-meter diameter. The 3D shape of the tornadic vortex in the mid-level is illustrated in Fig.5. The results for four events demonstrate that the vortex shape varies significantly around 4km height. The vortex tends to be stretched horizontally below 4km, but is vertical above 4km. The horizontal scale of the vortex below a height of 4km is 3 to 7km. The vortex is shaped roughly like Fig. 7 in Davies (2006). A large vertical wind shear and a low vertical velocity at low levels are important in making this shape because the stretching vortex needs to be deformed by the vertical wind shear for a long time at low levels. We will also show other results such as the minute-by-minute temporal evolution of the diameter, the vortex shape, and the vortex volume. From the 3D vortex shape, the axis of radar reflectivity field can

Table 1 Four tornadic cases utilized for validation.

	F-scale	TVP+LFC wind (ms-1)	CAPE (J kg-1)	SReH (m2 s-2)	EHI	VGP	STPC
1994.04.26	0	18.0	2971.0	73.7	1.36	0.160	0.230
1994.04.30	1	34.9	1368.0	211.4	1.81	0.169	0.777
2005.11.27	3	59.4	456.7	429.3	1.23	0.154	1.240
2007.10.19	2	50.7	905.3	405.6	2.30	0.227	0.851

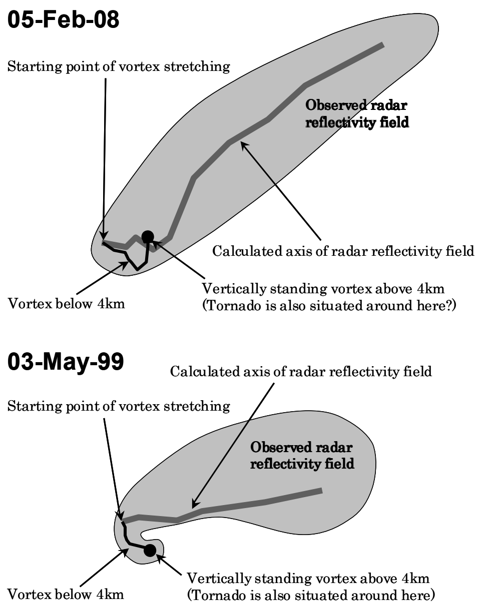

Fig.6 The difference between the tornadoes with and without hook echo.

also be calculated. Overlapping the top view of vortex shape with the axis of radar reflectivity field, we can distinguish the tornadoes with and without hook echo. The difference between the tornadoes with and without hook echo is illustrated in Fig.6.

6. CONCLUSION

The estimated wind speed and the diameter of tornadoes using our method are in good agreement with those obtained from damage surveys. For this calculation, the stretching in the mid-level is important. We investigated the validity of 1D vertical convection model used for the calculation of stretching, and described the results derived from this calculation. Our 1D model reconstructed some features of tornadoes and storms such as hook echoes. Our validation did not show any evident discrepancies with compared materials. This result supports the validity of our 1D model and our new tornado forecast parameters. Comparison with high-resolution cloud resolving model and a Doppler radar observation of the tornadic vortex in the mid-level is necessary for further research.

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