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SIMULATIONS OF INTERNAL WAVES APPROACHING A CRITICAL LEVEL

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1. BACKGROUND

Internal gravity waves exist abundantly in our world in stably-stratified fluids, such as the ocean and atmosphere. A stably-stratified fluid is one where the density of the fluid continuously decreases with increasing elevation. When a finite amount of fluid from a stratified environment is removed from its equilibrium density location by moving it upward or downward, buoyancy forces will compel the fluid to return to its original location. If the fluid is forced to oscillate at any frequency below the buoyancy frequency, it may oscillate in the horizontal direction in addition to the vertical direction. These multidimensional oscillations evolve into internal gravity waves, which will propagate throughout the fluid like regular surface water waves, but different in that they may propagate in three directions, and are not restricted to the horizontal plane of a liquid - gas interface. The wave phase and group speeds propagate orthogonally.

These waves are created through disturbances or perturbations in any stratified fluid, which occur regularly through natural forces, such as air or water currents moving over topography, interactions of fluid flows, or the breakdown of large waves into smaller scale waves or turbulence. Internal waves range in size, with wavelengths measured in scales of tens or hundreds of kilometers, and as such carry significant kinetic energy. These waves significantly affect the dynamic flows of the ocean and atmosphere. They contribute to the mixing of pollutants, organisms, and heat in the ocean and atmosphere, and transport momentum. Figures 1 and 2 illustrate how gravity waves may become visible in our atmosphere. As the fluid oscillates, water condenses and forms visible clouds in the crests of waves.

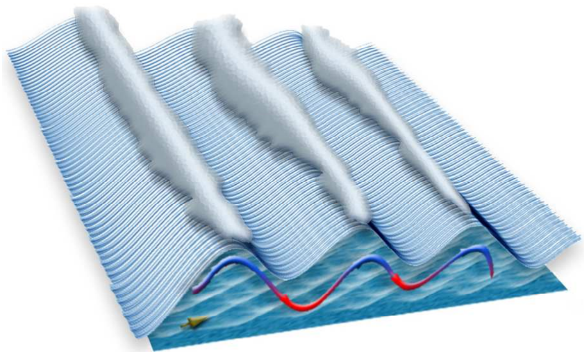

Figure 1 Internal gravity waves propagate in a stratefied fluid, sometimes leading to visible wave clouds. www.weathervortex.com

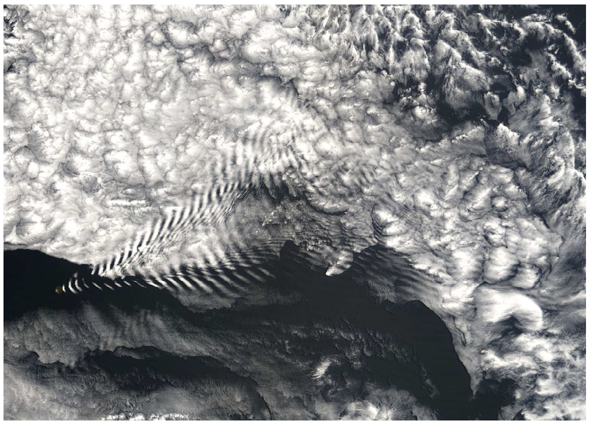

Figure 2 Gravity wave clouds formed by winds over Amsterdam Island. visibleearth.nasa.gov

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The scales of practically applicable flows in the atmosphere range in size from kilometers to millimeters or smaller. This creates a series of problems regarding observational data collection as well as resolution of numerical models. Due to computational cost, numerical models are often simplified by adopting linearized equations or two-dimensional simulation. These simplifications result in limitations, in that with the onset of turbulence, linear models break down and two-dimensional simulations lose validity. However, simplified models are usually preferred when numerous flows are analyzed. Nappo (2002) explained that linear simulations were generally adequate and often preferred for many internal wave simulations in the middle and upper atmospheres.

Of particular interest are interactions of internal waves with other geophysical flows, because of their regular occurrence in the environment. Winters and D'Asaro (1989) numerically studied the interaction between small scale internal waves and a strong steady shear, where the background wind continuously increases with altitude. The result is a critical level, where the phase speed of the wave matches the mean velocity of the background flow, and the relative frequency of the wave approaches zero. Critical levels are common in natural environments, such as where a wind blows above a region of topography where internal waves are generated. This point represents an imaginary boundary where instability is likely to occur, and critical level theory predicts that internal waves are incapable of propagating past this point. The background shear, or the rate of horizontal velocity change with elevation, causes the direction of the wave propagation to become more horizontal as it approaches the critical level. Winters and D'Asaro showed that if the amplitude of the internal waves were sufficiently small, energy from the waves approaching a critical level was lost to the background flow, accelerating the flow near that region. If the amplitude of the waves were larger, the waves would steepen and could potentially overturn and break, as discussed in later works of Winters and D'Asaro (1994). As these waves would steepen and overturn, analogous to water waves approaching a shore, energy was lost to turbulence and viscous dissipation. Perturbations from the overturning waves resulted in the formation of new, smaller internal waves that propagated past the critical level. Some waves were reflected from the critical level and propagated away in the direction from whence they appeared. Thorpe (1981) researched this interaction experimentally and found that wave behavior was surprisingly accurate for his numerical model, for which later researchers compared and validated their results.

Linear models are advantageous for critical level simulations, due to significantly increased computational speed. Winters and D'Asaro showed that two-dimensional simulations are adequate prior to wave breaking and turbulence. This research is helpful in understanding large scale energy budgets and knowing where the energy is distributed or lost. Steady critical levels have been extensively studied, yet less research has been done with time dependent critical levels, which are more prevalent in the environment.

2. METHODS

A linear ray-tracing program was used to simulate the interaction of internal waves approaching a critical level, as well as waves approaching a time dependent critical level and a large-scale inertial wave. The ray tracing program was written in MATLAB and used an R-K 4 method for solving differential equations.

The program uses linear WKB theory to calculate the dynamic properties of the interacting waves, and used the following assumptions:

- The interaction of the small wave does not affect or change the background winds.
- Buoyancy frequency is constant and average fluid density is treated as constant.
- Fluid viscosity is neglected.
- The WKB approximation is valid prior to wave breaking or inflection.

Many inputs are taken into the ray-tracing program, and for the atmospheric simulations discussed in this paper, many of the values were kept constant such as the following variables. These were the same variables used by other authors. (Sartelet 2003) (Vand 2008)

Coriolis frequency at mid-latitude: $f = 0.0001t^{-1}$

Buoyancy frequency: $N = 0.02t^{-1}$

Density of air at about 25-30 km altitude:
$\rho = 0.034\ kg/m^3$

Density gradient approximated from NRLMSISE Standard Atmosphere Model

$$\frac{d\rho}{dz} = -0.0001386\rho$$

For two-dimensional waves, the wavenumbers are defined as:

$$m = \frac{2\pi}{\lambda_V} \qquad k = \frac{2\pi}{\lambda_H}$$

2.1 Mid-frequency Approximation

Assuming $N^2 >> \omega^2 >> f^2$, which is known as the mid-frequency approximation, the WKB equations simplify to:

$$\omega = \pm\left(\frac{f^2m^2 + N^2k^2}{k^2 + m^2}\right)^{1/2}$$

$$\omega \approx \pm\frac{Nk}{m}$$

$$c_{gz} = \frac{dz}{dt} = \frac{\partial\omega}{\partial m} = \frac{-\left(N^2 - f^2\right)k^2m}{\left(f^2m^2 + N^2k^2\right)^{1/2}\left(k^2 + m^2\right)^{3/2}}$$

$$c_{gz} \approx \frac{\pm Nk}{m^2}$$

For a wave passing through a steady background flow with a differentiable flow field and constant k, N, and f, and initial wavenumber m, each value of horizontal velocity throughout the field produces a distinct wavenumber m.

A critical layer is an area where the phase speed of the small wave matches the mean background velocity, and the relative frequency goes to zero, or *f* if the fluid is rotating, such as the earth. This is defined mathematically as:

$$U_{CL} = \frac{\omega_0}{k} = \frac{1}{k}\left(\frac{f^2m_0^2 + N^2k^2}{k^2 + m_0^2}\right)^{1/2}$$

Simplifying the above equation using the mid-frequency approximation results in:

$$U_{CL} \approx \frac{N}{m_0}$$

From these simplified equations, we can determine the approximate value of m at any point within the background flow given the horizontal velocity, and it is shown that for a given velocity U:

$$\frac{\partial m}{\partial z} \approx \frac{m^2}{N}\frac{\partial U}{\partial z}$$

$$U_{CL}m_0\frac{\partial m}{m^2} \approx \partial U$$

$$m_0\frac{1}{m} \approx -\frac{U}{U_{CL}} + c$$

When $U = 0$, then $m = m_0$, so $c = 1$:

$$\frac{m}{m_0} = \frac{1}{\left(1 - U/U_{CL}\right)}$$

This simple model is valid for values where the buoyancy frequency is much larger than the Coriolis frequency (which is generally true for any stable region of the Earth's atmosphere) and when the value of m is greater than k. The following figures show the error associated with the mid-frequency approximation.

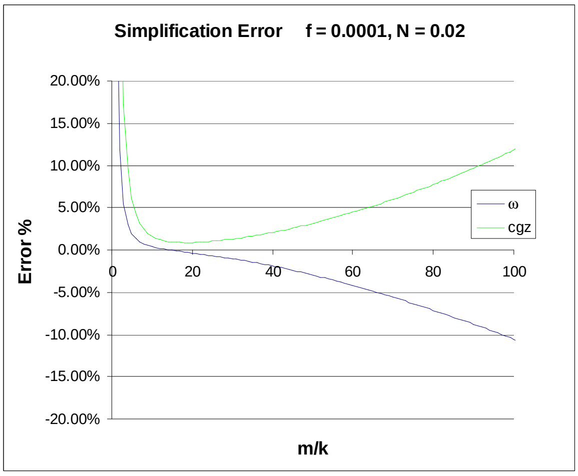

Figure 3 Error associated with the mid-frequency approximation. Typical errors in the mid-frequency range are less than 5%.

The ray tracing program does not use the mid-frequency approximation, but rather uses the full WKB approximation. However, the mid-frequency approximation is useful in understanding the dynamics of the wave parameters more simply.

The ray tracing program easily calculates the wave action density, proportional to the amplitude *A*, and the wave steepness ζ_z, using the following WKB equations:

$$A = \frac{1}{\forall}$$

$$\left(\frac{\partial(\forall)}{\partial t}\right) = \frac{k^2\left(f^2 - N^2\right)\left(1 - m^2\left(\frac{f^2}{k^2N^2 + m^2 f^2} + \frac{3}{k^2 + m^2}\right)\right)}{\left(k^2N^2 + m^2 f^2\right)^{1/2}\left(k^2 + m^2\right)^{3/2}} m_{z0}$$

$$\left(\frac{\partial m_{z0}}{\partial t}\right) = -k\frac{\partial^2 U}{\partial z^2}\forall$$

$$\left|\varsigma_z\right| = k\left(\frac{2A}{\omega\rho}\right)^{1/2}$$

The local Richardson number is calculated to determine instability due to shear forces, this is defined as:

$$Ri_g = \frac{N^2}{\left(\frac{dU}{dz}\right)^2}$$

And the wave induced shear:

$$\left|u'_z\right| = Nk\left(\frac{2A}{\omega\rho}\right)^{1/2}\left(1 - \frac{f^2}{\omega^2}\right)^{-1/2}$$

3. SIMULATIONS

3.1 Mean Wind

As waves approach a mean wind, ray theory suggests that waves will not be able to pass through the wind if a critical level is present. At the point where waves pass the critical level, ray theory predicts an infinite action density as well as zero vertical group velocity. For this reason, the waves will not actually reach the critical level, but rather approach it asymptotically, with its action density and wave steepness approaching infinity.

If a critical level is not present, the waves will pass through the mean wind. The action density of the waves and wave steepness will increase as the waves approach the maximum velocity of the mean wind, or when the wave is passing through a positive velocity gradient. The wave steepness and action density decrease as the wave passes through the negative velocity gradient, or shear. If the conditions on either side of the wind layer are identical, then the conditions of the small wave, including wave numbers, action density, steepness, and frequency, will all return to their original values. The wave steepness and action density reach a maximum as the waves pass through the maximum velocity. There is a solid relationship between the wave steepness, action density, and the ratio of the maximum wind velocity to critical layer velocity. The equations are shown below, the first confirmed by the mid-frequency approximation and then all confirmed with the ray tracing program.

$$\frac{m}{m_0} \approx \frac{1}{\left(1 - U/U_{CL}\right)}$$

Wave action density:

$$\frac{A}{A_0} \approx \frac{1}{\left(1 - U/U_{CL}\right)^2}$$

Wave Steepness:

$$\frac{\varsigma_z}{\varsigma_{z0}} \approx \frac{1}{\left(1 - U/U_{CL}\right)^{3/2}}$$

These relationships are valid for different values of the wave number k, maximum wind velocities, and wave number ratios m/k, provided that the ratio is larger than 1.0, with larger ratios fitting the curves more tightly.

Figures 4 and 5 show two simulations of small waves approaching a mean wind, the first with no critical level present and the second with a critical level present. The wave that doesn't approach a critical level never becomes unstable, but the wave that approaches a critical level becomes unstable near the critical level.

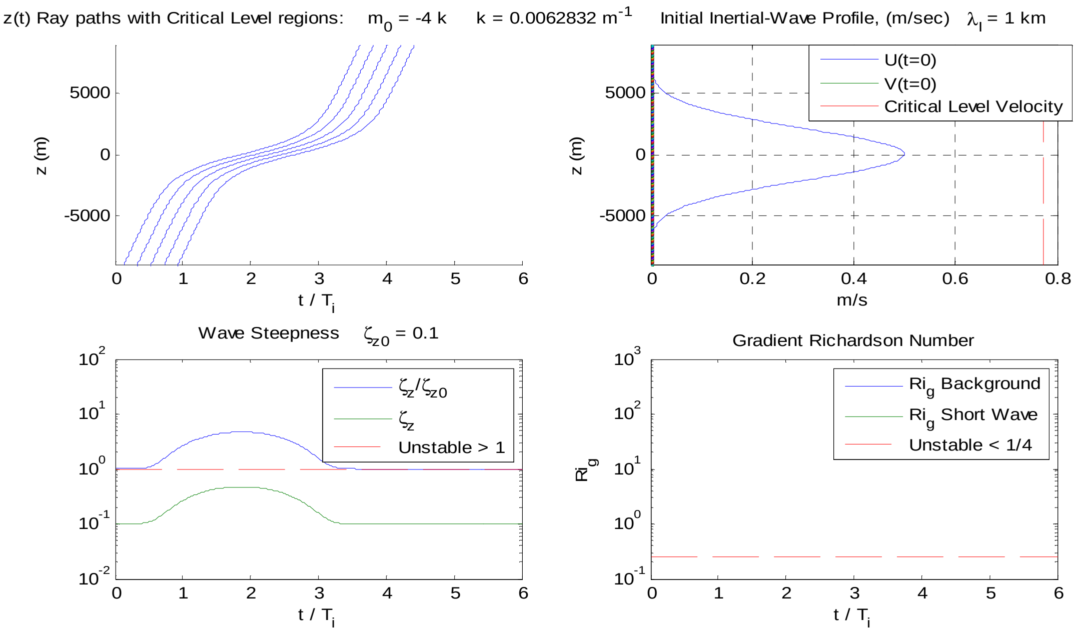

Figure 4 Wave approaches a mean wind with no critical level. The wave passes through the mean wind, and the steepness of the wave increases during the interaction, but returns to its original conditions as the wave exits the wind. Upper left: wave rays plotted in elevation versus time. Upper right: Initial background velocity profile. Lower left: Relative and actual wave steepness of first ray versus time. Lower right: Gradient Richardson number versus time for background with (and without) induced shear of small wave. The time is nondimensionalized against an inertial wave period, defined as $T_i = 2\pi/f$

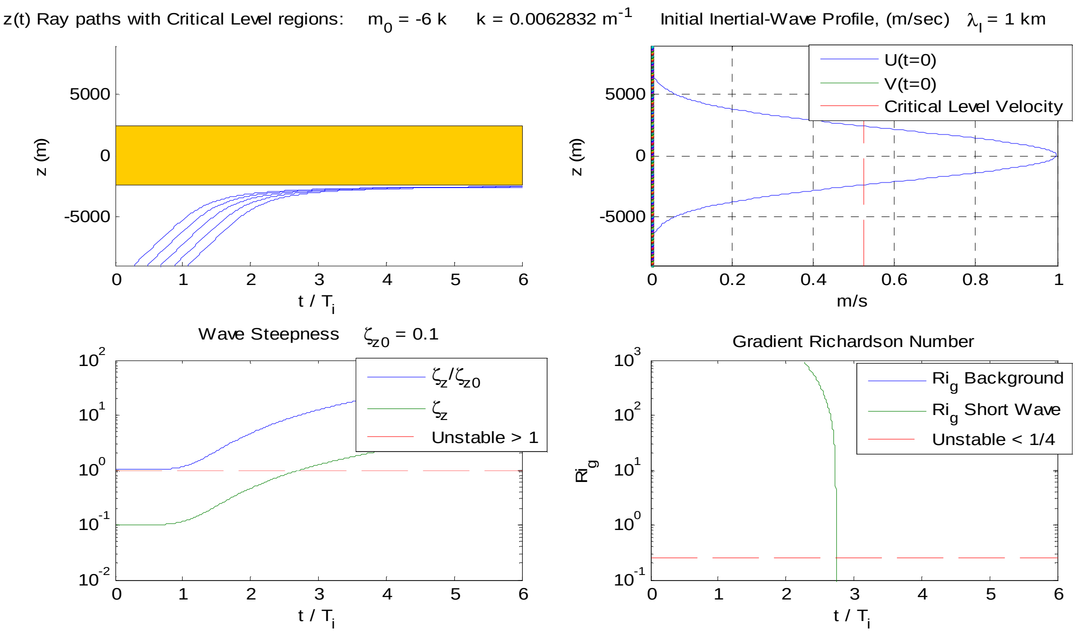

Figure 5 An internal wave approaches a critical level and the vertical group velocity decreases asymptotically. The steepness increases and the wave becomes unstable.

3.2 Moving Mean Wind

An understanding of the mean wind scenario provides the knowledge necessary to understand a wind envelope moving in the vertical direction, similar to the phase speed of an inertial wave. This provides a step to understanding the interaction between a small-scale wave propagating through an inertial wave.

In this scenario a small scale internal wave approaches a Gaussian envelope of wind, just like in the previous scenario. However, this envelope of wind, although it has no vertical component to its wind velocity, moves as an envelope toward the approaching wave at a constant speed. In this way, the wave not only approaches the critical level, but the critical level then moves through it. Figure 6 shows the simulation of a wave passing through a moving mean wind. From this and other simulations it is concluded that once the packet of wind has moved through the wave, the wave parameters return to their original values before they had approached the packet. This scenario, although not necessarily prevalent in the real world, leads to a better understanding of the interaction with inertial waves, where multiple packets of wind will pass through an internal wave.

The variable parameters in this scenario are m, k, U_0, and M, referring to the speed of the wind packet. We use the term M because in the inertial wave interactions, the wave number M and the Coriolis frequency determine the speed at which the wind packets, or wave phases, will pass through the wave.

In the previous scenario we saw that m was dependent on the local background velocity, and the same is true in this scenario, only that a new variable M is added to the equation. To find the changing wavenumber m:

$$\frac{\partial m}{\partial t} = -k \frac{\partial U}{\partial z}$$

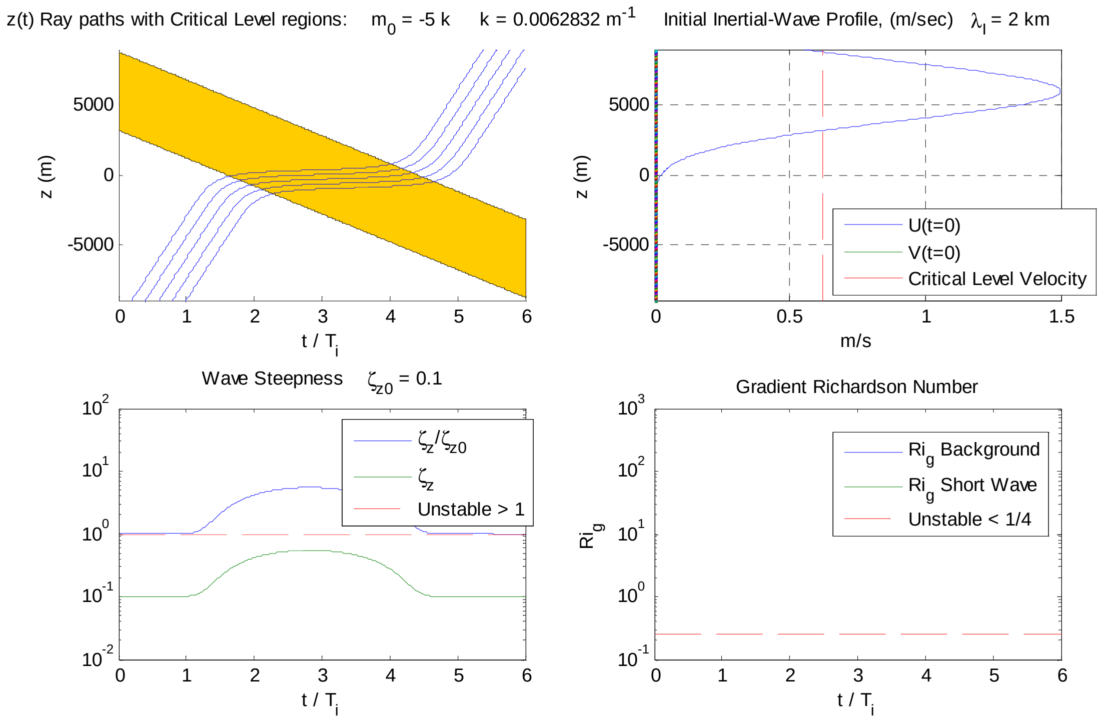

Figure 6 Wave approaches a moving mean wind. Although a critical level is present, the wave passes through the critical level region and returns to its original parameters. The wave behaves as if it isn't even approaching a critical level.

$$\frac{\partial z}{\partial t} \approx \pm \frac{Nk}{m^2}$$

For a downward moving background wind, we can simply add the speed of the background wind envelope ‘c’ to the vertical velocity of the approaching wave.

$$\frac{\partial z}{\partial t} \approx \frac{Nk}{m^2} + c$$

Combining the two allows us to find the change in wavenumber with changing velocity

$$\frac{\partial m}{\partial z} = \frac{\partial m}{\partial t}\frac{\partial t}{\partial z} \approx -k\frac{\partial U}{\partial z}\left(\frac{1}{\frac{Nk}{m^2} + c}\right)$$

$$\left(\frac{Nk}{m^2} + c\right)\frac{\partial m}{\partial z} \approx -k\frac{\partial U}{\partial z}$$

$$\left(-\frac{N}{m^2} - \frac{c}{k}\right)\partial m \approx \partial U$$

$$\frac{N}{m} - \frac{c}{k}m \approx U + C_1$$

Using the condition that $m = m_0$ at $U = 0$, we find that:

$$C_1 = \frac{N}{m_0} - \frac{c}{k}m_0$$

Combine to yield:

$$U \approx \frac{N}{m} - \frac{c}{k}m - \frac{N}{m_0} + \frac{c}{k}m_0$$

$$U \approx \frac{N}{m} - \frac{c}{k}(m - m_0) - \frac{N}{m_0}$$

The vertical velocity that comes from the downward propagating background wind can be related to the downward propagating phase speed of the inertial wave, which is *f/M*, where *M* is the vertical wavenumber of the inertial wave.

Also, the background velocity U can be non-dimensionalized against the critical layer velocity, where

$$U_{CL} = \frac{\omega_0}{k} = \frac{\left(N^2k^2 + f^2m_0^2\right)^{1/2}}{k\left(k^2 + m_0^2\right)^{1/2}}$$

$$U_{CL} \approx \pm\frac{N}{m_0}$$

Combining these with the equation above we get:

$$\frac{U}{U_{CL}} \approx -\frac{m_0}{m} + \frac{m_0 f}{MkN}(m - m_0) + 1$$

$$\frac{U}{U_{CL}} \approx -\frac{1}{m/m_0} + \frac{fm_0^2}{MkN}(m/m_0 - 1) + 1$$

From the above equation, as m becomes large relative to its initial value, it increases linearly with background velocity.

As the wave passes through the packet of wind (or the wind passes through the wave) m continually increases until it passes the peak wind velocity. After which it decreases in the negative background shear and returns to its original value.

In the previous scenario, where the background did not change with time, the wave action density, directly related to the wave amplitude, would increase and decrease with an increasing or decreasing wavenumber m. However, with moving critical levels, the wave action density reaches a maximum value near a velocity at about twice the critical layer velocity. This is shown in figures 8a-8d on the following page.

All action density figures appear similar, though not necessarily with the same length scales. As different simulations were run with a different inertial wave number M, the action density versus the velocity increased linearly. Similarly, as the wavenumber k of the short wave is increased, the change in wave action density is linearly decreased. This fully agrees with the previous scenario of the waves approaching a mean wind. In those cases, the speed of the envelope of wind is zero, which corresponds to an infinite value of ‘M.’ In these cases, the wave does not pass through the wind when the background velocity is greater than the critical layer velocity, and the action density approaches infinity at the critical layer.

In addition to the wave action density, it is important to know the wave steepness, which will cause the internal wave to theoretically overturn when the steepness is equal to unity. (Vand 2008) Steepness plots are shown in figures 7a-7b.

Many tests were run with moving envelopes of wind, and qualities were found. As mentioned before, the action density increases with larger values of M, as well as smaller values of k. This is true when the background velocity is significantly larger than the critical layer velocity. For velocities much lower than the critical layer velocities, the action density diminishes.

Theoretically, the action density and steepness from this moving wind should be a good estimate of the action density and steepness from an interaction with an inertial wave.

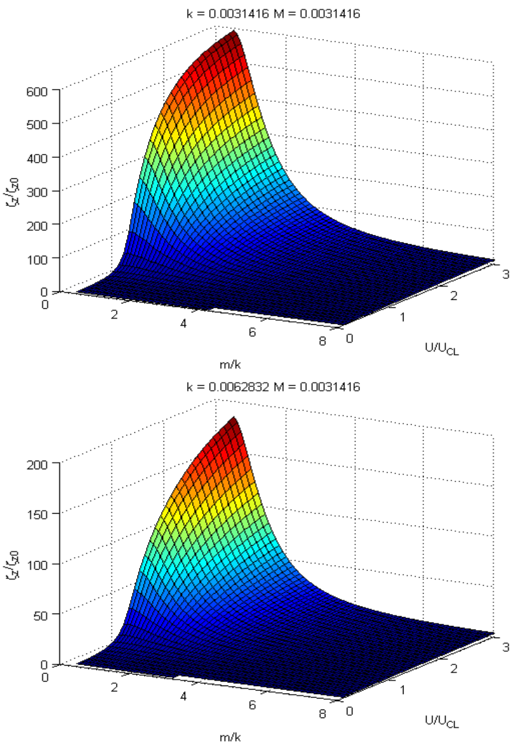

Figure 7 The wave steepness depends on the wave action density and the frequency of the wave. The steepness is therefore more dependent on the wavenumber k than the speed of the moving wind packet.

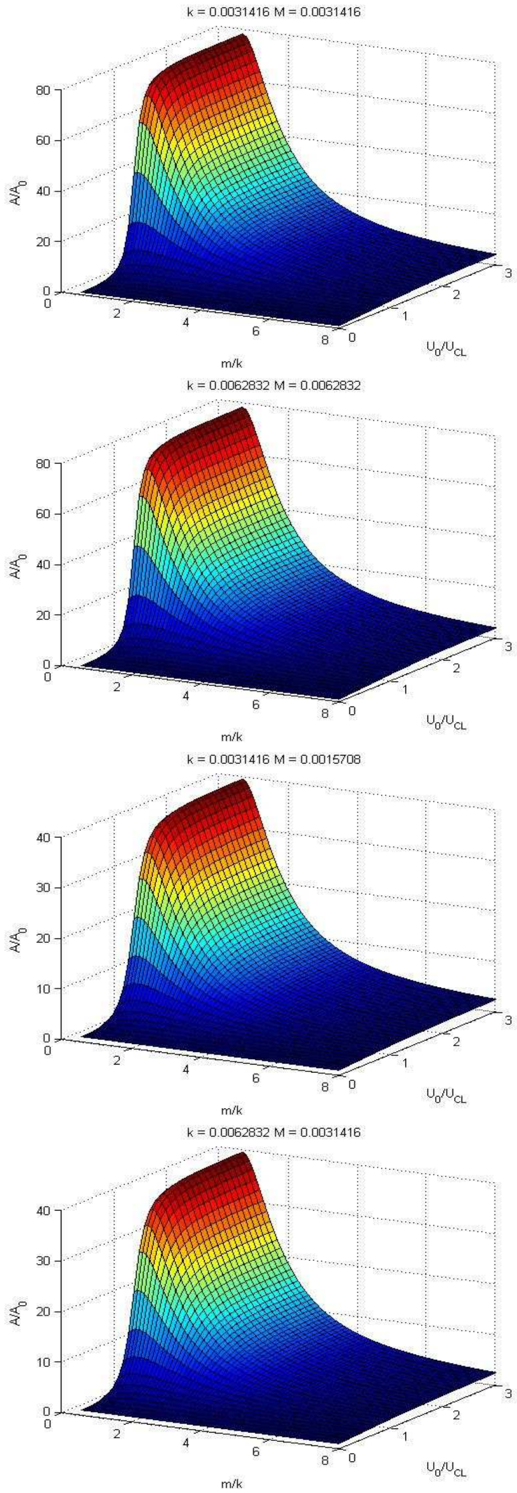

Figure 8 The wave action density increases based on the relative background velocity and speed of the moving wind packet. During an interaction, the wave action density doesn't increase much with winds greater than two times the critical level velocity.

3.3 Reverse Mean Wind

When a small amplitude wave interacts with a large inertial wave, not only will it encounter moving critical levels from envelopes of positive horizontal wind, but also envelopes of negative horizontal wind. If the negative wind is sufficiently strong, it may cause the wave to change its vertical group speed direction. If the wave is able to propagate past the region of inflection, the vertical wavenumber becomes imaginary and the disturbance decays exponentially. (PedloskyThe inflection occurs when the frequency of the small wave matches the local value of N. By the dispersion relation:

$$\omega = \left(\frac{f^2 m^2 + N^2 k^2}{k^2 + m^2} \right)^{1/2}$$

It can easily be shown that when $m = 0$, then $\omega = N$, and as m approaches ∞, then ω approaches f.

$$\frac{\partial \omega}{\partial t} = -k \frac{\partial U}{\partial z} c_{gz}$$

The point at which the frequency comes closest to the buoyancy frequency is the point at which m changes signs. The chance of inflection increases with increased maximum negative background velocity, larger k, smaller m, and smaller wavenumber M of the inertial wave.

3.4 Inertial Gravity Waves

Inertial gravity waves are internal waves which exist in any rotating fluid and have a frequency close to the Coriolis frequency. At 10 to 50 km above sea level. R. O . R. Y. Thompson (1978) showed that inertial waves are not present in the troposphere, but are common in the stratosphere though their source often comes from below. Sato, O'Sullivan, and Dunkerton (1997) found inertial gravity waves over Japan with a period of about 20 hr, and are dominant at an altitude of about 22 km, where background winds are small. Guest, Reeder, Marks, and Karoly (1999) determined the properties of internal waves in the stratosphere south of Australia. Sato, O'Sullivan, and Dunkerton detected inertial gravity waves 20 – 24 km over Shigaraki, Japan which had wind oscillations of 2.5 to 3 m/s. All sources found similar properties of inertial waves, with a vertical wavelength between 1 and 7 km, and horizontal wavelengths around 1000 km. The frequency of the waves are near the Coriolis frequency, and the phases of the waves propagated downward.

3.5 Inertial Wave interaction

Many experiments were run where small internal waves propagated through an inertial wave, with the inertial phases propagating downward. The inertial waves were given a vertical wavelength from 0.5 km to 7 km, with an infinite horizontal wavelength. As the wave propagated through phases where the background velocity is positive in the horizontal direction, the wave steepness and the wave action density increase, while the vertical group velocity decreases. As the small waves propagate through phases of negative velocity, the steepness and action density decrease, while the vertical group velocity increases. Excepting cases of inflection or overturning, the steepness, action density, and group velocities return to their original values once the small waves have propagated through the inertial wave. This assumes that the background wind velocities are identical on either side of the inertial wave packet.

With some small variation, the wave action density and steepness are similar to the scenarios of the moving wind envelopes. The variation arises from the non uniform phases within the inertial wave envelope.

Wave instability arises when the steepness of the wave exceeds unity, or when the background shear causes the local Richardson number to decrease below a value of 0.25. (Winters, D'Asaro 1994) Figures 9-10 illustrate two unstable interactions resulting in instability principally from shear and steepness, respectively.

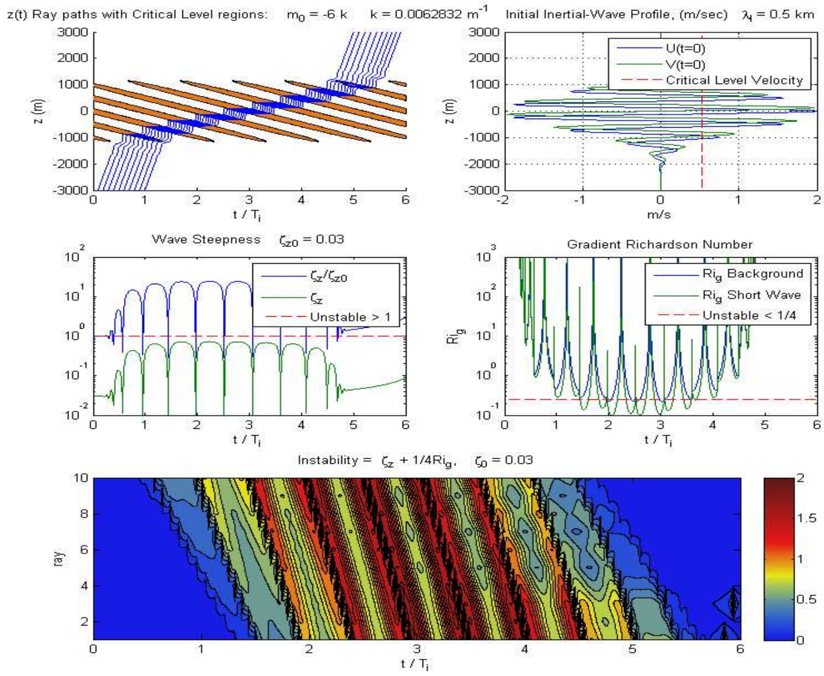

Figure 9 An internal wave interacts with an inertial wave. The lower plot shows an illustrated instability approximation using the wave steepness and local Richardson number. Orange and red regions represent areas of instability.High background shears cause the small scale wave to become unstable, even though the wave steepness remains low.

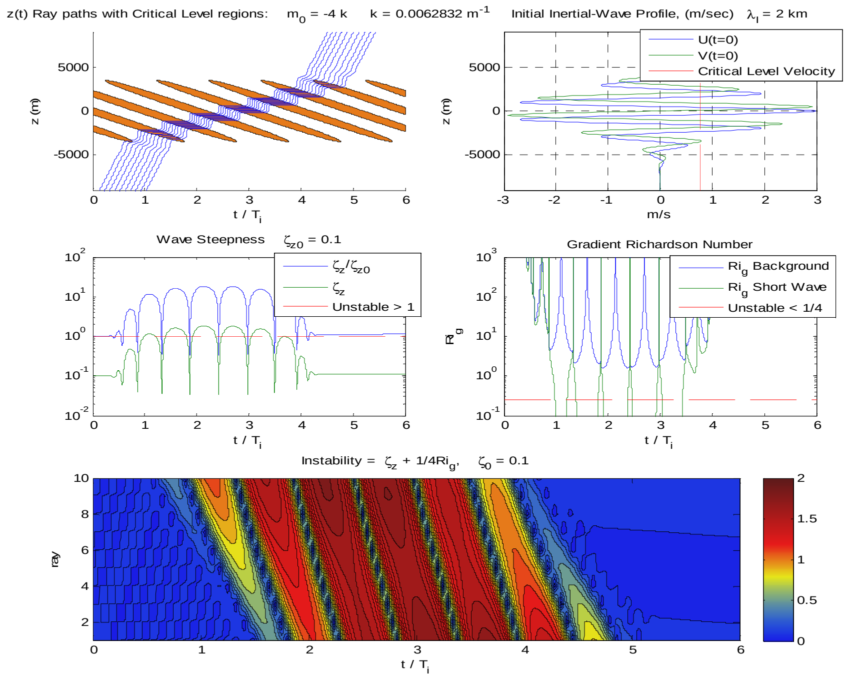

Figure 7 An internal wave interacts with an inertial wave. The wave becomes unstable due to excessive wave steepness.

4. CONCLUSIONS

Time-dependent critical levels affect small scale waves differently than stationary critical levels. Although critical level regions may be regions of instability, they do not permanently affect internal waves as much as stationary critical levels. Specifically, time-dependent critical levels decrease chances of wave instability during the interaction compared to stationary critical levels.

Wave instability, which may lead to breaking, arises from background shear and wave steepness, although one may dominate the approach to overturning. Detected inertial waves in the atmosphere typically do not produce low Richardson numbers (high shears) capable of instability. If wave instability arises during an inertial wave interaction, it is typically due to an excessive wave steepness of the approaching wave. This is dependent on the initial wave steepness before the interaction.

Waves propagating through larger inertial waves are less likely to become unstable and break compared to waves approaching stationary critical levels.

5. FUTURE WORK

These scenarios will be validated against non-linear simulations or observational data. Other parameters may be necessary to amend the ray-tracing program to increase its accuracy.

Wave energy parameters will be incorporated into the simulations. It is of particular interest to determine how much energy is transported or lost during an interaction. The ray-tracing program will be enhanced to calculate the energy loss due to wave breaking and energy loss due to transfer to the mean flow.

6. ANTICIPATED CONTRIBUTIONS

This research on internal waves has many large scale global applications, and an understanding is necessary in creating global meteorological models. As internal waves break, they become a significant source of mixing within the middle and upper atmosphere, influencing the amount of pollutants or organisms in any given area. Internal waves maintain environmental energy budgets by transporting mass, momentum, and heat throughout the global spectra. Because these waves are found in abundance over vast volumes of space, much energy is constantly being deposited or accumulated due to their propagation and breaking. An understanding of these processes is necessary to understand internal waves' effect on global processes.

5. REFERENCES

Guest, Reeder, Marks, and Karoly, (2000) Inertia-Gravity Waves Observed in the Lower Stratosphere over MacQuerie Island, J. Atmos. Sci., 57, 737-752.

Nappo CJ (2002) An Introduction to Atmospheric Gravity Waves, Vol.85 of *International Geophysics Series*. Academic Press, San Diego, 276 pp

Pedlosky, Joseph (2003) Waves in the Atmosphere and Ocean, Springer-Verlag Berlin Heidelberg New York

Sartelet, K. N., (2003) Wave Propagation inside an Inertia Wave. Part I: Role of Time Dependence and Scale Separation. *J. Atmospheric Sciences*, 60, 1448-1455.

Sartelet, K. N., (2003) Wave Propagation inside an Inertia Wave. Part II: Wave Breaking. *J. Atmospheric Sciences*, 60, 1448-1455.

Sato, O'Sullivan, and Dunkerton, (1997) Low-fequency inertia-gravity waves in the stratosphere revealed by three-week continuous observation with the MU radar. *Geophysical Research Letters*, 24, 1739-1742

Thompson, R. O. R. Y., (1978) Observation of inertial waves in the stratosphere. *Quart. J. R. Met. Soc.,* 104, 691-698

Thorpe, S. A. (1981) An Experimental Study of Critical Layers. *Journal of Fluid Mechanics, 195*, 321-344.

Vanderhoff, J. C., K. K. Nomura, J. W. Rottman, and C. Macaskill (2008), Doppler Spreading of internal gravity waves by an inertia-wave packet, *J. Geophys. Res.* 113, C05018, doi:10.1029/2007JC004390

Winters, Kraig B. & D'Asaro, Eric A (1989) Two-Dimensional Instability of Finite Amplitude Internal Gravity Wave Packets Near a Critical Level. *Journal of Geophysical Research, 94*, 12 709-12,719.

Winters, Kraig B. & D'Asaro, Eric A (1994) Three-Dimensional Instability Wave Instability Near a Critical Level. *Journal of Fluid Mechanics, 272*, 255-284.