RETRIEVAL OF MEAN VERTICAL AIR MOTION IN STRATIFORM SNOW USING DOPPLER RADAR WIND MEASUREMENTS

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1. INTRODUCTION

A better estimation of the vertical air motion from observations, both at the synoptic scale and the mesoscale, may bring significant improvement in the quality of analysis and numerical forecasts. Weather radars are often the preferred tools for mesoscale studies because of its relatively extended 3D coverage. In case of a stratiform snow system, the vertical air motion is of comparable magnitude to the terminal fall speed of snow particles, thus difficult to measure directly by any instrument.

In the present study, for one stratiform snow event: the 26th of October 2005, 00-03UTC, timely and spatially averaged vertical air motion profiles are estimated from three different techniques: the vertical motion can be obtained through integration of horizontal divergence (Browning and Wexler, 1968), using velocity volume scans of a single Doppler radar; a deterministic equation of the vertical motion can be derived from conservation of the potential temperature, assuming geostrophy; radar wind measurements and conservation laws can act as constraints, to retrieve an optimized solution to the vertical air motion using a variational method. Verification is possible by estimating the fall speed of snow, and extract the air motion component from the Doppler vertical velocity measurements of a high-resolution vertically pointing radar, such as the X-band (referred to as VertiX) of the Radar Observatory of McGill University.

Sections 2.1 to 2.3 will present, respectively, the two techniques of retrieval using radial wind volume scans and temperature measurements, and the algorithm of the variational method. Section 3 will present the results and the verification from the VertiX velocity measurements, followed by discussions on the potential of the different techniques of retrieval to estimate an average vertical air motion. Our aim is to assess the potential added value of the various sources of information to the Mesoscale Analysis System (Zawadzki et al. 2009).

2. METHODOLOGY

2.1 *From radial wind of Single Doppler radar*

The S-band single Doppler radar of McGill, situated at the McGill Radar Observatory in Ste-Anne-de-Bellevue, Montreal, provides volume scans of radial Doppler velocity and reflectivity every five minutes, at elevation angles ranging from 0.5 to 34 degrees.

The radial wind field V_r has little contamination from the falling precipitation particles, since the scans are performed at low elevations. The small contribution of the fall speed V_f is accounted for by taking a linear function of height, derived from a smoothed vertical velocity field of the VertiX. Equation 2.1.1 shows the expression of V_r as a function of the 3D wind field (*u,v,w*), the precipitation fall speed, the azimuth angle α (0^o toward North) and the elevation angle β.

$$Vr(r,\alpha,\beta) = [u\sin(\alpha) + v\cos(\alpha)]\cos(\beta) + [w + V_f]\sin(\beta) \quad (2.1.1)$$

The average wind flux through a circular region (i.e. the average of a Velocity-Azimuth Display as shown in Figure 1), centered about the radar, defines the mean horizontal divergence *D(z)* over that given circular region. A two-harmonic fitting was performed on the velocity as a function of azimuth, in the attempt of compensating for measurements errors and the missing or contaminated data: its average value gives the mean horizontal divergence at the given range and height level. This method for calculation of horizontal divergence allows for non-linearity in the horizontal wind field (Caya and Zawadzki, 1992). In this case, two harmonics were judged sufficient. The non-uniform power distribution within the radar beam is taken into account by deconvolution of the vertical motion profiles, assuming that the beam power follows a Gaussian distribution.

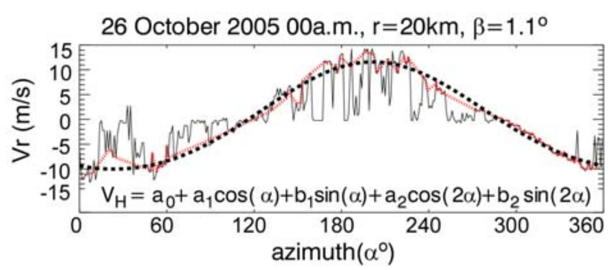

Figure 1. Black line is the radial wind as a function of azimuth, scanned by the S-band Doppler radar at 1.1^o of elevation and 20km range. The red dotted line is what is considered as the air motion, after masking most of the contamination of measurements by ground, and the component of the fall speed of particles subtracted. The thick black dotted line is the two-harmonics Fourier fit (as written in equation) to the air motion. Its average value (a_0) gives the horizontal divergence.

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Vertical air motion is defined as the vertical integral of the horizontal divergence, given a boundary condition. The uncertainties associated with the estimates of horizontal divergence are accumulated in the integration; thus, more weight should be given to the vertical motion computed at the beginning of the integration. The weighted average of a two-way vertical integration of divergence from ground and from top, density ρ-weighted, assuming zero vertical motion at ground and the echo top z_T (Protat and Zawadzki, 1990), gives the mean vertical wind $\overline{w}(z)$ as a function of height:

$$w_{up}(z) = \frac{1}{\rho(z_T)} \sum_{z=0}^{z_T} \rho(z)(-D(z)) \qquad (2.1.2)$$

$$\overline{w}(z) = \frac{z_T - z}{z_T}(w_{up}(z)) + \frac{z}{z_T}(w_{down}(z)) \qquad (2.1.3)$$

The region where the radial velocities of the scanning radar are used for computations is restricted to ranges between 40 to 50 kilometers radius away from the radar, to avoid most of the contamination by ground at near ranges, and still not too far to have an acceptable vertical resolution.

2.2 Synoptic advective vertical wind

Some commercial aircrafts take measurements of the temperature and the dew point temperature during their ascent and descent (Figure 2). The total change in the potential temperature θ of an air mass, which includes the local change and the 3D advection, is mainly due to the latent heat released from water vapor condensation, or sublimation in this case. Using the horizontal wind, temperature and dew point measurements by six aircrafts, all trajectories within 100km range with respect to the radar and within 00-02 UTC, the only variable that remains unknown is the advective, and geostrophic vertical wind *w*. Equation 2.2.1 is the expression that solves *w* in pressure coordinates.

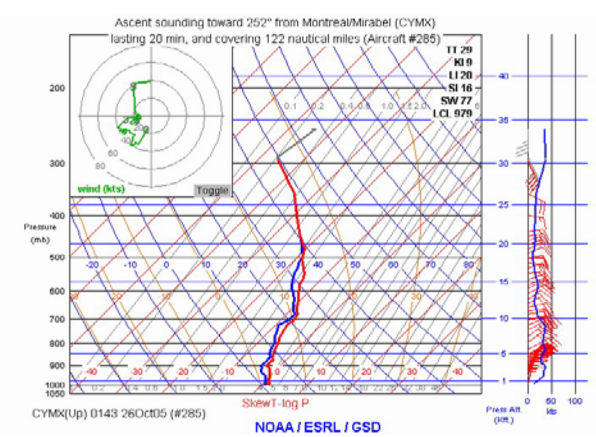

Figure 2. Temperature and wind profiles measured by a commercial aircraft (NOAA AMDAR online database).

$$\varpi\left[\frac{\partial\theta}{\partial p} + \frac{\theta}{C_p T}\left(L\frac{Dr_{(s)}}{Dp}\right)\right] = -\frac{\partial\theta}{\partial t} - \left(\frac{p}{p_0}\right)^{\frac{R}{C_p}} \cdot \frac{f}{R_{dry}} \left[v\frac{\partial u}{\partial(\ln p)} - u\frac{\partial v}{\partial(\ln p)}\right] \qquad (2.2.1)$$

The parameters are defined as follows: the Coriolis parameter $f \sim 10^{-4}\ s^{-1}$ at 45°N, R_{dry} the dry gas constant ~ 278.05 J/(kg*K), R the gas constant ~8.314472 J/(mol*K), C_p the heat capacity of moist air ~29.1 J/(mol*K), L the latent heat of sublimation ~2.83E06 J/kg, and r the mixing ratio, calculated from dew point temperatures. Here we assumed that for this particular snow case, the geostrophic component is dominant in the synoptic scale, and there exists a mean vertical air motion that is representative of the two-hour time period. Although this method involves important assumptions, it gives a good qualitative image of the vertical wind pattern in the synoptic scale.

2.3 Minimization

When redundant and independent information are available, it is interesting to use a variational method to combine the information if we know the relative weight of each contribution. The control variables here are the 3D wind field (*u,v,w*). The difference between the retrieved variables (analysis) and observations is expressed by a total cost function *J*. Through minimization of this cost function by iteration, the variational method finds a solution that best satisfies all constraints (I-IV listed below), based on radar observations and physical assumptions, in the same way as a least square fit. The weight of each constraint is inversely proportional to its associated uncertainty. If *J* has multiple minima, the initial guess of the control variables is very important. In this experiment, the uniform horizontal wind from aircraft measurements is used to initialize (*u,v*) over the whole domain; $w_{initial}$=0.

I) The uncertainties in radial Doppler velocities are taken as 1m/s. The radar measures in a spherical coordinate system. Data at three consecutive times are projected, by distance-weighted interpolation, to a regular 3D grid compatible with numerical models. At each grid point, V_r is considered as a function of (*u,v,w*) (Eq. 2.1.1). The control variables (*u,v,w*) are found so that $\frac{dJ_{Vr}}{dV_r} \to 0$, J_{Vr} is the cost function for this constraint:

$$J_{Vr} = \sum_{x,y,z} weight \cdot (V_{r\ obs.} - V_{r\ analysis})^2 \qquad (2.3.1)$$

II) The horizontal wind field of a stratiform system is often close to linear:

$$u(x,y,z,t) = u_0 + \frac{\partial u}{\partial x}x + \frac{\partial u}{\partial y}y \qquad (2.3.2)$$

$$v(x,y,z,t) = v_0 + \frac{\partial v}{\partial x}x + \frac{\partial v}{\partial y}y \qquad (2.3.3)$$

III) Conservation of air mass in its anelastic form is applied as a strong constraint:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial \rho}{\partial z}w \qquad (2.3.4)$$

IV) Even though the case was chosen in order to have extended precipitation coverage over the analyzed domain, a smoothness constraint is added to deal with discontinuities in measurements. This constraint minimizes the spatial gradients of the retrieved wind field $\vec{u}$. Equation 2.3.5 is the expression of the smoothness cost function, k for index in space.

$$J_{smooth}=\sum_{x,y,z} weight\cdot\frac{dx^2}{(z_{k+1}-z_{k-1})^2}\left(\frac{\vec{u}_{k+1}-\vec{u}_k}{z_{k+1}-z_k}-\frac{\vec{u}_k-\vec{u}_{k-1}}{z_k-z_{k-1}}\right)^2 \qquad (2.3.5)$$

3. VERIFICATION AND RESULTS

The Vertically-pointing X-band radar of McGill is located at ground near the S-band, and provides high-resolution data in height and time. It measures the vertical Doppler velocities (Figure 3), which is the resultant of vertical air motion acting on snow particles, falling under gravitational force. It also measures the reflectivity, that is, the sixth moment of the particle melted size distribution.

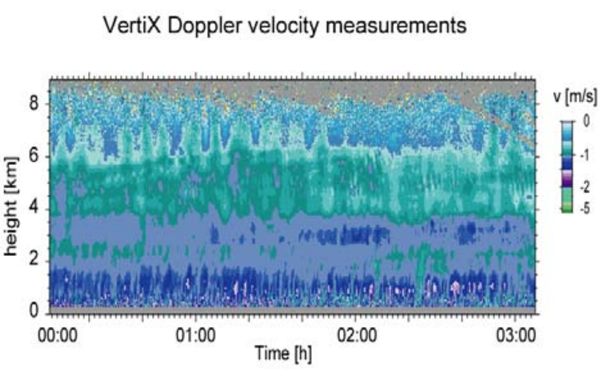

Figure 3. Time series of the vertical Doppler velocities (m/s), as measured by the VertiX. Note the presence of gravity waves, appearing as time periodicity as well as a vertical periodicity suggesting large scale of vertical motions.

Snow particles fall at a speed that is dictated by their mass and microphysical properties, which can be estimated from the reflectivity and particle size distribution. Various empirical relationships that express the mass *M* of snow particles and its fall speed as a power function of the snow diameter D_{snow} can be found in literature. The general form of these power laws is as follows:

$$M(D)=a_m {D_{snow}}^{b_m} \qquad (3.1)$$

$$V_{f\ snow}(D)=a_u {D_{snow}}^{b_u} \qquad (3.2)$$

Assumptions on the particle size distribution allow to relate mass to the radar reflectivity, and to find an empirical relationship between the mass of snow particles and their fall speed as a function of height, based on measurements of their fall speed at the ground (Equations 3.3-3.4, Szyrmer et al., 2009). Szyrmer and Zawadzki (2009) give an equation for the radar reflectivity-weighted fall speed V_{f_radar} (in SI units) as a function of reflectivity and snow mass, for non-rimed snow:

$$V_{f\_radar}=\frac{\int_D V_{f\,snow}(D)Z(D)dD}{\int_D Z(D)dD}$$

$$=[3.65+0.004835Z_{dBZ}+1.204\log(a_m)]\cdot\sqrt{\frac{\rho_0}{\rho}} \qquad (3.3)$$

$$\log(a_m)=-2.92+0.00558a_u \qquad (3.4)$$

$$a_u=(0.67\pm0.049)+(4.6\pm1.2)\cdot10^{-5}H+(0.006\pm0.003)(T-273) \qquad (3.5)$$

This reflectivity-weighted fall speed is measured combined with the vertical air motion, and subtraction of this component from the VertiX velocity measurements gives the vertical air motion.

Profiles of vertical air motion, averaged over space and over the first two hours 00-02 UTC (overlapping time period with the aircraft measurements), are shown in Figure 4. The timely averaged profile of the synoptic pattern, retrieved from conservation of potential temperature, is encouragingly similar to the vertical wind from averages of VertiX data, representative of the large scales. Unlike these two latter techniques, which both use very localized measurements, the vertical wind from integration of horizontal divergence is largely dependent of the analyzed region with respect to the radar. The fact that the volume scans are sampled both in time and space, and the likely underestimation of the echo-top height from the S-band measurements, are factors that prevent us from having a fair comparison with the results of the two previous techniques. The average over 40 to 50km in range, however, should be comparable to the profile of average vertical motion retrieved from the variational method analysis, which uses as well the radial wind of the S-band and the anelastic continuity equation as constraints, and which result was averaged over the same space domain.

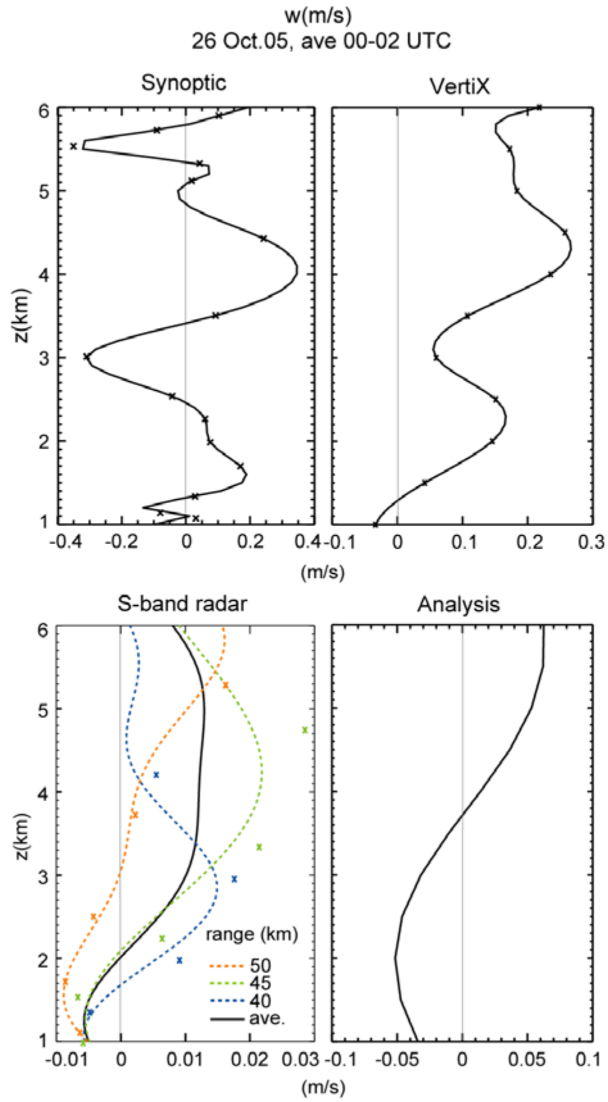

Figure 4. Average vertical air motion obtained by, respectively, conservation of potential temperature, subtraction of the fall speed of particles from VertiX measurements, integration of horizontal divergence, and by the variational method. The two top ones are time averages over 00-02 UTC, of narrow air columns close to the scanning radar; the two below are averaged for the same time period, and also averaged over the domain of 40-50km in range, with respect to the S-band radar.

The resulting radial wind field from the variational analysis, calculated from its control variables (u,v,w), captures well the interpolated observations (Figure 5). Since no additional constraint was given on the vertical motion, it is derived from the horizontal wind field of the analysis, obeying the air mass conservation. This method of analysis is an alternative to the VVP (Volume Velocity Processing) method, solving for a linear wind field that has the radial wind observed by the volume-scanning radar.

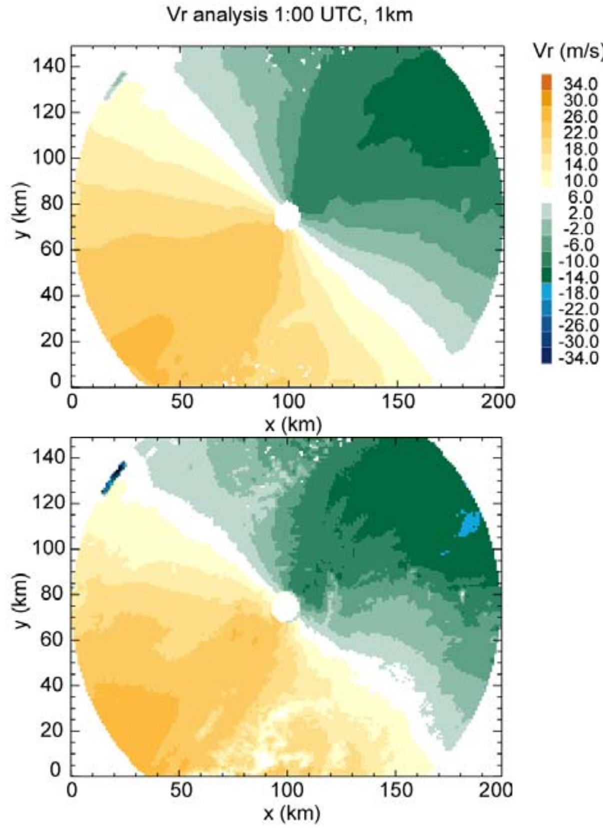

Figure 5.
Top: Radial wind field retrieved from variational analysis.
Bottom: Radar radial velocity observations, interpolated to the 1-km height level, at 01:00 UTC of 26th of October 2005, over the analysis domain.

4. CONCLUSION

The VertiX measurements are too localized to be representative of the whole domain of the analysis, that is why it should be taken as verification instead of a constraint. The vertical motion derived from energy conservation, in theory, does not have the potential of capturing mesoscale processes, considering the lack of sampling in time and space, and the implied synoptic assumptions. However, its result is very consistent with the profile from VertiX measurements. Both are estimations of the large-scale vertical air motion. Therefore, the results of *w* from conservation of potential temperature could be a good additional background value to the Mesoscale Analysis System. It would be consistent as well with the horizontal wind field measured by the aircrafts, and could be used to initialize *w* in the very simple variational analysis presented in this study.

The variational analysis of the 3D wind field presented above is a system with a unique solution: it could be solved by iteration and analytically. The advantage of the variational method is that it can make use of all the available information, including observations and our physical understanding of the system, as long as they are used properly balanced by their errors and are consistent with each other. However, in order to have a system that allows more degrees of freedom, and really exploit the potential of the variational algorithm, additional constraints will have to be added. The conservation of water mass content is a candidate for future work.

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