13B.3 QUANTIFICATION OF ERROR FOR POLARIMETRIC RAINFALL ALGORITHMS USING THE CP2 10CM RADAR IN SOUTH-EAST QUEENSLAND, AUSTRALIA.

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1. INTRODUCTION AND BACKGROUND

Polarimetric radar is now widely recognised as a tool for measuring the variability in rainfall drop-size distribution and improving estimates of the resultant rain rate (e.g. Ryzhkov et al., 2005; Bringi & Chandrasekar, 2001). However, the accuracy varies as a function of rain rate, with some polarimetric estimates exhibiting high error at low rainrates. Consequently, various methods have been developed to optimise such estimates, typically using decision-tree logic to determine which algorithm to use in specific conditions (e.g. Ryzhkov et al., 2005) or highly complex algorithms based on dropsize distribution characteristics (e.g. Bringi et al., 2004; Brandes et al., 2004).

This paper considers polarimetric estimation of rainfall in Queensland, Australia, suggesting an alternative approach for improving estimation methods. This is investigated using 2DVD disdrometer data and T-matrix scattering calculations before validation against radar and gauge data. Standard algorithms are tuned using the disdrometer data, with a number of methods of combining algorithms investigated in both a traditional decision-tree logic manner and by applying weightings to algorithms based on the inverse of theoretical errors. Results are presented with some comparison to relationships such as the Marshall-Palmer R-Z relation, existing CP2 codes and the Ryzhkov et al. (2005) Oklahoma study.

2. DATA AND METHODOLOGY

The CP2 10cm polarimetric radar is located in Redbank Plains, Australia, at 27°40.0' S, 152°51.5' E, close to Brisbane. Typical volumetric operation uses a maximum range of 142.35km (Figure 1), sampling every 1° of azimuth and 0.15km range. For this study, polarimetric radar data was retrieved from the first scan at 0.5° elevation at six-minute intervals for seven high-rainfall events between February 2008 and February 2009, totalling 20 days or parts thereof and 665 complete half hours of radar data. A simple quality control procedure was applied, which removed all data with ρ_{HV}<0.8 or $\sigma(\varphi_{DP})$>10 over ten range gates, K_{DP}>5°.km^{-1} or Z_{DR}>5dB. Due to noisiness of data, variables are first-order smoothed over three range gates and the two adjacent azimuths prior to further analysis.

The data is supplemented by 30-minute rainfall totals from a dense tipping-bucket rain-gauge network (figure 1), with 293 stations within the range covered by the radar, concentrated to the east along the coastline, including ten high-resolution AWS stations which also record rainfall at 1 minute intervals. Drop size distributions (DSDs) are also available from a 2D video disdrometer located at 63° azimuth, 16.6km range, which additionally gives information on the shape and orientation of individual drops (for example Schönhuber et al. 2008). The DSDs were used to determine the mass-weighted median drop size (D_M), the total number of droplets (N_T) and rain rate. Simulated radar variables were derived using T-matrix scattering calculations, assuming (i) normalised gamma distributions, (ii) the oblate approximation of the Beard and Chuang (1987) shapes, (iii) with maximum drop diameter 2.5D_M, (iv) standard deviation of canting angle of 7° and (v) water temperature of 20°C and (vi) for an elevation of 0°. Radar data are compared with ground-based measurements by averaging the radar data over the 7 range gates and two azimuths surrounding the gauge location, and over five 6-minute scans to provide a half-hourly dataset.

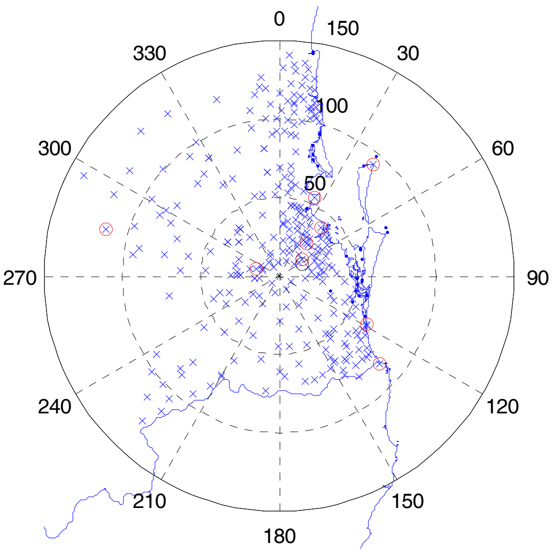

Figure 1. Location of gauges used in this study relative to the CP2 radar, with the Queensland coastline marked. Red circles identify those gauges which record 1-minute data, with a black circle indicating the disdrometer.

Error characteristics will be analysed in terms of the mean and standard deviation of the fractional bias (FB), and the fractional root mean squared error (FRMSE), defined below. To minimise the impact of outliers, a 'trimmed' FRMSE using only data with

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fractional squared error between the 10th and 90th percentiles will also be used.

$$FB_i = \frac{R_i - R_{Ti}}{R_{Ti}} \quad (1)$$

$$FRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{R_i - R_{Ti}}{R_{Ti}}\right)^2} \quad (2)$$

3. RADAR-RAINFALL ALGORITHMS

A total of 18 significant rainfall events which occurred over the disdrometer between 20/11/2008 and 19/02/2009 were used, with a total of 2299 minutes of non-zero disdrometer rain data and derived radar variables. Only 64 measurements exceeded 35mm.hr^{-1}, to a maximum one-minute rain rate of 131mm.hr^{-1}. This data was used to determine optimised constants for each the four standard polarimetric relationships from bootstrapping regression over 1000 samples. The algorithms are given by:

$$Z_h = 200R_D^{1.36} \quad (3)$$

$$R_D(K_{DP}) = 44K_{DP}^{0.8} \quad (4)$$

$$R_D(Z_h, Z_{dr}) = 0.017Z_h^{0.84}Z_{dr}^{-4.47} \quad (5)$$

$$R_D(K_{DP}, Z_{dr}) = 88.9K_{DP}^{0.88}Z_{dr}^{-2.51} \quad (6)$$

When applied to simulated radar variables and compared with the rain rate derived from the same DSD data, each relationship had a mean bias less than ±6% at rainrates between 1mm^{-1} and 35mm.hr^{-1}. The lowest biases were for multiparameter algorithms R(Z_h,Z_{DR}) and R(K_h,Z_{DR}), both of which had FB of +2%. However, the standard deviation of bias varied significantly between methods, reaching 0.55 for R(Z_h) due to several very high outliers, whereas all other algorithms had standard deviations lower than 0.30. Trimmed FRMSE was also low for all algorithms at rainrates <35mm.hr^{-1}, decreasing slightly with rain rate for multiparameter algorithms but increasing slightly for the R-Z relation (figure 2a). In all cases the multiparameter algorithms outperform their single parameter counterparts in the absence of measurement error.

This disdrometer data was then used to simulate real-world radar data by simulating expected measurement errors. This was achieved by applying to each data-point a set of 100 normally-distributed random numbers with mean zero and standard deviation defined by theoretical calculations (as given in Bringi & Chandrasekar 2001) using CP2 characteristics and an assumed $\rho_{HV}(0)$ of 0.98 for rain:

$$\sigma(Z_h) = 0.479Z_h \quad (7)$$

$$\sigma(Z_{dr}) = 0.1360Z_{dr} \quad (8)$$

$$\sigma(K_{DP}) = 0.745^{\circ}.km^{-1} \quad (9)$$

This created a larger dataset of 229900 simulated datapoints to use for validation. Using this data, very similar error characteristics were seen to that anticipated from literature research, with K_{DP}-based algorithms exhibiting high error at low rainrates. R(Z_h) and R(Z_h,Z_{DR}) perform best at rainrates <40mm.hr^{-1}, with FB only +2% for R(Z_h) at these rainrates. Similarly, where R(K_{DP}) and R(K_{DP},Z_{DR}) are most valid, at rainrates >40mm.hr^{-1}, R(K_{DP}) has FB of only +3%. In both cases, the higher measurement error in multiparameter algorithms appears to counteract the lower parameterisation error seen in Figure 2a.

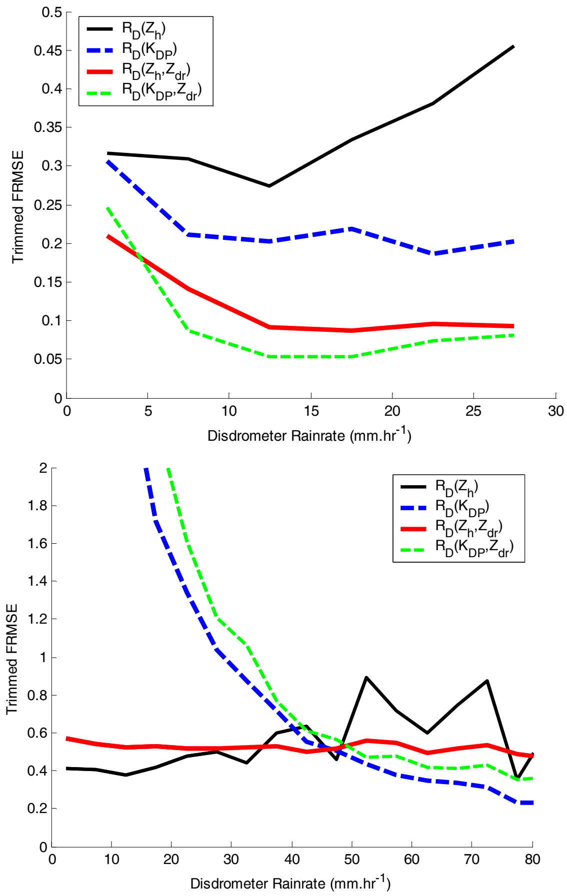

Figure 2. Trimmed FRMSE for disdrometer-derived algorithms using a) 1-minute disdrometer data and b) disdrometer data with propagated theoretical errors, using bins of 5mm/hr.

4. SYNTHETIC ALGORITHMS

4.1 Methodology

4.1.1 Decision-tree logic method

In order to optimise rain rate estimations at all rates, polarimetric algorithms may be combined, generally using simple decision-tree logic. Using disdrometer data, we therefore examined FRMS error after propagation as a function of reflectivity Z, in order to identify which disdrometer-tuned algorithm has the lowest error at each reflectivity and thus determine reflectivity-based decision criteria. For each algorithm we then re-performed a regression as in section 3 utilising only the disdrometer data with reflectivities in the range at which that algorithm will be applied. This allowed us to develop a decision-tree logic based combination algorithm for non-hail cases given by:

$$R_s = 0.0144 Z_h^{0.8474} \quad \text{if } Z_h < 25\text{Dbz} \qquad (10)$$

$$R_s = 0.0131 Z_h^{0.9148} Z_{dr}^{-7.2283} \quad \text{if } 25 \le Z_h < 40\text{dBZ} \qquad (11)$$

$$R_s = 73.5 K_{DP}^{0.968} Z_{dr}^{-1.4238} \quad \text{if } Z_h \ge 40\text{dBZ} \qquad (12)$$

4.1.2 Theoretical error weighting method

In comparison, a potentially more robust combination method is proposed, weighting the algorithms by the inverse of the theoretical error. On an individual point basis, this involves determining the theoretical errors of measurement $\sigma(\varepsilon_M)$, as derived from Bringi & Chandrasekar (2001), and the parameterisation error $\sigma(\varepsilon_P)$. Although traditionally the variance or square error would be used for such calculations, results using standard deviation were generally superior. Parameterisation error can either be treated as zero or represented by the FRMS error as derived from disdrometer data. The best results were found by approximating the derived FRMSE of parameterisation for each algorithm as a linear function of the derived rain rate.

The combined rain rate for each datapoint is then estimated by summing the derived rain rate from each disdrometer algorithm weighted by the inverse of its total error, i.e.

$$R = \sum_{i=1}^{4} \frac{a}{\sigma_i} R_i \text{ , where } \frac{1}{a} = \sum_{i=1}^{4} \frac{1}{\sigma_i} \qquad (13)$$

4.1.3 Discrete error weighting method

A third method of algorithm combination utilised the propagation of error through disdrometer data to combine both the measurement and parameterisation errors. This method used the simulated error dataset to calculate the FRMS errors for each disdrometer algorithm in discrete boxes as a function of one or more radar variables, where a box contained at least five measurements. These errors were used to create a set of matrices of weighting functions in a similar method to 4.1.2, which could then be applied to radar measurements with the correct weighting factors applied dependant on the discrete 'box' the variables fell in.

The best results were found using either all variables (in boxes of 5dBZ Z_H, 0.5mm.hr^{-1}K_{DP} and 0.5dB Z_{DR}) or just reflectivity and specific phase difference (in boxes of 5dBZ Z_H, 0.2mm.hr^{-1}K_{DP}). Box sizes were decided to optimise the range of data represented while remaining robust to variations in real algorithm behaviour with variables. However, the discrete nature of the boxes results in a potential problem at values infrequently sampled in the disdrometer data set, particularly at very high rainrates.

4.2 Validation against disdrometer data

These three methods were applied to both raw disdrometer data and in the presence of simulated error, and compared with the Ryzhkov (2005) decision-tree method, the Bringi et al. (2004) DSD-derived complex method and the Bringi (unpublished) CP2 code.

Using raw disdrometer data at rates less than 35mm.hr^{-1}, the decision-tree logic method tuned by disdrometer data significantly outperformed both the polarimetric algorithms and the similar Ryzhkov and Bringi methods, with a mean bias of -0.2% and a standard deviation of 0.15. Using the Ryzhkov or Bringi decision-tree steps optimised to disdrometer data provided similar results to our method but no improvement (not shown). However, the more complex DSD-tuned method of Bringi et al. (2004) performs fairly well, although still worse than our weighted methods. Both the methods of weighting combination performed similarly at these rates, with low biases (< ±1% for all cases), but significantly higher FRMSE than the decision-tree method (figure 3a).

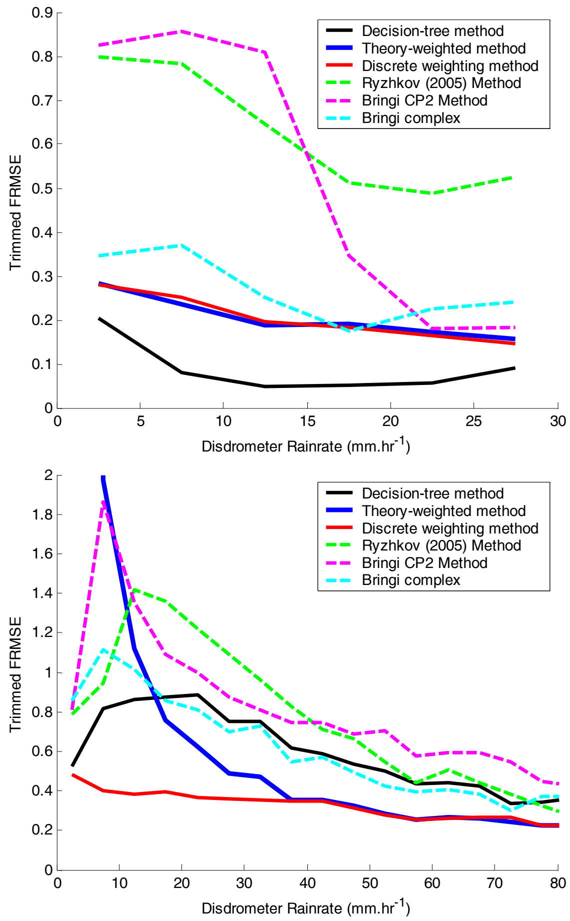

Figure 3. Trimmed FRMSE for combination algorithms using a) 1-minute disdrometer data and b) disdrometer data with propagated theoretical errors, using bins of 5mm/hr.

Under simulated error conditions, combinations optimised to disdrometer data continue to have lower errors than the Ryzhkov or Bringi CP2 methods at most rainrates (figure 3b). However, the FRMS error for the decision-tree method increased substantially, with a mean bias of +50% and high susceptibility to outliers using untrimmed FRMSE (not shown), performing generally similar to or worse than the Bringi complex DSD-based method. The method weighted by theoretical error also exhibits rapidly increasing errors at low rainrates, with neither outperforming the disdrometer-derived $R(Z_h)$ relationship at rainrates < 25mm.hr^{-1}. In comparison, the method using discrete weightings exhibits consistently low trimmed FRMS error at all rainrates, with biases remaining below ~±10% after propagation for all combinations except (Z_{DR},K_{DP}).

5. VALIDATION AGAINST CP2 DATA

5.1. Disdrometer-tuned algorithms

The disdrometer-tuned algorithms were then applied to the filtered radar data and compared with the Bureau of Meteorology gauge network for all rainrates exceeding 1mm.hr^{-1}, below which tipping bucket gauges have poor resolution. These were compared to the standard Marshall-Palmer R-Z relation, and the polarimetric algorithms found most accurate in a study by Ryzhkov et al. (2005) (Table 1).

Table 1: Mean FB and trimmed FRMSE (where R≥1mm.hr^{-1}) for disdrometer-tuned algorithms and literature algorithms of similar types.

	FB		FRMSE	
	Disd	Lit	Disd	Lit
$R(Z_h)$	-0.08	-0.30	0.60	0.59
$R(K_{DP})$	0.94	2.13	1.13	2.19
$R(Z_h,Z_{dr})$	-0.00	0.80	0.59	1.01
$R(K_{DP},Z_{dr})$	1.63	0.19	1.76	0.66

The fractional mean bias for the disdrometer-tuned $R(Z_h)$ remained very low, with -8% compared to -30% for the Marshall-Palmer method, indicating an improved representation via tuning. However, the standard deviation of FB was significantly higher for the disdrometer-tuned algorithm, 1.4 compared to 0.8, indicating a strong influence of high outliers. This resulted in significantly worse FRMSE for the disdrometer-tuned method at all rainrates, but fairly similar values for trimmed FRMSE (figure 4a), the Marshall-Palmer method only superior at rainrates less than 10mm.hr^{-1}.

In comparison, the disdrometer-tuned $R(K_{DP})$ and $R(Z_h,Z_{dr})$ algorithms were found to perform significantly better than their literature counterparts, with FRMSE approximately halved and low bias, nearly nil for $R(Z_h,Z_{dr})$. However, the Ryzhkov (2005) $R(Z_h,Z_{dr})$ algorithm has lower trimmed FRMSE than the disdrometer-tuned version at rates between 5 and 20mm.hr^{-1} (figure 4a). The disdrometer-tuned $R(K_{DP},Z_{dr})$ appears to have significantly higher bias than its literature counterpart; however, this is due to very high error at low rates, and at rates exceeding 10mm.hr^{-1} this exhibits significantly lower FRMSE than its literature counterpart (figure 4b), with a mean FB of just -9%.

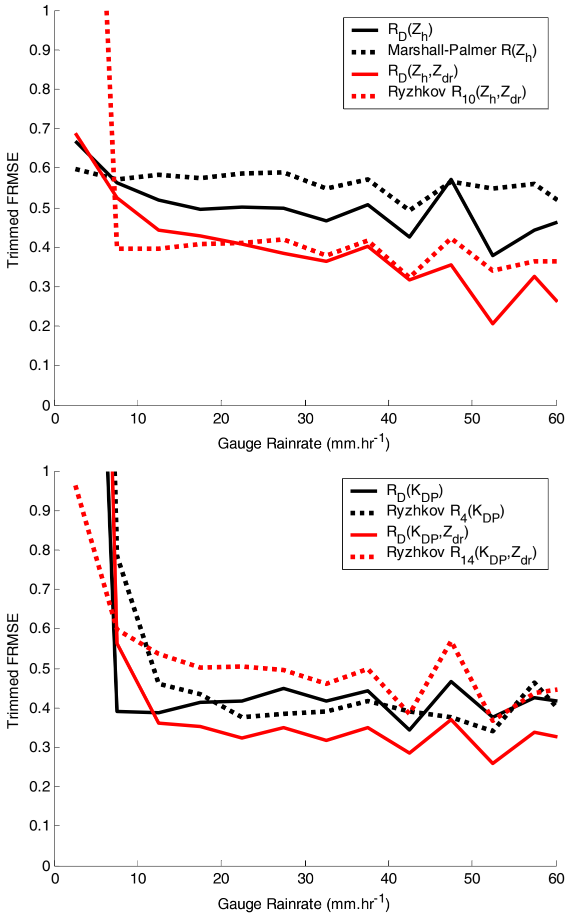

Figure 4. Trimmed FRMSE error where R>=1 using radar and gauge data for both disdrometer and literature algorithms by type: a) Z_h-based algorithms b) K_{DP}-based algorithms, with dotted lines representing the literature version.

5.2. Combination algorithms

We also applied all of the combination algorithms specified in section 4 to the same CP2 data. Of the decision-tree methods, the lowest overall error was found for that tuned against disdrometer data, so this will be used to represent this methodology. However, tuning the Bringi CP2 methodology to disdrometer data outperformed our method at rates >=30mm.hr^{-1} by ~10%, although performing worst of all the algorithms at lower rainrates, indicating some combination of these may be viable. The Bringi et al. (2004) complex methodology will also be considered as an alternate methodology.

Using all rainrates in excess of 1mm.hr^{-1}, the decision-tree combination method has a very low fractional bias of +3%, significantly better than the bias of +24% for the theory weighted method using linear regression, the best of these methods, or the Bringi et al. (2004) method. However, the decision-tree method is also strongly affected by outliers, with σ(FB) of 1.45, compared to 1.25 for the theory weighted method, leading to significantly higher FRMSE at all rainrates. Of the discretely weighted algorithms, the lowest FB was found for the Z-K combination, at -4%. However, the lowest standard deviation was found for the three-dimensional combination at just 1.00, leading to very similar values for FRMSE of ~1 but potentially different error characteristics.

However, with the removal of outliers using trimmed FRMSE the errors were fairly consistent between our derived methods, all having trimmed FRMSE within ±0.02 of 0.55 (table 2). This is too insignificant to indicate superiority of any method, indicating that the predominant difference between methods is in their sensitivity to outliers, with the discrete weighted method showing significantly better untrimmed FRMSE. Trimming also appeared to change the mean bias by ~ -20% for all algorithms, indicating that the majority of outliers were due to overestimation in each case. The smallest trimmed bias is +0% for the Bringi et al. (2004) method; however, this method has a significantly higher trimmed and untrimmed FRMSE. This suggests that although this method should represent a better representation of actual dropsize characteristics, it appears more sensitive to measurement error than our weighted methods.

Table 2: Errors using radar data (where R≥1mm.hr^{-1}) for two combination algorithms.

	Untrimmed		Trimmed	
	FB	FRMS	FB	FRMS
Decision-tree	0.03	1.4	-0.16	0.57
Theory Weight	0.24	1.3	0.04	0.54
Discrete (Z,K)	-0.04	1.1	-0.20	0.55
Discrete (All)	-0.09	1.0	-0.24	0.55
Bringi (2004)	0.22	1.6	+0	0.67

The theoretically weighted method has the lowest trimmed FRMSE at all rainrates greater than 10mm.hr^{-1} but poorer estimation at low rainrates, probably due to overrepresentation of K_{DP}-based algorithms at these ranges (figure 5). This is closely followed by the discretely weighted method using Z_h and K_{DP}, while error in the decision-tree method increases at high rainrate. However, the variation between the two weighting methodologies is low, and all three methods have lower trimmed FRMSE than the Bringi et al. (2004) methodology and all disdrometer algorithms for R>=1mm.hr^{-1}.

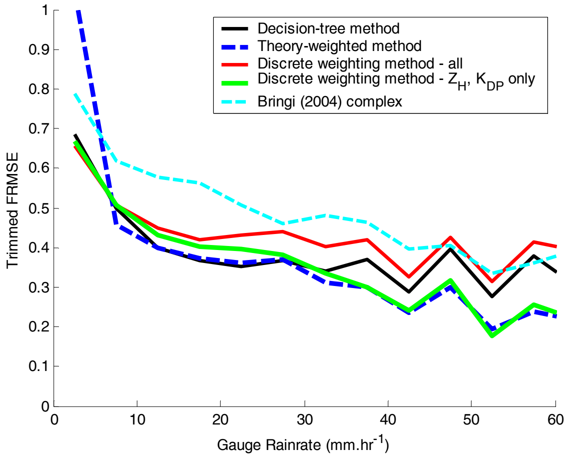

Figure 5. Trimmed FRMSE where R>=1 using radar and gauge data for five methods of synthetic algorithm combination.

6. DISCUSSION AND CONCLUSION

An investigation of a number of methods of rainfall determination from polarimetric radar data has reiterated the known error characteristics of various polarimetric radar-rainfall relations as a function of rain rate. Parameterisation of algorithms to disdrometer data for a specific location has been shown to generally produces superior results to algorithms derived from literature, due to the strong variability of rainfall dropsize characteristics in various areas. However, the presence of measurement and comparison errors mean this is not necessarily constant at all rainrates.

Several methods of combining algorithms to create a more robust estimation have also been attempted. All of these have slightly lower trimmed FRMSE than individual polarimetric algorithms or two decision tree methods derived from other studies. However, accuracy is poor at very low rainrates, indicating a continued need for simple Z-R relations at these rates. Nonetheless, using weighted averages of algorithms is found to consistently outperform decision-tree logic for both trimmed and untrimmed error, although it is yet unclear whether the discrete or theoretical weightings are most accurate. These algorithms also outperform a more complex DSD-based algorithm derived by Bringi et al. (2004), to a greater extent with radar data than disdrometer-derived, indicating this may be more susceptible than measurement error. However, this may be a consequence of the applicability of assumed DSD relationships to this area and warrants further study. These results indicate that more robustly weighted algorithm combinations may be a superior method for rainfall estimation at rates exceeding 5mm.hr^{-1}.

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