1. Introduction

This paper describes the procedures of recombining super resolution dual polarization data into "legacy" (1 degree) resolution data. Because the recombination will be done on spectral moment data, it is natural to recombine the polarimetric data in the same manner for processing by the algorithms on the RPG and recording. The Open Radar Data Acquisition (ORDA) will produce the following polarimetric variables: reflectivity factor at horizontal polarization $Z_h$, differential reflectivity $Z_{DR}$, cross-correlation coefficient $\rho_{hv}$, and differential phase $\Phi_{DP}$. The reflectivity factor will be computed from the power at horizontal polarization after subtracting the receiver noise power. Similarly, the differential reflectivity and cross-correlation will be computed from the powers after elimination of the noise contribution. Thus these quantities will not be biased by noise. We are using these unbiased variables in the functional description herein.

2. Inputs and outputs

**Input variables (at super resolution):**
- $Z(i,j)$: Censored and quantized reflectivity at gate $i$ and beam $j$ in dBZ;
- $Z_{dr}(i,j)$: Censored and quantized differential reflectivity at gate $i$ and beam $j$ in dB;
- $\rho_{hv}(i,j)$: Censored and quantized correlation coefficient at gate $i$ and beam $j$;
- $\phi_{dp}(i,j)$: Censored and quantized differential phase at gate $i$ and beam $j$;
- $N_h$: Noise power in the horizontal channel in internal processor unit;
- $N_v$: Noise power in the vertical channel in internal processor unit;

**Output variables (at legacy resolution):**
- $Z_{c}(i,jc)$: Quantized and recombined reflectivity at gate $i$ and beam $jc$;
- $Z_{dr}(i,jc)$: Quantized and recombined differential reflectivity at gate $i$ and beam $jc$;
- $\rho_{hv}(i,jc)$: Quantized and recombined correlation coefficient at gate $i$ and beam $jc$;
- $\phi_{dp}(i,jc)$: Quantized and recombined differential phase at gate $i$ and beam $jc$.

3. Procedures

Herein we present the functional description of the recombination procedure. Note that if $\phi_{dp}$ is folded at super resolution, unfolding $\phi_{dp}$ is needed before the recalculation of complex covariance.

1) Recalculate powers $P_h(i, j)$ and $P_v(i, j)$, signal-to-noise ratios $snr_h(i, j)$ and $snr_v(i, j)$ in horizontal ($h$) and vertical ($v$) channels, and complex covariance $reR_{hv}(i, j)$ and $imR_{hv}(i, j)$ from dual polarization level II data at super resolution.

\[
\text{snr}_h(i, j) = 10^{0.1(Z(i, j) - C + \text{Att} \times R - 20 \log R)},
\]

\[
P_h(i, j) = N_h \times \text{snr}_h(i, j),
\]

\[
P_v(i, j) = \frac{P_v(i, j)}{10^{0.1Z_{dr}(i, j)}},
\]

\[
\text{snr}_v(i, j) = P_v(i, j) / N_v,
\]

\[
reR_{hv}(i, j) = \rho_{hv}(i, j) [P_h(i, j)P_v(i, j)]^{1/2} 
\cos(-\phi_{dp}(i, j) \times \pi / 180),
\]

\[
imR_{hv}(i, j) = \rho_{hv}(i, j) [P_h(i, j)P_v(i, j)]^{1/2} 
\sin(-\phi_{dp}(i, j) \times \pi / 180).
\]
2) Recombine them back at 1° beamwidth

\[ P_{hc}(i, jc) = 0.5 \times (P_h(i, j) + P_h(i, j+1)), \]  
\[ P_{vc}(i, jc) = 0.5 \times (P_v(i, j) + P_v(i, j+1)), \]

\[ reR_{hvc}(i, jc) = \frac{reR_h(i, j) + reR_h(i, j+1)}{2}, \]  
\[ imR_{hvc}(i, jc) = \frac{imR_h(i, j) + imR_h(i, j+1)}{2}, \]

where \( jc \) is the index for recombined beams.

Rules for recombination of missing data with valid data:
For powers in horizontal channel:
According to “Super Resolution Base Data Recombination Algorithm”, a non-zero value PBG is given as

\[ PBG = 10^{0.1 \times (10 \log_{10} (0.7) + 10 \log_{10} (Nh) + THZ)}, \]

where THZ is the censoring threshold for reflectivity.

a) If both \( P_h(i, j) \) and \( P_h(i, j+1) \) are missing, then \( P_{hc}(i, j+1) \) is set to be missing.

b) If \( P_h(i, j) \) or \( P_h(i, j+1) \) is missing, then

\[ P_{hc}(i, jc) = 0.5 \times (PBG + P_h(i, j+1)), \]

or

\[ P_{hc}(i, jc) = 0.5 \times (P_h(i, j) + PBG), \]

where recombined power \( P_{hc} \) in horizontal channel is for the calculation of recombined reflectivity only.

For dual-polarization variables, the rules are

a) If \( P_h(i, j) \) is missing and \( P_h(i, j+1) \) is valid, then \( P_{hc}(i, j+1) \) is equal to \( P_h(i, j+1) \);

b) If \( P_h(i, j) \) is valid and \( P_h(i, j+1) \) is missing, then \( P_{hc}(i, j) \) is equal to \( P_h(i, j) \);

c) If \( P_v(i, j) \) is missing and \( P_v(i, j+1) \) is valid, then \( P_{vc}(i, j+1) \) is equal to \( P_v(i, j+1) \);

d) If \( P_v(i, j) \) is valid and \( P_v(i, j+1) \) is missing, then \( P_{vc}(i, j) \) is equal to \( P_v(i, j) \).

The rules for real and imaginary parts of correlation function are:

a) If \( reR_{hv}(i, j) \) is missing and \( reR_{hv}(i, j+1) \) is valid, then \( reR_{hvc}(i, j) \) is equal to \( reR_{hv}(i, j+1) \);

b) If \( reR_{hv}(i, j) \) is valid and \( reR_{hv}(i, j+1) \) is missing, then \( reR_{hvc}(i, j) \) is equal to \( reR_{hv}(i, j) \);

c) If \( imR_{hv}(i, j) \) is missing and \( imR_{hv}(i, j+1) \) is valid, then \( imR_{hvc}(i, j) \) is equal to \( imR_{hv}(i, j+1) \);

d) If \( imR_{hv}(i, j) \) is valid and \( imR_{hv}(i, j+1) \) is missing, then \( imR_{hvc}(i, j) \) is equal to \( imR_{hv}(i, j) \).

Time series data (in phase I, and quadrature phase Q) were obtained with the KOUN radar as a large mesoscale convective system was passing by the radar site on June 29, 2007. This versatile data contains echoes in clear air (from insects), stratiform precipitation, and convective comprised of few growing cells, few decaying, and an active squall line.

Classification field generated by using all the radar variables is the best to show the results of recombination. It is displayed in Fig. 1 with classification based on legacy resolution data. As expected, the main feature in the classified field based on recombined data does not show significant difference from the one base on legacy data. It demonstrates that recombination procedure does not essentially affect the classification results.
3) Recalculation of radar variables:

Calculate recombined Signal-to-Noise Ratio $SNR_{hc}$ and $SNR_{vc}$ in dB as follows:

$$SNR_{hc}(i, jc) = 10\log(P_{hc}(i, jc)/N_h),$$  \hspace{1cm} (11)

$$SNR_{vc}(i, jc) = 10\log(P_{vc}(i, jc)/N_v).$$  \hspace{1cm} (12)

$$Z_{c}(i, jc) = C + 20\log(R(i)) + SNR_{hc}(i, jc).$$  \hspace{1cm} (13)

Note that recombined differential reflectivity $Z_{drc}(i, jc)$ will be calculated only if both $P_{hc}(i, jc)$ and $P_{vc}(i, jc)$ are not missing, otherwise these variables are set to be missing. Recombined correlation coefficient $\rho_{hvc}(i, jc)$ will be calculated only if $P_{hc}(i, jc)$, $P_{vc}(i, jc)$, $reR_{hvc}(i, jc)$, and $imR_{hvc}(i, jc)$ are not missing, otherwise these variables are set to be missing. Recombined differential phase $\phi_{dpc}(i, jc)$ will be calculated only if both $reR_{hvc}(i, jc)$ and $imR_{hvc}(i, jc)$ are not missing, otherwise these variables are set to be missing.

Recombined differential reflectivity $Z_{drc}$ can be obtained as follows:

$$Z_{drc}(i, jc) = 10\log\left(\frac{P_{hc}(i, jc)}{P_{vc}(i, jc)}\right).$$  \hspace{1cm} (14)

Then recombined correlation coefficient $\rho_{hvc}$ and differential phase $\phi_{dpc}$ are calculated based on the followed equations:

$$\rho_{hvc}(i, jc) = \frac{\left(reR_{hvc}^2(i, jc) + imR_{hvc}^2(i, jc)\right)^{1/2}}{\left((reR_{hvc}^2(i, jc) + imR_{hvc}^2(i, jc))\right)^{1/2}}$$  \hspace{1cm} (15)

$$\phi_{dpc}(i, jc) = -\frac{180}{\pi} \tan^{-1}\left(\frac{imR_{hvc}(i, jc)}{reR_{hvc}(i, jc)}\right)$$  \hspace{1cm} (16)

Rules for missing data:

a) If $P_{hc}(i, jc)$ or $P_{vc}(i, jc)$ is missing, then $Z_{drc}(i, jc)$ is set to be missing.

b) If $reR_{hvc}(i, jc)$ or $imR_{hvc}(i, jc)$ is missing, then $\rho_{hvc}(i, jc)$ is set to be missing.

c) If one of $reR_{hvc}(i, jc)$, $imR_{hvc}(i, jc)$, $P_{hc}(i, jc)$, and $P_{vc}(i, jc)$ is missing, then $\rho_{hvc}(i, jc)$ is set to be missing.

4) Quantization of recombined variables:
The quantization of recombined radar variables $V$ is performed based on the following formula:

$$V_{integer} = \text{round}(V\times scale + offset),$$  \hspace{1cm} (17)

$$V_{quantized} = \frac{(V_{integer} - offset)}{scale},$$  \hspace{1cm} (18)
where the values of scale and offset for different radar variables are listed in the table.

Table 1: Values of scale and offset for different radar variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$Z_c$</th>
<th>$Z_{drc}$</th>
<th>$\phi_{dpc}$</th>
<th>$\rho_{hvc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scale</td>
<td>2.0</td>
<td>16.0</td>
<td>2.8361</td>
<td>300.0</td>
</tr>
<tr>
<td>offset</td>
<td>66.0</td>
<td>128.0</td>
<td>2.0</td>
<td>-60.0</td>
</tr>
</tbody>
</table>

4. Comparison between recombined variables and variables in legacy resolution

The legacy and super resolution data sets we processed are generated from time series data observed by KOUN at 0216 UTC on 29 June 2007. 17 samples are used to produce the powers in horizontal and vertical channels and complex covariance in both legacy and super resolution mode. Rectangular window is applied to collect legacy resolution data. For super resolution data, von Hann window is used. The azimuthal layout of data collection is shown in Fig.2.

In legacy resolution, pulse #17 coincides with pulse #1 of next radial. Meanwhile, in super resolution, the centers of adjacent radials are at pulses #5 and #13.

Comparing the fields in the super resolution with legacy resolution (not shown), some fine features can be easily found in the super resolution, especially in the reflectivity field. On the other hand, super resolution fields are noisier than legacy resolution fields.

Following the procedures described in the first section, the recombined variables are calculated. Then they are compared with the variables in legacy resolution. The differences between recombined and legacy resolution variables are estimated at the gate with valid observation. The histograms of the differences ($V_{\text{recombined}} - V_{\text{legacy resolution}}$) of radar variables between recombined and legacy resolution are displayed in Fig.3. Here the letter "V" represents the radar variable. The average differences of the radar variables over the entire tilt at 0.5° elevation angle are shown in Table 2.

The recombined reflectivity, differential reflectivity, correlation coefficient, and differential phase are displayed with corresponding variables in legacy resolution in the Fig.4. It can be seen that the features in both fields are similar. Combined with results in Fig.3 and Table 2, we conclude that the recombination does not distort or bias the fields.
Fig. 2. Layout of data processed in LegacyResolution (NR) and SuperResolution (SR) modes. (Upper panel): Azimuthal layout of data collected with the rectangular window in NR and the vonHann window in SR (semi arcs in the figure). Azimuthal layout is shown for azimuth from 0° to 2°. (Lower panel): Radar pulse layout for the number of samples M = 17.

Table 2: Average difference ($V_{\text{recombined}} - V_{\text{legacy resolution}}$) for tilt at 0.5° elevation angle.

<table>
<thead>
<tr>
<th>Average Differences</th>
<th>Reflectivity (dBZ)</th>
<th>$Z_{\text{dr}}$(dB)</th>
<th>$\rho_{hv}$</th>
<th>$\phi_{dp}$(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over a tilt</td>
<td>-0.024</td>
<td>-0.044</td>
<td>0.0067</td>
<td>-0.16</td>
</tr>
</tbody>
</table>
Fig. 3. Histograms of differences between recombined and legacy (a) reflectivity, (b) differential reflectivity, (c) correlation coefficient, and (d) differential phase at each gate with valid observations.

5. Conclusion and discussion

A functional description of how to recombine super resolution (0.5 deg) polarimetric level II data into regular (legacy 1 deg) resolution is presented. Then the classification algorithm has been applied on these data. Comparing the classification results, it demonstrates that it is essentially not affected by the recombination procedure. This is because there is no substantial difference between polarimetric data computed in the super resolution and legacy modes. Nonetheless, testing was done on one data set hence it is premature to accept this finding in general. Additional testing should be made on a variety of weather radar data.

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Fig. 4. Recombined reflectivity (a), differential reflectivity (c), correlation coefficient (e), and differential phase (g) are on left side. Reflectivity (b), differential reflectivity (d), correlation coefficient (f), and differential phase (h) in legacy resolution are on the right side.