P5.10 GENERALIZED SZ PHASE CODES TO MITIGATE RANGE AND VELOCITY AMBIGUITIES

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1. INTRODUCTION

It is well known that for Doppler radars transmitting uniformly spaced pulses there is a coupling between the maximum unambiguous range and velocity. That is, one can only be increased at the expense of a proportional decrease of the other. Because this fundamental limitation hinders observation of severe weather phenomena, the Radar Operations Center of the US National Weather Service has undertaken the implementation of evolutionary signal processing techniques to mitigate the effects of velocity and range ambiguities on the NEXRAD network. The first technique that was targeted for operational implementation is referred to as Sachidananda-Zrnić (SZ)-2 and has been in use since 2007.

The SZ-2 algorithm is based on systematic phase coding of the transmitted pulses with the SZ(8/64) code (Torres 2005). Although the SZ(8/64) phase code results in a very effective recovery of weak overlaid signals, it leads to optimum performance only if the overlaid signal trip numbers differ by one. However, in the current operational implementation of the SZ-2 algorithm, overlaid signals can exhibit trip differences of up to three.

This paper introduces a family of systematic phase codes of the form SZ(n/64). A closer look into the performance of these generalized codes reveals a number of omissions in the early research work. Further, no single code is optimum for all overlay cases, and, surprisingly, the best overall phase code in the SZ family is not the SZ(8/64).

2. THE PHASE CODING TECHNIQUE

In the phase coding technique, the transmitted pulses are phase shifted using a systematic code sequence given by $\psi(m)$, where $m = 0, 1, \ldots, M-1$. If received echo samples are multiplied by $exp[-j\psi(m-k+1)]$, intrinsic phases of the signal from trip k are restored. Consequently, the k-trip signal is made coherent and out-of-trip overlaid signals are phase-modulated by the code $\psi(m-k'+1) - \psi(m-k+1)$, where k' is the trip number of the overlaid signal. In general, any one of the overlaid trip signals can be cohered leaving the rest modulated by different codes. This is the fundamental principle behind these techniques.

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2.1. SZ Phase Codes

Sachidananda and Zrnić (1999) proposed the SZ phase code as a better alternative to random codes (e.g., Laird 1981). SZ phase coding is similar to random phase coding except that the transmitted pulses are phase-modulated with a systematic code consisting of M phases that repeat periodically. These codes exhibit properties that make them attractive for the separation of overlaid signals in the spectral domain. That is, if the received signal is cohered for a given trip, the spectra of all out-of-trip echoes are split into evenly spaced replicas and have zero lag-one autocorrelation. Hence, out-of-trip echoes do not bias the mean Doppler velocity estimate of the coherent signal. Once the velocity is recovered for the strong-trip, the coherent signal is notched out such that the two least contaminated "replicas" of the out-of-trip (i.e., the weak trip) echo remain. These two replicas are sufficient to reconstruct (or "recohere") the weak-trip echo and recover its mean Doppler velocity. From the family of SZ(n/M) codes, the SZ(8/64) code was selected for NEXRAD as it gives the best performance in terms of recovery of overlaid signals that are separated by one trip (Sachidananda et al. 1998).

2.2. The SZ-2 Algorithm

Recovery of strong and weak trip overlaid signals can proceed in a stand-alone manner (referred to as the SZ-1 algorithm) or with the aid of an extra scan at the same elevation angle using a long pulse repetition time (PRT) (referred to as the SZ-2 algorithm). Although the latter results in longer acquisition times due to the extra scan, long-PRT data provides non-overlaid power information that is essential in the determination of the location and strength of overlaid trips for the short-PRT scan. Having the long-PRT information available makes the SZ-2 algorithm computationally simpler and more effective than its stand-alone counterpart. Whereas the long-PRT data provides the reflectivity free of range ambiguities, the short-PRT data is used to compute Doppler velocities associated with the two strongest overlaid signals.

The SZ-2 algorithm, which has been implemented on the US network of weather surveillance radars since the Spring of 2007 (Saffle et al. 2007), incorporates a set of censoring rules to maintain data quality under situations that preclude the recovery of one or more overlaid echoes (Saxion et al. 2007, Ellis et al. 2005). Meteorological data displays characterize this failure by encoding those range locations where overlaid powers are present with a purple color, normally referred to as the "purple haze".

3. GENERALIZED PHASE CODES

As mentioned before, the SZ-2 algorithm is based on the SZ(8/64) phase code, which was deemed optimum in the early stages of this project. However, the methodology used to make this determination did not consider overlay situations with trip differences of more than one. With the current implementation of the SZ-2 algorithm, overlaid signals can exhibit trip differences of one, two, or three. Hence, it is natural to question whether the assessment done using only one overlay case still holds when we allow other overlay cases to occur. The main motivation for this work is the need to determine which phase codes might lead to better performance for overlay cases not considered before. In addition, we would like to explore the ability of other phase codes to extend the recovery of weak overlaid echoes to more trips, since the operational SZ-2 algorithm only provides recovery of weak overlaid signals up to four trips. Although this is not a limitation within the NEXRAD network, other radar systems, especially those operating at shorter wavelengths, might benefit from an approach that extends the recovery of overlaid echoes to more trips.

Herein, we look at switching codes in the SZ(*n*/64) family, where *n* is a positive integer. These are of the form

$$\psi(m) = -\sum_{p=0}^{m} \frac{n\pi p^2}{64}, \; m = 0, 1, 2, \ldots \quad (1)$$

These codes are attractive because they exploit the WSR-88D phase shifter resolution to the maximum. That is, because the WSR-88D phase shifter is controlled with 7 bits, its phase resolution is $\pi/64$. Hence, the phase shifter can realize any phase that is an integer multiple of $\pi/64$, and this is the exact same form of the code given in (1).

As with the SZ(8/64) code, the modulation codes for the family of SZ(*n*/64) codes are different for different overlay cases. Without loss of generality, assume that $k = 1$ (the first trip is coherent) and $t = k' - k$ is the trip difference between the modulated and coherent overlaid signals. Hence, the modulation code for an overlay trip difference *t* is given by

$$\phi(m) = \psi(m-t) - \psi(m) = \frac{n\pi}{64} \sum_{l=0}^{t-1} (m-l)^2 \quad (2)$$

which for $t = 1$ (i.e., $k' = 2$, which was the only case analyzed in the previous work) reduces to

$$\phi(m) = \frac{n\pi m^2}{64}. \quad (3)$$

3.1. Periodicity and performance of SZ(n/64) codes

In general, the performance of systematic phase codes is measured by the ability of recovering the velocity of the weaker overlaid signal after removing most of the stronger signal. In Sachidananda et al. (1998), it was established that recovery of weak-trip velocity is possible from at least two replicas of the modulated weak-trip signal. Thus, a contradiction arises. On one hand, a modulation code producing more replicas (i.e., one with shorter periodicity) allows for a wider processing notch filter (PNF) and therefore a more efficient suppression of the strong-trip signal. On the other hand, a modulation code producing fewer replicas (i.e., one with longer periodicity) would result in more accurate weak-trip velocity estimates since less overlap of the weak-trip replicas occurs. It would seem that the periodicity (or the number of replicas) of the modulation code determines its performance in terms of weak-trip velocity recovery. However, it can be shown with a simple counterexample that the performance of these codes is not dictated solely by their periodicity.

Let's first consider the codes SZ(8/64) and SZ(56/64). The spectra of the corresponding modulation codes are shown in Fig. 1, where it is evident that both would lead to the same number of replicas. The performance of these codes in terms of weak-trip velocity recovery is shown in Fig. 1 as the standard deviation of velocity estimates on the power-ratio/strong-trip spectrum width plane for a weak-trip spectrum width of 4 m/s and high signal-to-noise ratios. Evidently, these two codes have the same periodicity and the same performance.

Consider now the codes SZ(8/64) and SZ(24/64). Again, the modulation code spectra and performance charts are shown below in Fig. 2, where it is now obvious that same periodicity does not lead to same performance.

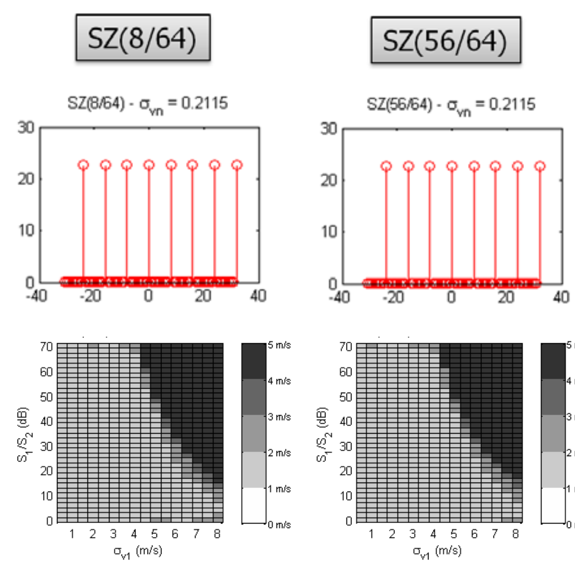

Fig. 1. (top) Spectra of the SZ(8/64) and SZ(56/64) modulation codes. (bottom) Statistical performance of weak-trip recovery corresponding to the SZ(8/64) and SZ(56/64) codes. The plots show the standard deviation of weak-trip velocity estimates as a function of the strong-to-weak trip power ratio and the strong-trip spectrum width. Strong and weak trips differ by one.

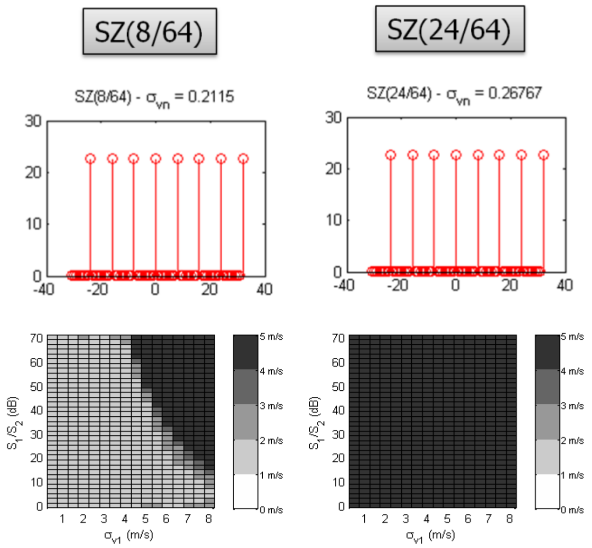

Fig. 2. Same as Fig. 1 for the SZ(8/64) and SZ(24/64) modulation codes.

Although the periodicity of the modulation code plays an important role in the performance of these codes, it is not enough to predict it. The reader might be wondering what is different between the two examples presented above. It is important to remember that weak-trip velocities are recovered after applying the PNF and re-cohering the weak trip signal. So it would make sense to look at the spectra of the modulation codes after the same process. Fig. 3 shows the spectra of the modulation codes after the SZ-2 process for the codes in the examples above. Note that the codes with the same performance have the same code spectrum after notching and re-cohering. This is not the case for the SZ(24/64) code, which, as shown above, does not exhibit the same performance.

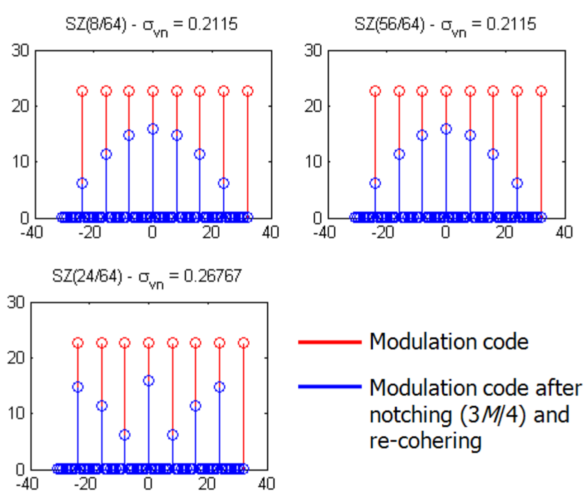

Fig. 3. Spectra of the SZ(8/64), SZ(56/64), and SZ(24/64) modulation codes (red) and same after notching and re-cohering (blue).

Therefore, not all codes with the same period (i.e., leading to the same number of modulated replicas) exhibit the same performance in terms of weak-trip velocity recovery. The performance of a given code depends on the structure of the sidebands after notching and re-cohering. But it is not clear at this time if there is a way to predict the performance of a given code based on its sideband structure.

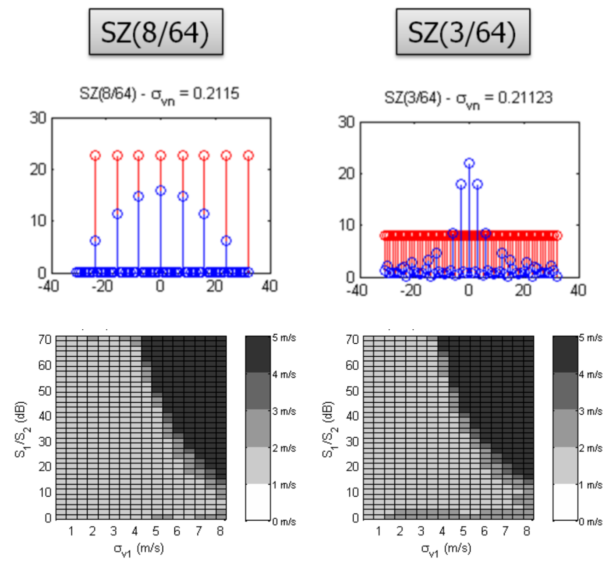

Fig. 4. Same as Fig. 1 for the SZ(8/64) and SZ(3/64) modulation codes.

The previous examples showed codes with the same periodicity and different performance. Are there codes with the same performance but different periodicity? Consider now the SZ(8/64) and SZ(3/64) codes. These codes have a periodicity of 8 and 64, respectively. Although the periodicity of these codes is very different, their performance in terms of weak-trip velocity recovery is very similar! (see Fig. 4) This example reinforces the idea that the performance of systematic phase codes is not uniquely related to the number of spectral “replicas” (or periodicity) of the code. In other words, as the modulation code exhibits more “replicas”, the performance in terms of weak-trip velocity recovery does not necessarily get worse as previously suspected. Another consideration is that the PNF width must be tailored to the specific code and cannot be designed with the idea of retaining spectral replicas since this concept of “replicas” stops working for longer code periodicities (i.e., when the number of “replicas” increases with respect to the normalized spectrum width of the modulated signal).

3.2. Performance of SZ(n/64) codes

Next, simulations are used to evaluate the performance of this family of codes in a systematic way. Once again, performance is gauged in terms of weak-trip velocity recovery, which depends on the switching code and the PNF width. The performance for any given code-PNF width combination is quantified in terms of the size of the “recovery region”. That is, on the power ratio vs. strong-trip spectrum width plane, we count the number of cases for which the standard deviation of weak-trip velocity estimates is less than 2 m/s for a true weak-trip spectrum width of 4 m/s (see Fig. 5). Note that the relaxed 2 m/s error benchmark reflects the recently established requirements for weak-trip velocity estimates obtained with the SZ-2 algorithm.

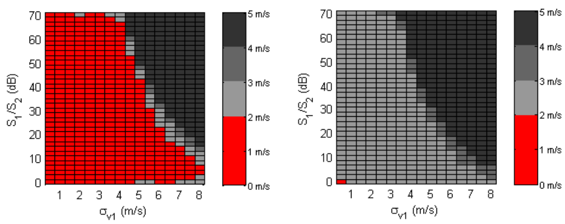

Fig. 5. Examples of good (left panel) and bad (right panel) phase code-PNF width combinations in terms of weak-trip recovery.

The simulation tested all codes in the SZ(n/64) family with two overlaid echoes and trip differences ranging from one to four. For each case, the PNF width was varied from 25% to 75% of the Nyquist co-interval. Signal parameters were varied as follows: the strong-to-weak signal overlaid ratio from 0 to 70 dB in steps of 2 dB; the strong-trip spectrum width from 0.5 to 8 m/s in steps of 0.5 m/s, and the overlaid signal velocities were chosen randomly in the Nyquist co-interval for each realization. The number of samples was M = 64, the weak-trip spectrum width was fixed at 4 m/s, the radar frequency was f = 2.8 GHz, the PRT was T = 780 μs, and the signal-to-noise ratio was high (more than 20 dB).

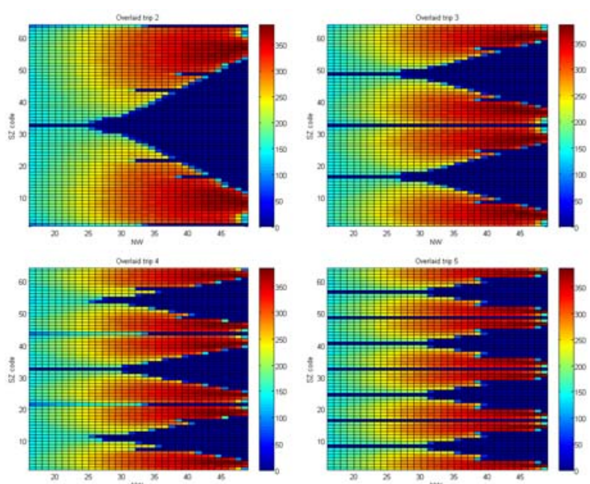

Fig. 6. Performance of SZ(n/64) codes for different PNF widths (NW) and overlaid cases with trip differences of 1 (top left), 2 (top right), 3 (bottom left), and 4 (bottom right). “Warmer” colors represent better performance.

The performance for every phase code-PNF width combination is plotted in Fig. 6 for overlaid signals with 1, 2, 3, and 4 trip differences. Larger numbers (“warmer” colors) represent better performance (i.e., a larger weak-trip velocity recovery region). Many interesting properties can be inferred from these plots. For example, the vertical symmetry about n = 32 implies that codes of the form SZ(n/64) and SZ[(64−n)/64] are equivalent in terms of performance. Also, it is easy to spot codes that are not suitable for weak-trip velocity recovery, such as the SZ(32/64), which has a null recovery region for all PNF widths and overlay cases.

The performance of the SZ-2 algorithm can be obtained from Fig. 6 by looking at the rows with n = 8. For an overlaid trip difference of one, two, and three, the SZ-2 PNF width is set at 48, 32, and 32, respectively. As expected, for an overlaid trip difference of four, no PNF width leads to recovery of the weak-trip velocity. Note that, as introduced earlier, SZ(8/64) is not the optimum phase code for all overlay situations. The question arises then as to which codes are the best for each overlay case. Table 1 lists the best code-PNF width combinations for each overlay case and compares their performance to the current SZ-2 algorithm. For overlaid signals with one trip difference, the best code is SZ(56/64), which is statistically equivalent to the familiar SZ(8/64) (symmetry property). For other overlay cases, the optimum code-PNF width combinations can extend the size of the recovery region by more than 50%! However, there is no single switching code that is optimum for all overlaid cases.

	SZ(*n*/64)			SZ(8/64)		
t	***n***	**NW**	**SRR**	**NW**	**SRR**	**Improv.**
1	56	48	388	48	382	2
2	28	47	384	32	298	29
3	3	47	384	32	246	56
4	62	47	386	N/A	0	∞

Table 1. Comparison of best SZ(n/64) codes-PNF width combinations and SZ-2 for different overlay cases. The table lists the PNF width (NW), the corresponding size of the weak-trip recovery region (SRR) and the improvement with respect to the SZ-2 algorithm.

	SZ(4/64)		SZ(8/64)		
t	**NW**	**SRR**	**NW**	**SRR**	**Improv.**
1	41	343	48	382	-10
2	47	382	32	298	28
3	43	363	32	246	48
4	35	310	N/A	0	∞

Table 2. Same as Table 1 but comparing single-code best combinations and SZ-2 for different overlay cases.

Although the performances of the best combinations are appealing, it is not practical to consider different phase codes for different overlay cases. Hence, we are interested in finding the best set of combinations based on a single phase code. These are listed in Table 2, where the phase code with best overall performance is SZ(4/64). For a trip difference of 1, the SZ(4/64) code is about 10% worse than the operation SZ(8/64). This is expected since the SZ(8/64) was chosen for this situation. However, for all other situations, SZ(4/64) results in significant improvements over the SZ(8/64). It is important to mention that the determination of single-code best combinations was done considering overlay cases with trip differences of 1, 2, and 3 only. A trip difference of 4 is not possible with the WSR-88D PRTs. Still, the SZ(4/64) code can handle the overlay case with a trip difference of 4, which might be of interest for shorter-wavelength radars, such as the TDWRs.

4. CONCLUSIONS

This paper introduced a family of systematic phase codes of the form SZ(n/64). A closer look into the performance of these generalized codes revealed a number of omissions in the early research work. Further, no single code is optimum for all overlay cases, and, surprisingly, the best overall phase code in the SZ family is not the SZ(8/64), which is currently used operational on the NEXRAD network.

This analysis is by no means comprehensive. However, these preliminary results justify further exploration of generalized phase codes. For example, performance should be assessed using the actual levels and types of phase errors encountered operationally on the NEXRAD network, which have not been measured systematically. Also, we plan to complement a simulation-based study with the analysis of multiple real-data cases collected with the KOUN research radar.

In summary, this work is not complete yet but has the potential to lead to an even greater improvement with respect to previous "legacy" algorithms to effectively mitigate range and velocity ambiguities on the US network of weather surveillance radars.

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