8A.6 SINGLE-SCAN RADAR REFRACTIVITY RETRIEVAL: THEORY AND SIMULATIONS

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1. INTRODUCTION

Surface refractivity measurements have been receiving increasing attention in recent years due to its close relation with surface moisture. Previous work has demonstrated that convective precipitation initiation is highly dependent on the surface moisture (e.g., Dabberdt and Schlatter, 1996; Koch et al., 1997; Bodine et al., 2009).

A radar method to retrieve near surface refractivity using ground clutter echoes was first developed by the work of Fabry et al. (1997). Field experiments were conducted and have shown success of the technique (Fabry and Pettet, 2002; Weckwerth and Parsons, 2003). A similar technique has been developed and implemented here at the University of Oklahoma (Cheong et al., 2008). The technique uses phase measurement from two scans to derive the change of refractivity between the scans. Absolute refractivity is obtained by choosing a condition where reference refractivity can be obtained from surface measurements from the Oklahoma Mesonet.

In this work, a technique to derive surface refractivity using phase measurements from *one scan* is presented. This technique is referred to as the single-scan radar refractivity retrieval (SR3). Currently, there is no radar suitable for the implementation of SR3 but its feasibility is explored here from theory and simulations.

2. THEORY OF SINGLE-SCAN RADAR REFRACTIVITY RETRIEVAL (SR3)

Theoretically, the received phase from stationary targets is an integral function of the refractive index, this quantity can be expressed as follows (Bean and Dutton, 1968)

$$\phi(r) = -\frac{4\pi}{\lambda}\int_0^r n(\gamma)d\gamma \tag{1}$$

where λ represents wavelength of the radar and r is the range. Because the value of n is close to 1 for applications near the earth's surface, a convenient term *refractivity* is frequently used. It can be represented as

$$N = 10^6(n-1) \tag{2}$$

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For most weather radars, the operation wavelength is on the order of cm and since $n \approx 1$, the phase wraps many times considering target range that may span up to 50 km. Therefore, deriving refractivity directly from a single scan using the *absolute* phase is problematic. However, from Eq. (1), one can also realize that refractive index is a local derivative of the phase as follows

$$n(r) = -\frac{\lambda}{4\pi}\frac{d}{dr}\phi(r) \tag{3}$$

The key problem is to unwrap the gate-to-gate phase change, rather than to retrieve the absolute phase. This problem is still a challenging problem but less problematic than retrieving the absolute phase of each gate. For a given discrete range sampling of a radar, Eq. (3) is applied as

$$\begin{aligned} n(r) &\approx -\frac{\lambda}{4\pi}\frac{\Delta}{\Delta r}\phi(r) & (4)\\ &= -\frac{\lambda}{4\pi\Delta r}\left[\phi(r+\Delta r) - \phi(r)\right] & (5)\end{aligned}$$

It should be emphasized here that Δr represents the centroid spacing between the two adjacent range gates, which is a function of the ground target distribution in those two cells. This parameter must be estimated accurately in order to successfully retrieve the refractive index.

Clearly, there are multiple solutions that can exist in unwrapping phase measurements. Fortunately, for the application near the earth surface, the refractive index can only be valid for $N \in [200, 400]$ and the separation of the multiple solutions are sufficient for us to always choose the unique solution. A numerical example is illustrated as follows.

For simplicity, parameters of an X-band radar are considered here. Let $\lambda = 3\,\mathrm{cm}$, $\Delta r = 90\,\mathrm{m}$ and $n(r) = 1.0003$, using Eq. (1), the theoretical phase change would be -37710.42 rad. from one gate to the next. Now, consider the possibility of an incorrect unwrapped phase, e.g., add $\pm 2\pi$ offsets to the theoretical number. These would be the next possible solutions that are closest to the truth. The estimates of $\hat{\phi} = -37710.42 \pm 2\pi = -37704.14\,\mathrm{rad}, -37716.70\,\mathrm{rad}$ would result in refractive index estimates of $1.000133, 1.000466$, which are invalid for the typical atmospherical conditions near the earth surface. In practice, when this scenario is encountered, the unique phase can be obtained by adding the appropriate integer multiples of 2π so that the solution of $n(r)$ falls within the valid range.

From Eq. (5), we can also solve for Δr if $n(r)$ is known, as

$$\Delta r = -\frac{\lambda}{4\pi n(r)} \left[\phi(r + \Delta r) - \phi(r)\right] \quad (6)$$

Similar to estimating $n(r)$, multiple solutions for Δr exist but unfortunately there is no distint signature that allows us to choose the unique solution. If two frequencies are available, however, then two phase measurements can be made to mitigate the limitation. Essentially, the so called frequency domain interferometry (FDI) is applied here (e.g., Kudeki and Stitt, 1987; Franke, 1990; Palmer et al., 1990). For the SR3, we are interested in the range centroid spacings (rather than the absolute range) so the effects of initial phase of the radar can be neglected. With two phase measurements, Eqs. (1) and (3) becomes

$$\phi_1(r) = -\frac{4\pi f_1}{c}\int_0^r n(\gamma)d\gamma \quad (7)$$

$$\phi_2(r) = -\frac{4\pi f_2}{c}\int_0^r n(\gamma)d\gamma \quad (8)$$

$$\phi_1(r) - \phi_2(r) = -\frac{4\pi(f_1 - f_2)}{c}\int_0^r n(\gamma)d\gamma \quad (9)$$

$$\frac{d}{dr}\left[\phi_1(r) - \phi_2(r)\right] = -\frac{4\pi(f_1 - f_2)}{c}n(r) \quad (10)$$

where c is the speed of light. Assuming the refractive index is uniform within the two sampling points in range, we can then derive the spacing between the two points if $n(r)$ is known.

$$\Delta r = -\frac{c}{4\pi(f_1 - f_2)n(r)}\Delta[\phi_1(r) - \phi_2(r)]_r \quad (11)$$

In practice, Δr may be estimated during a condition where $n(r)$ is less complex. In that case, objective analysis from surface stations can be used to derive $n(r)$. From Eq. (11), one can think of the dual-frequency combination effectively provides us a lower operating frequency $(f_1 - f_2)$ (longer wavelength). Multiple solutions still exist for Eq. (11) but the separation among the solutions become larger and thus, allows for finding a unique solution. Of course, with the ambiguity of phase wrapping, there is a limit to Δr estimation, which is

$$\Delta r \in \left[0, \frac{c}{2(f_1 - f_2)}\right] \mathrm{m} \quad (12)$$

Since it is very rare to have centroid spacing close to zero, The acquired phase can be *rewrapped* by a π-equivalent into the $[\pi, 3\pi]$ interval, which then maps the centroid spacing into

$$\Delta r \in \left[\frac{c}{4(f_1 - f_2)}, \frac{3c}{4(f_1 - f_2)}\right] \mathrm{m} \quad (13)$$

It should also be mentioned here that range oversampling may be applied to aid this process.

In the next section, a numerical simulation will be presented to demonstrate the process deriving refractive index from a single scan measurement using two frequencies.

3. RESULTS FROM SIMULATIONS

A simple refractivity distribution is used for simulation, illustrated in Figure 1(a). Here, we considered an X-band radar system with two frequencies, i.e., $f_1 = 9.55\,\mathrm{GHz}$, $f_2 = 9.5518\,\mathrm{GHz}$, $\Delta r = 90\,\mathrm{m}$ and $N(r) \in [200, 400]$.

3.1. Estimating Centroid Spacings

With the selected frequency separation, we have the ability to estimate centroid spacings within $[41.67, 125]\,\mathrm{m}$, according to Eq. (13). For simplicity, we assume that ground clutter is present in all range gates. Since only a pair of adjacent measurements are considered for each point of refractivity retrieval, regions without ground clutter simply may not be used in practice.

A 30-m uniformly distributed random perturbation is introduced to the range gate spacing to simulate random ground target distribution along the range. In addition, a 0.001° uniformly distributed random phase is added to simulate instrumental noise. For now, a quite precise equipment is assumed. Later in this paper, different phase noise will be presented to show the effects of such contamination to this technique. Using Eq. (3), the absolute returned phase for each frequency is shown in Figure 1(b). They are wrapped into the $[-\pi, \pi]$ range in Figure 1(c), which represent the actual phase measurements that would be obtained in practice.

Using the simulated phase measurements in Figure 1(c), the first step is to calculate the gate-to-gate phase difference for the two measurements from the two frequencies, i.e., the term $\Delta\left[\phi_1(r) - \phi_2(r)\right]_r$ in Eq. (11). The phase is then rewrapped into the $[\pi, 3\pi]$ interval. Using the rewrapped phase, the centroid spacing are derived and is shown in Figure 2(b). With a precise phase measurement (0.001° noise), the corresponding estimation error is small and shown in Figure 2(c).

3.2. Single-scan Radar Refractivity Retrieval

Again, for simplicity, the same radar system is used. Once the centroid spacing of ground targets are identified, which can be done ahead of time, radar refractivity can be retrieved by using the phase measurements from *one scan* using the method presented in Section 2. In this example, we assumed that the centroid spacings have been obtained with an accuracy on the orders of $10^{-4}\,\mathrm{m}$, as depicted in Figure 2(c), and proceed with refractivity retrieval based on phase measurement from one of the two frequencies. With the estimated centroid spacings, we can first derive the initial guess of the gate-to-gate unwrapped phase as shown in Figure 3(a). Proceed with refractivity derivation using Eq. (5) and Eq. (2), the initial estimated refractivity is shown in Fig-

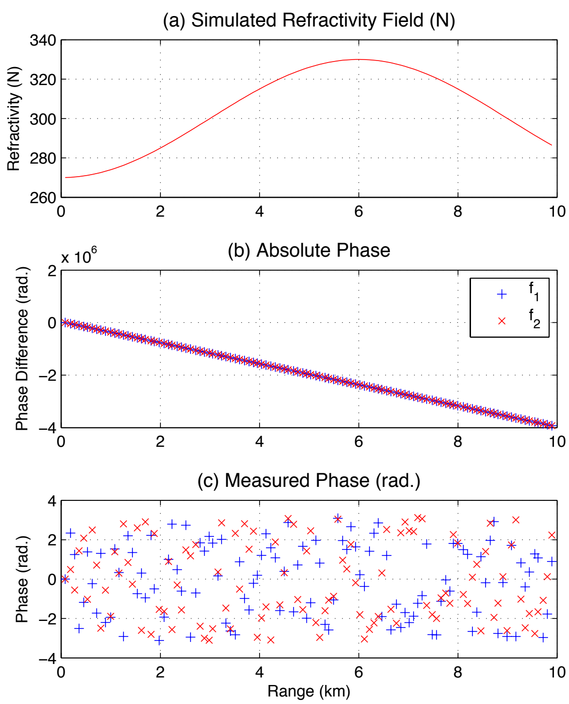

Figure 1: Simulation setup with an X-band radar with two frequencies at $f_1 = 9.55\,\mathrm{GHz}$, $f_2 = 9.5518\,\mathrm{GHz}$. Random range perturbations and measurement noise are added to simulate radar echoes. (a) shows the refractivity distribution in the experimental setup, (b) shows the theoretical absolute phase with range perturbation and added noise while (c) shows the wrapped phase measurements.

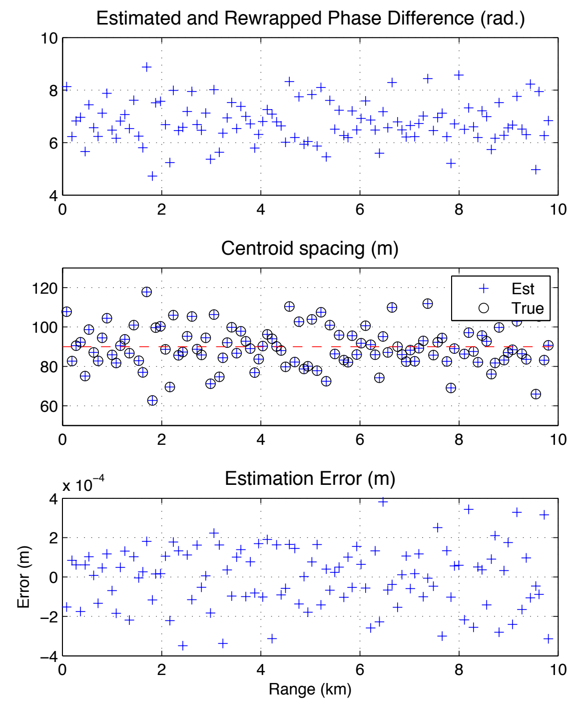

Figure 2: Range-gate centroid spacing can be estimated if Refractivity is known. In practice, this can be achieved by using a refractivity field from objective analysis using surface measurements, e.g., ASOS or Oklahoma Mesonet, during conditions where the field is less complex.

ure 3(b). Note that there are some refractivity values that can be considered "outliers". Using a simple interval check where $N \in [200, 400]$, we can recover these points into the valid range and the result is shown in Figure 3(c).

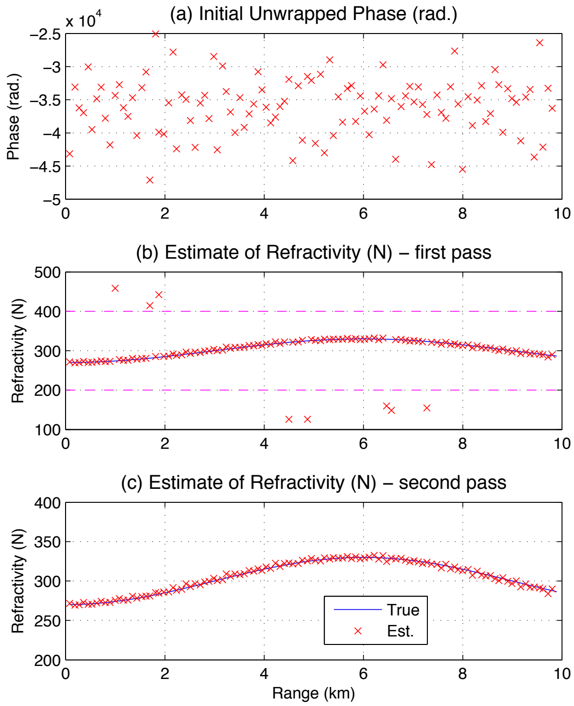

Figure 3: SR3 to retrieve refractivity using one phase measurement. (a) shows the initial unwrapped phase measurements and (b) shows the corresponding initial refractivity estimate. It can be seen that there are several "outliers". Since $N \in [200, 400]$, they can be corrected and (c) shows the results by re-mapping the "outliers" into the valid range.

4. INSTRUMENTAL PRECISION

The precision of refractivity retrieval directly depends on the precision of phase measurements, this can be realized from Eq. (5). Figure 4 illustrates the effects of phase noise on refractivity retrieval. It can be seen that the results of refractivity suffers tremendously even with phase noise of $0.005°$(uniformly distributed in $[-0.0025, 0.0025]°$).

There are several open questions on the practicality of the SR3 technique. The most important ones include the instrumental precision and the stability of centroid spacings of ground clutter. These parameters directly control the viability of the technique and we are currently exploring the practical implications of SR3.

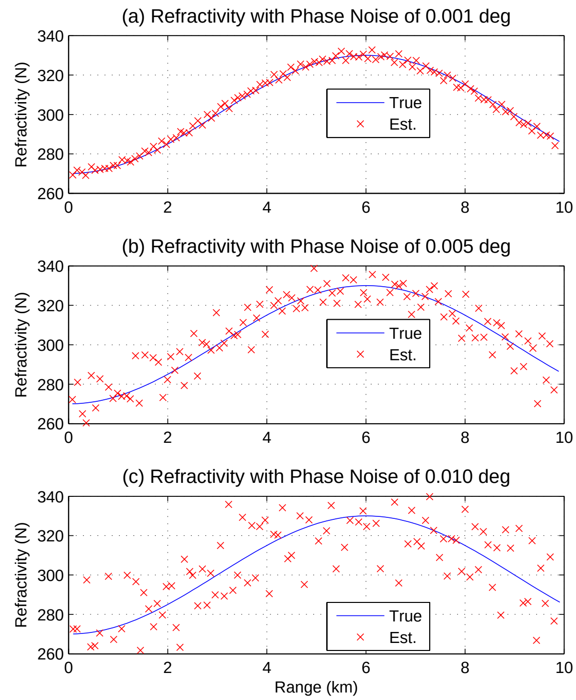

Figure 4: With noise added to the phase measurements, the performance of refractivity retrieval deteriorated.

5. FUTURE WORK

Here at the ARRC at OU, a travelling wave tube (TWT) based X-band radar is currently being developed. Simulation parameters presented in this paper have been chosen to mimic this system and this radar will be the first testbed for the SR3 technique. Future work also includes investigation of using multiple frequencies to solve for centroid spacings and refractive index simultaneously.

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