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# RETRIEVAL OF THE RAIN DROP SIZE DISTRIBUTION USING TELECOMMUNICATION DUAL-POLARIZATION MICROWAVE LINKS

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## 1 INTRODUCTION

The (rain)drop size distribution (DSD hereinafter) is a useful statistical way to summarize the large amount of information concerning the numerous drops within a given volume of rainy atmosphere (larger than about 1 $m^3$ to have enough drops). The focus is on the size, usually quantified as the equivolumetric diameter, because, for rain, both the shape and the terminal fall velocity of a drop can be accurately derived from its size (Andsager et al., 1999; Beard, 1976). The concept of DSD was introduced in the early ages of weather radar (Marshall and Palmer, 1948) and was shown to be crucial for the quantitative interpretation of weather radar measurements, in particular in terms of rain rate (the variable of interest for many applications). More generally, all variables describing rain integrated above the continuous scale (e.g., radar reflectivity, liquid water content) can be expressed as weighted statistical moments of the DSD. The concept of DSD has been further investigated and refined since then (e.g., Best, 1950; Fujiwara, 1965; Ulbrich, 1983; Jameson and Kostinski, 2001; Villermaux and Bossa, 2009).

DSD measurements are mostly collected using (ground-based) disdrometers, for which different measurement principles have been proposed (e.g., Joss and Waldvogel, 1967; Sheppard, 1990; Salles et al., 1998; Loffler-Mang and Joss, 2000; Kruger and Krajewski, 2002). Polarimetric weather radar (Seliga and Bringi, 1976; Zhang et al., 2001; Moisseev and Chandrasekar, 2007) can also be used to estimate DSD spectra, as well as Doppler profilers (Loffler-Mang et al., 1999; Williams et al., 2000).

The main objective of this paper is to present a new method for (parametric) DSD retrieval using microwave link measurements. A microwave link consists of a transmitter communicating with a receiver through a microwave signal. Because of the interactions between electromagnetic waves and raindrops, the link signal is attenuated when rainfall occurs along the path of the link (Atlas and Ulbrich, 1977). The potential of this feature for rainfall estimation has been investigated for dual-frequency (Holt et al., 2000; Rahimi et al., 2003) or dual-polarization (Ruf et al., 1996; Aydin and Daisley, 2002) dedicated microwave links. Recently, Messer et al. (2006) and Leijnse et al. (2007b) have demonstrated the possibility to use commercial telecommunication microwave links to estimate the path-averaged rain rate.

Microwave links are appealing as rain sensors because they provide measurements at the path scale (a few km), an intermediate scale between point measurements (from a rain gauge or a disdrometer) and radar measurements (sampling volume up to 1 $km^3$). Thanks to the recent development of telecommunications, microwave link networks are already deployed and operational, being a potential additional source of rain data with respect to conventional sensor networks for rainfall monitoring.

Most of previous work has focused on rain-rate retrieval. Only a few studies have investigated DSD retrieval from microwave link measurements. Using the ratio of attenuations at the two frequencies of dedicated dual-frequency microwave links, DSD information can be retrieved qualitatively (Holt et al., 2008) and quantitatively (Rincon and Lang, 2002). A similar approach is proposed in the present paper, but using the total attenuation measured by dual-polarization links, taking advantage of the oblateness of rain drops larger than 1 mm (Pruppacher and Klett, 1997). The paper is organized as follows: Section 2 is devoted to the description of the DSD retrieval method, Section 3 presents the data used for the application and the evaluation of the proposed method, described in Section 4.

## 2 DSD RETRIEVAL METHOD

When rainfall occurs along the path, there is an additional attenuation $A$ [dB] affecting the link signal

$$A_p = \int_0^L k_p(s)\, ds \tag{1}$$

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where $p$ denotes the polarization (horizontal or vertical), $L$ is the length of the link [km], and $k_p$ is the one-way specific attenuation [dB km$^{-1}$] at the considered frequency and polarization.

$$k_p(s) = \frac{1}{\ln(10)} \int_{D_{min}}^{D_{max}} \sigma_{E,p}(D) N(D,s)\, dD \qquad (2)$$

where $D$ is the equivolumetric drop diameter [mm] varying between $D_{min}$ and $D_{max}$; $\sigma_{E,p}$ is the extinction cross-section [cm$^2$] at the considered frequency and polarization, and $N(D,s)dD$ [m$^{-3}$] is the number of drops with a diameter between $D$ and $D+dD$ per cubic meter at a location $s$. Combining Eq.(1) and Eq.(2), and invoking Fubini's theorem to change the order of integration leads to

$$A_p = \frac{1}{\ln(10)} L \int_{D_{min}}^{D_{max}} \sigma_{E,p}(D) \bar{N}(D)\, dD \qquad (3)$$

where $\bar{N}$ denotes the average DSD along the link path:

$$\bar{N}(D) = \frac{1}{L} \int_0^L N(D,s)\, ds \qquad (4)$$

$\bar{N}$ is supposed to be adequately described by a truncated gamma model, similarly to point-scale DSD (Ulbrich, 1983):

$$\bar{N}(D) = \alpha_e N_{te} D^{\mu_e} e^{-\Lambda_e D} \qquad (5)$$

where $\alpha_e$ is a normalization factor to take into account the finite range of possible drop diameters between $D_{min}$ and $D_{max}$, $N_{te}$ [m$^{-3}$] is the drop concentration, $\mu_e$ [-] the shape parameter and $\Lambda_e$ [mm$^{-1}$] the scale parameter. Note that the effective average DSD parameters $N_{te}$, $\mu_e$ and $\Lambda_e$ are different from the path average of the point values of $N_t$, $\mu$ and $\Lambda$ because of the non-linearity of Eq.(5). Combining Eq.(5) and Eq.(3) gives

$$A_p = \frac{L \alpha_e N_{te}}{\ln(10)} \int_{D_{min}}^{D_{max}} \sigma_{E,p}(D) D^{\mu_e} e^{-\Lambda_e D}\, dD \qquad (6)$$

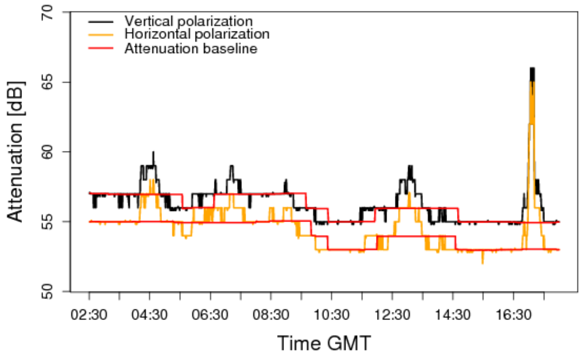

Figure 1: Raw attenuation [dB] in the horizontal (orange) and vertical (black) polarization along the 2.4-km link path, during the rain event on the 22nd of August 2008. The red lines indicate the attenuation baselines.

A dual-polarization link provides measurements of the total attenuation at the horizontal and vertical polarizations. Because of the oblateness of big raindrops, the ratio between the attenuation (in dB) at the vertical and horizontal polarization is smaller or equal to one and reads

$$\frac{A_v}{A_h} = \frac{\displaystyle\int_{D_{min}}^{D_{max}} \sigma_{E,v}(D) D^{\mu_e} e^{-\Lambda_e D}\, dD}{\displaystyle\int_{D_{min}}^{D_{max}} \sigma_{E,h}(D) D^{\mu_e} e^{-\Lambda_e D}\, dD} \qquad (7)$$

Assuming an axis ratio relationship (e.g., Andsager et al., 1999) and using the T-matrix code (Mishchenko and Travis, 1998), $\sigma_{E,h}$ and $\sigma_{E,v}$ can be computed at a given temperature and a given frequency. In addition, some studies (e.g., Zhang et al., 2001; Brandes et al., 2004) suggest a deterministic relationship between the scale and shape parameters of the gamma DSD model. Assuming $\Lambda_e = f(\mu_e)$, $\mu_e$ remains the only independent parameter in Eq.(7) which can hence be numerically solved. Once the value of $\mu_e$ has been obtained, $\Lambda_e$ is directly calculated as $f(\mu_e)$ and $N_{te}$ is derived using Eq.(6) for one or the other polarization.

## 3 DATA

### 3.1 LINK DATA

Bouygues Telecom, a french telecommunication company, provided instantaneous measurements of the transmitted and received power every 30 s for a 2.4-km dual-polarization (H and V) link at about 26 GHz near Paris, France. Power measurements are rounded at 1 dB. The rain event that occured on the 22nd of August 2008 is used to illustrate our approach (see Fig. 1).

In order to apply the retrieval method, the attenuation solely due to rain must be extracted from the total attenuation affecting the link signal. Consequently, attenuation due to losses in transmission/reception, to atmospheric gases, to multipath effects, also occuring during dry weather and named attenuation baseline (Rahimi et al., 2003), must be quantified. Taking advantage of the different statistical behaviour of the link signal during dry or rainy periods, a procedure to identify the attenuation baseline has been developed (for more details, see Schleiss and Berne, 2009).

In addition to the attenuation baseline, the attenuation due to a thin film (not necessarily continuous) of water when rain falls on the antenna(s) of a link, named wet-antenna attenuation, must also be removed from the total measured attenuation. We use the exponential model (not shown) proposed by Kharadly and Ross (2001) with the parameters from Leijnse et al. (2007a) and Minda and Nakamura (2005). The obtained wet-antenna attenuation estimates are supposed to be a fair approximation of the real attenuations affecting the considered link signal.

### 3.2 RADAR DATA

As there are no direct DSD observations in the vicinity of the studied link, the quality of the retrieved DSDs is indirectly evaluated using polarimetric radar measurements from an operational C-band polarimetric weather radar managed by Météo France and located about 30 km of the link. The equivalent radar reflectivity $Z_h$, the differential reflectivity $Z_{dr}$ and the rain rate $R$ are calculated using the retrieved DSDs from link data (as weighted moments of the DSD) and compared to the radar-measured ones (at the lowest elevation of 0.4 deg). It must be noted that the radar rain-rate values are estimated according to the methodology described in Tabary (2007), correcting for the main sources of error (ground echoes, VPR, advection). The difference in sampling volume and altitude between the link and the radar introduces an additional uncertainty in the comparison.

## 4 RESULTS

To apply the proposed method, a relation between the scale parameter $\Lambda_e$ and the shape parameter $\mu_e$ is established. To do so, we use a large DSD data set collected in Lausanne, Switzerland, during 15 months with a Parsivel disdrometer. Assuming Talyor hypothesis and a mean advection velocity of about 5 m s$^{-1}$, $\Lambda_e$ and $\mu_e$ values are derived from the observed DSD spectra at a 8-min resolution (2400 observations), in order to be representative of the path-averaged DSD. The following linear relationship fits quite well the scatter (correlation coefficient of 0.93):

$$\Lambda = -0.96 + 1.65\mu \qquad (8)$$

Assuming the rain climatology is similar, Eq.(8) will be used for DSD retrieval in Paris.

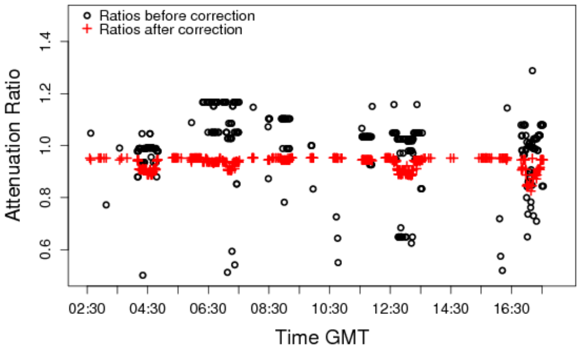

Figure 2: Raw and corrected attenuation ratio values, during the rain event on the 22nd of August 2008.

Once the attenuation solely due to rainfall has been quantified, the ratio of attenuation at vertical and horizontal polarization can be estimated. The 1-dB rounding has a significant effect on the derived ratio values, especially for limited attenuation values (light rain). In order to compensate for this effect, a correction based on the distribution of true ratio values estimated from the DSD data collected in Lausanne and conditioned on the attenuation at horizontal polarization is applied (see Fig. 2).

Using the computed values of extinction cross-sections for both polarizations (at 20°C, the temperature having a limited influence), Eq.(7) can be numerically solved to estimate the value of $\mu_e$. The corresponding value of $\Lambda_e$ is then derived using Eq.(8). Finally, the value of $N_{te}$ is calculated using Eq.(6) for the horizontal polarization because the attenuation is larger, hence minimizing the effect of the 1-dB rounding. In this way, the path-averaged DSD $\bar{N}(D)$ can be retrieved from telecommunication dual-polarization microwave link measurements.

In order to (indirectly) evaluate the quality of the retrieved DSDs, the corresponding path-averaged rain variables are computed using the following equations

$$\bar{R} = 6\pi 10^4 \int_{D_{min}}^{D_{max}} v(D) D^3 \bar{N}(D)\, dD \quad (9)$$

$$\bar{Z}_{h|v} = \frac{10^4 \lambda^4}{\pi^5 |K|^2} \int_{D_{min}}^{D_{max}} \sigma_{B,h|v}(D) \bar{N}(D)\, dD \quad (10)$$

where $v(D)$ denotes the terminal fall velocity [m s$^{-1}$] of a drop with an equivolumetric diameter $D$, calculated using Beard's model (Beard, 1976); $K$ is the dielectric factor of liquid water, $\lambda$ is the wavelength [cm] and $\sigma_{B,h|v}$ denotes the backscattering cross section [cm$^2$] in the horizontal (vertical) polarization. $\bar{R}$ is expressed in mm h$^{-1}$ and $\bar{Z}_{h|v}$ in mm$^6$ m$^{-3}$. The path-averaged differential reflectivity $\bar{Z}_{dr}$ is calculated as $\bar{Z}_{h,dB} - \bar{Z}_{v,dB}$ and is expressed in dB.

The time series of the estimated $\bar{R}$, $\bar{Z}_h$ and $\bar{Z}_{dr}$ values together with radar observations are displayed in Figure 3. There are discrepancies, in particular for light rain, but overall the agreement is quite good and the temporal dynamics is well captured.

To better quantify this agreement, $\bar{R}$, $\bar{Z}_h$ and $\bar{Z}_{dr}$ values from 5 different rain events are first averaged at a 5-min time resolution and then compared to radar observations in the scatter plots in Figure 4. Because the retrieval is uncertain for light rain, a threshold of 0.1 mm h$^{-1}$ for $\bar{R}$, of 20 dBZ for $\bar{Z}_h$ and of 0 dB for $\bar{Z}_{dr}$ is applied. For $\bar{R}$, the correlation coefficient is 0.82, the ratio of means is 1.07 and the intercept is 1.1 mm h$^{-1}$ (i.e., about 29% of the mean radar estimates). In terms of total rain amount for the 5 rain events, the link estimate is about 108 mm while the radar estimate is about 113.5 mm, i.e., 5% difference. For $\bar{Z}_h$, the correlation coefficient is 0.74, the ratio of means is 1.07 and the intercept is 10 dBZ$^{-1}$ (i.e., about 30% of the mean radar estimates). Finally, for $\bar{Z}_{dr}$, the correlation coefficient is 0.81, the ratio of means is 1.01 and the intercept is negligible. For the three variables, the relatively high correlation values indicate that the temporal dynamics is well captured by the link estimates. Similalry, the ratio of means close to 1 and the limited intercept values indicate a limited bias in the link estimates. It must be noted that a perfect agreement cannot be achived because of the difference in the sampling volume and altitude of the two sensors.

## 5 CONCLUSIONS

The raindrop size distribution is crucial for the quantitative interpretation of radar measure-

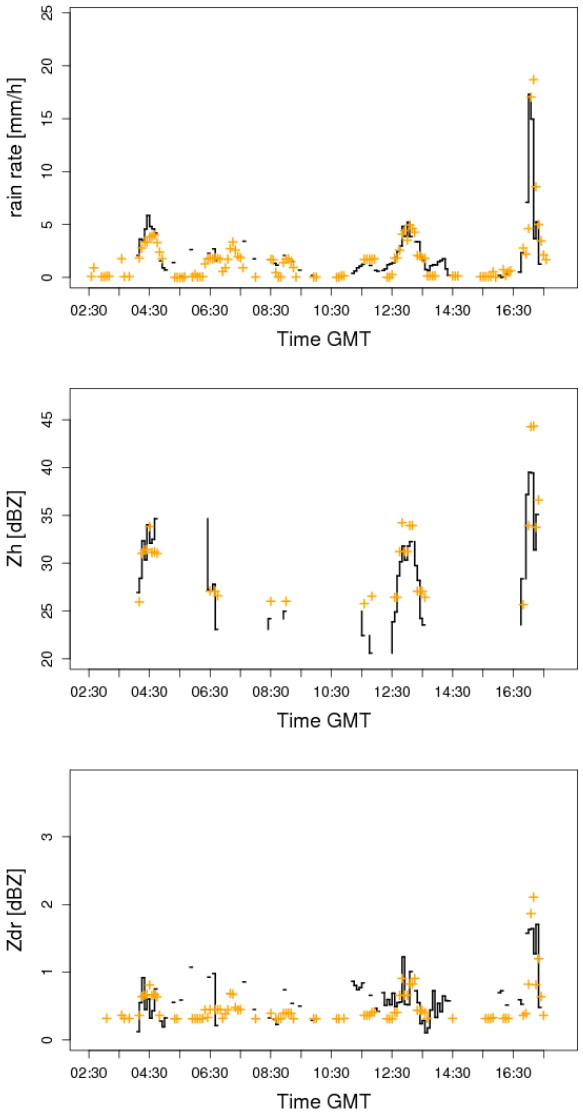

Figure 3: Time series of path-averaged rain rates (top), horizontal reflectivity (middle) and differential reflectivity (bottom) measured/estimated from the operational radar (black) and derived from the retrieved DSDs using link data (orange), for the rain event on the 22nd of August.

ments. In this paper, a method to retrieve the path-averaged DSD using measurements from dual-polarization microwave links is presented. It estimates the three parameters of a gamma DSD model, assuming a deterministic relationship between the scale and shape parameters. The shape parameter is obtained from the ratio of attenuations at vertical and horizontal polarizations. The scale parameter is then directly calculated through its relationship with the shape parameter. Finally the drop concentration is derived using the attenuation at one or the other polarization.

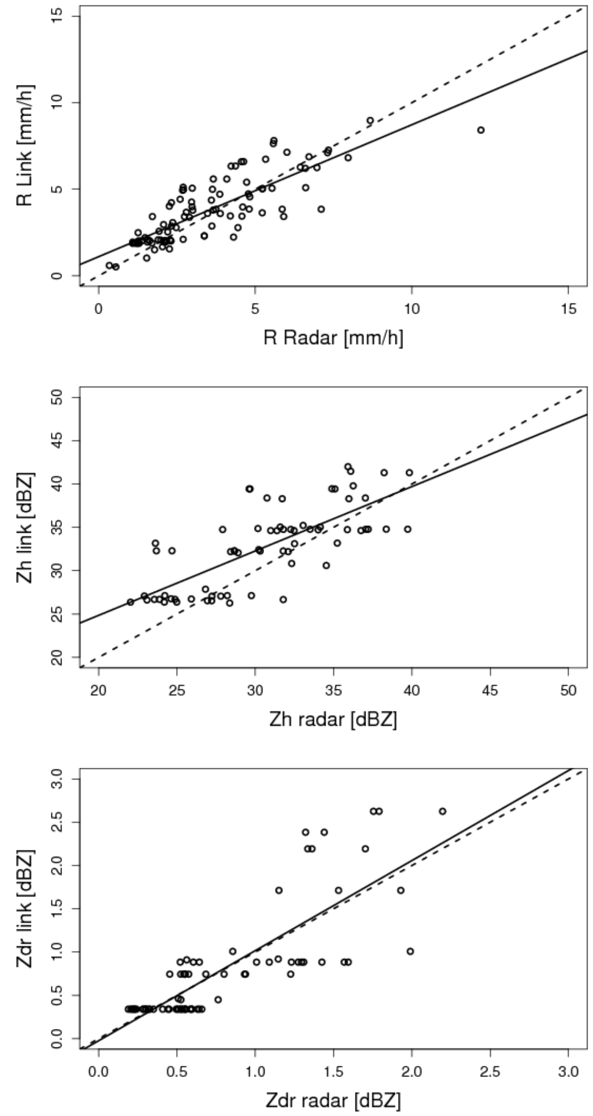

Figure 4: Scatter plots for rain rates (top, threshold at 0.1 mm $h^{-1}$), horizontal reflectivity (middle, threshold at 20 dBZ) and differential reflectivity (bottom, threshold at 0 dB) measured/estimated from the operational radar and derived from the retrieved DSDs using the link data, for 5 rain events.

The DSD retrieval method is applied to operational data from a telecommunication dual-polarization microwave link. The quality of the retrieved DSDs is evaluated by comparing the rain rate, the horizontal and differential reflectivity derived from the retrieved DSDs with measurements from a nearby operational C-band polarimetric weather radar. Despite discrepancies for light rain, the good agreement between these independent estimates demonstrates the feasibility of retrieving the path-averaged DSD using dual-polarization microwave links from telecommunication networks. The main limitation of this method is the difficulty to obtain reliable DSD estimates for light rain because of the rather large uncertainty in the attenuation measurements, in particular for short links.

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