INVERSE PROBLEM OF PARAMETERS IDENTIFICATION: RADAR WEATHER EQUATION

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1. INTRODUCTION

Radar is used to detect the presence of rain and hail (referred to as "precipitation") and to estimate its intensity from the strength of the received echoes. The information is presented to forecasters on computer displays in the form of maps that show the location of the rain in relation to local features such as the coastline, with different colors used to depict intensity. When displayed in this way, it is often found that particular weather situations give rise to radar echoes with distinctive patterns or shapes which help forecasters to identify them Because of radar's ability to estimate the intensity of rainfall over wide areas, it has a useful role to play in monitoring weather situations which might result in serious flooding. Also, by replaying a series of recorded images after the event, radar can often provide useful evidence or insights into the meteorological situation at the time of incidents such as storm damage or aircraft accidents. Radar is able to depict light rain to distances beyond about 100 km from the radar and to indicate the possibility of severe storms to 250 km or more, limited in the main by earth's curvature. Wherever possible, Weather Watch radars are located on high ground in order to clear local obstructions and give the best possible coverage of the surrounding area.

Several empirical values to the coefficients a and b, from Z-R relationship, *i.e.*, the radar measurements and the rainfall rate values, have been proposed. The IPMet radar use the values determined by Marshal-Palmer (1948), $a = 200$, $b = 1{,}6$, for the stratiform rain. Calheiros and Zawadzki (1987), used a likelihoods sum method to determine $Z - R$ relation from data collected by Bauru's radar (C band), for the convective rain. They presented different values to a, b parameters, depending on the radar position and the localization of the rain. Microphysical features related to Z were been discussed be Steiner et al (2004) using inverse method. They define three microphysical conditions to the raindrop size distribution. First condition define that all variability of the distribution is controlled by variation of the raindrop concentration number; the second, the control depends on the size raindrop characteristics and the last, the control is influenced by both conditions. They conclude that even using modern mathematical and statistical techniques, how inverse problem, the uncertainties of measurement will always be there due the limitation concerned to comparison between reflectivity measured by radar and rain measured by gauge. Fiser (2004), using radar data collected by a year in Prague city, Czech Republic, tries to improve the Z-R relationship using power law and second order polynomial. He concludes that, although without getting a better performance of the weather radar equation, using of the technique of comparison of the raindrop size distribution with rain collected by gauge led to a better estimation of the rain. This paper is concerned to fit Z-R relationship, used in the operational center of the Institute of Meteorological Researches, IPMet/UNESP to rainfall event. It´s performed as inverse problem to identification of parameters.

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First, a direct theoretical model relating radar return to rainfall rates measured from raingauge localized in Botucatu city, far 95 km from radar position, is presented. Second, the inverse algorithm of Levenberg Marquardt, available in computational library (Press et al., 1992), used to identify parameters of the radar returns model is described and its application is discussed. Third, a sensitivity analysis, follow Beck and Arnold (1977), as the performed to test the influence of the parameters on the rainfall calculated by model. The sensitivity analysis allows to establish the application conditions of the method. Last, a evaluating of de method is presented, through using rainfall data and reflectivity measured to retrieve a and b parameters values. The study case confirms the utility of the proposed method and its capability to provide a better performance of the studied model.

2. DATA AND METHODOLOGY

The data used in the study are from the Bauru S-band Doppler radar (Altitude 620 msl, Latitude 22°35´S and Longitude 49°03´W, Figure 1). This radar forms part of IPMet´s (Instituto de Pesquisas Meteorológicas) network of two S-band Doppler radars, being operated continuously with volume scans every 7.5 – 15min, if rain occurs within the 240km range, comprising 11 PPIs between 0.3° and 34.9° elevation. The beam width is 2° and the resolution is 1km^2 in range and 1° in azimuth. The study period was from 1994 to 2004, with 27.072 CAPPIs collected during January and 24.942 CAPPIs during February.

Figure 1. Map of the State of São Paulo, showing the main cities, rivers and the position of the Bauru radar (BRU) with 240 and 450 km range rings
(Source: http://www.guianet.com.br/sp/mapasp.htm)

Levenberg-Marquartd optimization is a virtual standard in nonlinear optimization, which significantly outperforms gradient descent and conjugate gradient methods for medium sized problems. It is a pseudo second order method which means that it words with only function evaluations and gradient information but it estimates the Hessian matrix using the sum of outer products of the gradients. To the merit function case χ^2 :

$$\chi^2(\beta) = \sum_{i=1}^{N}\left[\frac{Y_i - \eta_i(\gamma,\beta)}{\sigma_i}\right]^2 \qquad (1)$$

where, Y_i are the experimental values of the variable Y, σ_{di} corresponding to standard deviation, $\eta_i(\gamma,\beta)$ variable values calculated by theoretical model and $\beta=(\beta_1,\beta_2,\beta_3,...\beta_m)$ the known parameters of the model, Press et al (1992).

This process is repeated until convergence. The two methods we described above have problems. (1) The steepest descent method has no good way to determine the length of the step. (2) Newton's method is based on solving a linear system. The matrix to be inverted can be singular. (3) Moreover, unless it is started close to the minimum, Newton's method sometimes leads to divergent oscillations that move away from the answer. That is, it overshoots, and then overcompensates, etc.

The inverse formulation is given by relation

$$R^i = f(Z^i, a, b) \qquad (2)$$

The adjust of the merit function S^2 can be written for the rainfall in the form

$$S^2(\beta) = \sum_{i=1}^{N}\left[\frac{R_{model}{}^{i} - R_{measured}{}^{i}}{d_i}\right] \qquad (3)$$

The data obtained were submitted to consistency analysis based on the conventional criteria within the practical norms of the operational meteorology. The process of the identification requires a preliminary analysis of the sensibility of the variable obtained by model in terms of the parameters and objectives of the estimation. The coefficient of reduced sensibility is used and represented graphically to evaluate the possibility of the satisfactory application of the method of identification.

The direct formulation of the problem is represented by the empirical equation (4), which related the radar signal to the rainfall rate as follows:

$$Z = aR^b \qquad (4)$$

Where,
Z (dBZ) is a reflectivity measured by the radar and R is the corresponding rainfall rate ($mm.h^{-1}$) measured by raingauge.

A numerical code is developed in FORTRAN language to calculate the rainfall from model and to calculate the coefficient of sensibility. Figure 1 shows smaller sensitivities to low frequencies. The sensibility coefficient, defined by partial derivate relate to the parameters a and b, here represented by $\beta_i, i = 1,2,...k$, means (5) the feedback of the theoretical model to calculation the rainfall rate (R), due a finite variation of the parameters.

$$\chi_i(\beta_i) = \frac{\partial R(\beta_i)}{\partial \beta_i} \qquad (5)$$

β is a vector of n components, χ_i indicates the variation of R when β_i vary of the finite way. The sensibility coefficient depends on the each parameter values. To comparison different values of χ_i, one should use an admensional sensibility coefficient, defined as:

$$\chi_i^*(\beta_i) = \beta_i \chi_i(\beta_i) = \frac{\partial R(\beta_i)}{\left(\dfrac{\partial \beta_i}{\beta_i}\right)} \qquad (6)$$

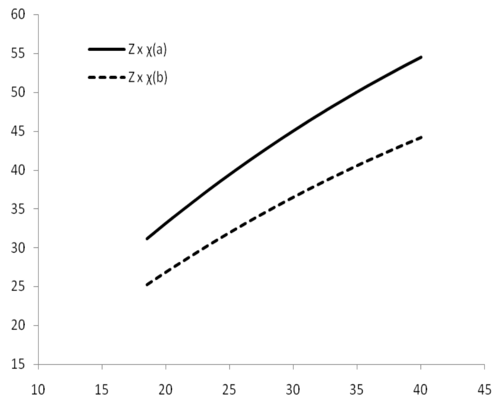

Figure 2. Sensibility coefficients, a, b.

The Figure 2 shows observed and modeled values of rainfall rate after identification, taking into consideration that the conditions of the measurement are different in the cases of radar and raingauge, the are satisfactory. We always expect with this model a better fit to the radar measurement.

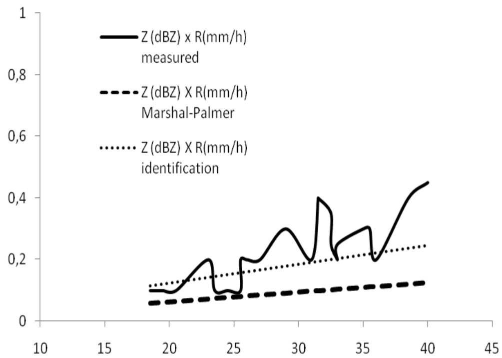

Figure 3 - Modeled values of rainfall rate before (Marshal-Palmer) and after identification, raingauge measured values.

4. CONCLUSIONS

Although there are an inherent bias error due the known limitations of the theoretical model and to uncertainties in the experimental values, the results have shown the efficiency of the inverse analysis where the values identified for $a = 200$ and $b = 0{,}81$ permit visualize that theoretical model is well behaviored. Taking into considerations that the conditions of the measurement the rainfall rate are different in the case of radar and raingauge, the result is satisfatory and and one can expect a better fit of the model with a better precision of the radar measurement.

6. REFERENCES

CALHEIROS, R. V.; ZAWADZKI, I., 1987: Reflectivity-rain rate relationships for radar hydrology in Brazil. *Journal of Climate and Applied Meteorology,* Boston, v. 26, p. 118-132.

BECK, J. V. ARNOLD, K. J., Parameter estimation in engineering and sciences, *Jonh Wiley and Sons,* Ney york, 1977.

FISER, O., 2004: Z-R radar Reflectivity-Rain rate relationships derived from Czech distrometer data. In: EUROPEAN CONFERENCE ON RADAR IN METEOROLOGY AND HYDROLOGY – ERAD, 3., 2004, Visby. *Proceedings…* Visby: [s.n.], p. 233-236

MARSHALL, J. S.; PALMER, W. M. K.,1948:The distribution of raindrops with size. *Journal of Applied Meteorology,* Boston, v. 5, p. 165-166.

PRESS, W. H., TEUKLSKY, S. A., VETTERLING, E., and FLANNERY, P. B., (1992) The art of scientific computing, second edition, Cambridge University Press, New York.

STEINER, M.; SMITH, J. A.; UIJLENHOET, R., 2004: A microphysical interpretation of radar reflectivity-rain rate relationships. *Journal Atmospheric and Oceanic Technology,* v. 61, p. 1114-1131.