P12.5 SPACED ANTENNA INTERFEROMETRIC MEASUREMENTS OF CROSSBEAM WIND USING A MONTE CARLO SIMULATOR BASED ON THE CONFIGURATION OF THE NWRT

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1. INTRODUCTION

The National Weather Radar Testbed (NWRT) is a facility used to test and evaluate phased array technology. The phased array radar could replace the current network of WSR-88D radars. The NWRT includes a AN/SPY-1A antenna and beamsteering controller and a WSR-88D transmitter. The transmitter is modified with a new Klystron tube that can transmit at 3.2 GHz (Forsyth et al., [2005]). The NWRT can simultaneously measure crossbeam wind and monitor the weather. Longer dwell times for radar interferometry can be interlaced with shorter ones for normal weather surveillance. The installation of the NWRT in NSSL, Norman, OK can be seen from the following picture:



Figure 1: Installation of the phased array antenna during construction of the NWRT (Courtesy of NSSL). L denotes virtual left side; R denotes virtual right side, they are overlapped. The left side is defined to be the left side of an observer looking outward along a direction perpendicular to the array.

The antenna of the NWRT is an early model for one

face of the four-face antenna used for the AN/SPY-1A radar of the Aegis system. Signals from each face of the antenna originate from a pair of two halves (i.e., left and right, lower and upper halves) of the array elements. The electronically steered beam for one face can cover 90° sector in azimuth and to 55° in elevation angles. This single face phased array antenna is mounted on a rotatable pedestal, thus a full hemisphere can be scanned. Putting phased array antenna on a turn-table allows us to determine the azimuth shape of a electronically steered beam and its sidelobes by rotating the turn-table while the electronically steered beam is fixed in azimuth and elevation.

There are 68 receive sub arrays and 68 weights, which means for each sub-array there is a single weight and weights will vary from sub-array to sub-array. In each receive sub-array, there are 64 array elements (horns). There are 32 array elements in each of two modules that comprise the sub-array. We use different weightings for sum and difference signal and the purpose is to provide an aperture distribution function that provides the narrowest sum beam width for a designed sidelobe level, and the maximum slope of the received difference signal across the principal null of the pattern. Thus Taylor aperture distribution for sum signal and Bayliss aperture distribution for

There are no ports on the NWRT where signals from left and right halves of the array are available to directly implement SAI using existing formulations based on spaced receiving antennas. For the NWRT, there are three channels: sum, azimuth difference, and

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elevation difference channel. In the following section we will discuss how to relate sum and difference signals to the signals received by virtual spaced receiving antennas.

2. Relating sum and difference signals to the virtual signals from the left and right sides of the array

Throughout this paper, we do not consider differential phase shift and loss between the sum and difference channels. Because existing SAI theory deals with processing signals from spaced receivers, we derive apparent signals received from the left- and right-sides (or top- and bottom-sides) of the antenna using the sum and difference signals. In this paper, we only consider azimuth SAI. First we need to distinguish the signals from left and right halves of the array from those from apparent left and right sides of the array. The apparent left or right side signal of the array is not entirely from the left or right half signal of the array. This is because the magnitudes of the Taylor and bayliss aperture distributions are not identical, thus the apparent left and right sides signals of the array come from both halves of the array. Although the left side signal is mainly from the left half of the array, some of the left side signal is from right half of the array. Thus, the apparent left and right side apertures are overlapped. In terms of the virtual signals from the left and right sides of the antenna, we have:

$$\Sigma = \frac{v_l + v_r}{\sqrt{2}} \tag{1a}$$

$$\Delta_a = \frac{v_l - v_r}{\sqrt{2}} \tag{1b}$$

Where v_l and v_r are voltages of the virtual overlapped apertures on the left and right sides of the

array. Σ is the sum signal, Δ_a is the azimuth difference signal. In order to have the sum power of the left and right sides of the array equal to the sum power of the sum and azimuth difference channels, we choose the coefficient $\sqrt{2}$. From Eqs. (1a) and (1b), we can obtain apparent left- and right-side signals of the array using the sum and difference signals:

$$v_l = \frac{\Sigma + \Delta_a}{\sqrt{2}} \tag{2a}$$

$$v_r = \frac{\Sigma - \Delta_a}{\sqrt{2}} \tag{2b}$$

According to the definition of correlation function, we have:

$$C_{ss}(\tau) = \langle s(t+\tau)s^*(t) \rangle$$
(3a)

$$C_{sd}(\tau) = \langle s(t+\tau)d^*(t) \rangle$$
(3b)

$$C_{ds}(\tau) = \langle d(t+\tau)s^*(t) \rangle$$
(3c)

$$C_{dd}(\tau) = \langle d(t+\tau)d^*(t) \rangle$$
(3d)

If we substitute Eqs. (1a) and (1b) into Eqs. (3a), (3b), (3c), and (3d), we have:

$$C_{ss}(\tau) = \frac{1}{2} (C_{ll}(\tau) + C_{rr}(\tau) + C_{lr}(\tau) + C_{rl}(\tau))$$
(4a)

$$C_{dd}(\tau) = \frac{1}{2} (C_{ll}(\tau) + C_{rr}(\tau) - C_{lr}(\tau) - C_{rl}(\tau))$$
(4b)

$$C_{sd}(\tau) = \frac{1}{2} (C_{ll}(\tau) - C_{rr}(\tau) - C_{lr}(\tau) + C_{rl}(\tau))$$
(4c)

$$C_{ds}(\tau) = \frac{1}{2} (C_{ll}(\tau) - C_{rr}(\tau) + C_{lr}(\tau) - C_{rl}(\tau))$$
(4d)

Similar formulas of Eqs. (4a)-(4d) have already been derived by Doviak and Zhang [2006]. In that paper, 1 and 2 are used instead of L and R to denote spaced receiving antennas. Given that the power patterns are symmetrical and matched, we have $C_{ll}(\tau) = C_{rr}(\tau)$ and $C_{sd}(\tau) + C_{ds}(\tau) = 0$. From Eqs. (4a)-(4d), we

can obtain the auto- and cross-correlations of apparent left- and right-side signals from the auto- and cross-correlations of sum and difference signals:

$$C_{ll}(\tau) = \frac{C_{ss}(\tau) + C_{dd}(\tau)}{2}$$
(5a)

$$C_{rr}(\tau) = \frac{C_{ss}(\tau) + C_{dd}(\tau)}{2}$$
 (5b)

$$C_{rl}(\tau) = \frac{C_{ss}(\tau) - C_{dd}(\tau)}{2} + C_{sd}(\tau)$$
(5c)

$$C_{lr}(\tau) = \frac{C_{ss}(\tau) - C_{dd}(\tau)}{2} + C_{ds}(\tau)$$
(5d)

Current SAI techniques are all based on spaced receiving antennas. Using Eqs. (5a)-(5d), the autoand cross-correlations of apparent left- and right-side signals can be obtained, then FCA (Full Correlation Analysis, Briggs [1984]), CCR (Cross-Correlation Ratio, Zhang et al. [2003]), SZL (Slope at Zero Lag, Lataitis et al. [1995]), and INT (Intersection Method, Holloway et al. [1997]) methods can be applied. Or we can directly use the auto- and cross-correlations of sum and difference signals to estimate apparent baseline wind which will be discussed in details in the next section.

3. Use $c_{dd}(\tau)$ and $c_{sd}(\tau)$ to estimate apparent baseline wind

Since Zhang and Doviak [2007] have already derived the analytical expression for $c_{12}(\tau)$ ($c_{rl}(\tau)$ if expressed in left and right instead of 1 and 2), and given the solutions of how to derive $c_{22}(\tau)$ and $c_{11}(\tau)$, we can use Eqs. (4a-4b) to derive the analytical expressions of the auto- and cross-correlations of sum and difference signals. If the power patterns of the array are matched and symmetrical, $c_{ll}(\tau) = c_{rr}(\tau)$. The analytical expression of $c_{ll}(\tau)$ is shown below:

$$c_{ll}(\tau) = c_{rr}(\tau) = \exp(-2jkv_{x'}(0)\tau - 2k^2(\sigma_R^2 S_{x'}^2 + \sigma_{tx'}^2)\tau^2 - 2k^2 \sigma_{e\phi}^2 v_{ay}^2 \tau^2 - 2k^2 \sigma_{e\phi}^2 v_{az}^2 \tau^2)$$
(6)

Eq. (6) is normalized by the receiving power of virtual left- or right-side signal. $v_{r'}(0)$ is the radial mean wind velocity. σ_R^2 is the second central moment of the range weighting function. $S_{x'}$ is the radial wind shear in the radial direction. $\sigma_{tx'}$ is the standard deviation of turbulence velocity in the radial $\sigma_{e\phi}$ is the azimuth effective x'direction. beamwidth, the definition of it can be seen in Zhang et al. [2007]. $v_{qy'}$ is the apparent baseline wind velocity, which is equal to $r_0 S_{v'} + v_{v'}(0)$. $S_{v'}$ is the radial wind shear in the baseline wind direction, r_0 is the distance between the transmitter and the center of resolution volume. $\sigma_{e heta}$ is the elevation beamwidth. $V_{az'}$ is the effective apparent cross-baseline wind velocity, which is equal to $r_0 S_{z'} + v_{z'}(0)$. $S_{z'}$ is the radial wind shear on cross-baseline wind direction. x' is along the beam axis direction; the y', z' plane is parallel to the array plane containing the transmitting and receiving antennas with y' parallel to the baseline and z'parallel to the cross-baseline. Δy_{12} is the distance between two phase centers alone the baseline direction. If we only consider azimuth SAI, $\sigma_{e\theta}$ is equal to 0.0085radian assuming transmitting 3-dB one way elevation beamwidth 1.53° and sum pattern receiving 3-dB one way elevation beamwidth 1.72°. $\sigma_{e\phi}$ is equal to 0.0097radian assuming transmitting 3-dB one way azimuth beamwidth 1.53° and virtual left (or right) side receiving 3-dB one way azimuth beamwidth 2.50°. $\Delta y'_{12}$ is equal to 1.22m.

The auto-correlation of difference signal can be derived according to Eq. (4b). The analytical expression of $c_{dd}(\tau)$ can be written as:

$$c_{dd}(\tau) = \exp(-2jkv_{x'}(0)\tau - 2k^{2}(\sigma_{R}^{2}S_{x'}^{2} + \sigma_{tx'}^{2})\tau^{2} - 2k^{2}\sigma_{e\theta}^{2}v_{az'}^{2}\tau^{2} - 2k^{2}\sigma_{e\phi}^{2}v_{ay'}^{2}\tau^{2})\bullet$$

$$(1 - \exp(-\frac{k^{2}\sigma_{e\phi}^{2}\Delta y_{12}^{'2}}{2})\cosh(2k^{2}\sigma_{e\phi}^{2}v_{ay'}\tau\Delta y_{12}^{'}))$$
(7)

Because the hyperbolic cosine function is symmetric about zero lag, and because the square of lag appears in the exponential functions, it is concluded that the magnitude of $c_{dd}(\tau)$ is symmetrical. The amplitude

of $c_{dd}(\tau)$ is shown in the following figure:



Figure 2: Theoretical results of the normalized amplitude of $c_{dd}(\tau)$, $v_{ay'} = 20m / s$, $v_{az'} = 0$, $s_{x'}$

=0, $v_{x'}(0) = 0$, and $\sigma_{tx'} = 1 \text{ m/s.}$ $\lambda = 0.09375 \text{ m}$, T_s

=800
$$\mu$$
s, Δy_{12} =1.22m, $\sigma_{e\theta}$ =0.0085rad and $\sigma_{e\phi}$ =0.0097rad.

The way to calculate $c_{sd}(\tau)$ has already been introduced in Eq. (4c). If the power patterns of the antenna array are matched, we have

$$c_{sd}(\tau) = \frac{1}{2} (c_{rl}(\tau) - c_{lr}(\tau))$$
(8)

In Eq. (8), $c_{rl}(\tau)$ and $c_{lr}(\tau)$ are normalized by the signal power received by the virtual left (or right) side of the array, thus the cross-correlation function of sum and difference signals $c_{sd}(\tau)$ is normalized by it

too. The analytical expression for $c_{rl}(\tau)$ $(c_{12}(\tau))$ has already been introduced in Zhang et al. [2007], and according to the properties of correlation functions, we can obtain $c_{lr}(\tau)$, thus the cross-correlation coefficient of sum and difference signals can be written as:

$$c_{sd}(\tau) = \frac{1}{2} [\exp(-2jkv_{x'}(0)\tau - 2k^2(\sigma_R^2 S_{x'}^2 + \sigma_{tx'}^2)\tau^2 - 2k^2 \sigma_{e\theta}^2 v_{az'}^2 \tau^2)(\exp(-2k^2 \sigma_{e\phi}^2 (v_{ay'}\tau - 0.5\Delta y_{12}')^2) - \exp(-2k^2 \sigma_{e\phi}^2 (v_{ay'}\tau + 0.5\Delta y_{12}')^2))]$$
(9)

The amplitude of $c_{sd}(\tau)$ is shown in the following figure:



Figure 3: Theoretical results of the normalized amplitude of $c_{sd}(\tau) \cdot v_{ay'} = 20m / s, v_{az'} = 0$, $s_{x'}$ =0, $v_{x'}(0) = 0$, and $\sigma_{tx'} = 1m/s$. $\lambda = 0.09375m$, T_s =800µs, $\Delta y'_{12} = 1.22m$, $\sigma_{e\theta} = 0.0085rad$ and $\sigma_{e\phi}$ =0.0097rad.

When the magnitude of $c_{sd}(\tau)$ is equal to the magnitude of $c_{dd}(\tau)$, we have:

$$\exp(2k^2\sigma_{e\phi}^2 v_{ay'}\tau_i \Delta y_{12}) = \exp(\frac{k^2\sigma_{e\phi}^2 \Delta y_{12}^{'2}}{2}) \qquad (10)$$

In Eq. (10), there is only one variable: the apparent baseline wind velocity. Thus we have:

$$v_{ay'} = \frac{\Delta y_{12}}{4\tau_i}$$
(11)

Eq. (11) has the same form with the Intersection Method (INT) introduced by Holloway et al. [1997]. The apparent baseline wind velocity is proportional to the separation of two virtual receiving antennas and inversely proportional to the intersection lag. Because $|c_{sd}(\tau)|$ and $|c_{dd}(\tau)|$ are symmetrical, we can't decide

the apparent baseline wind direction from the magnitude. However we can decide the wind direction from the amplitude of $c_{sd}(\tau)$ which is shown in Fig. 3. We would expect there are two intersections of $|c_{sd}(\tau)|$ and $|c_{dd}(\tau)|$. Due to the effect of turbulence, $|c_{sd}(\tau)|$ might not be symmetrical about zero lag, thus the two intersections might not have the same distance from zero lag. In order to minimize the estimating error, we assume:

 $\hat{\tau}_i = \frac{\tau_r - \tau_i}{2}$. $\hat{\tau}_r$ is the right intersection lag which

is bigger than zero, τ_l is the left intersection lag which is less than zero. However, this method would have poor performance when the intersections locate at which the magnitudes of $|c_{dd}(\tau)|$ and $|C_{sd}(\tau)|$ are small and when the apparent baseline wind velocity is small. The analytical expression of the variance of apparent baseline wind measured by the INT method using $c_{sd}(\tau)$ and $c_{dd}(\tau)$ is given below:

$$\operatorname{var}[v_{ay'(INT)}] = \left(\frac{\Delta y_{12}}{4\tau_i^2}\right)^2 \operatorname{var}[\tau_i]$$
(12a)

$$\operatorname{var}\left[\stackrel{\circ}{\tau}_{i}\right] = \frac{T_{s}^{2}}{2(\Delta_{1} - \Delta_{2})^{2}} (\operatorname{var}\left[\stackrel{\circ}{c}_{dd}(\tau_{r})\right] + \operatorname{var}\left[\stackrel{\circ}{c}_{sd}(\tau_{r})\right])$$
(12b)
$$-2 \operatorname{cov}\left[\stackrel{\circ}{c}_{dd}(\tau_{r})\right], \stackrel{\circ}{c}_{sd}(\tau_{r})\right])$$

While

$$\Delta_1 = \left| c_{dd}(\tau_1) \right| - \left| c_{sd}(\tau_1) \right| \tag{13a}$$

$$\Delta_2 = \left| c_{dd}(\tau_2) \right| - \left| c_{sd}(\tau_2) \right| \tag{13b}$$

$$\operatorname{var}\left[\left|\hat{c}_{dd}(\tau_{r})\right|\right] = \frac{1}{2M_{I}}\left[1 + \frac{\rho_{12}^{2}(\tau_{p})}{2}(1 + \exp(-\frac{\tau_{p}^{2}}{4\tau_{c}^{2}})) - (14)\right]$$
$$2\rho_{12}(\tau_{p})\exp(-\frac{\tau_{p}^{2}}{4\tau_{c}^{2}}) + \left|c_{dd}(\tau_{r})\right|^{2}\right]$$

$$\operatorname{var}[\left|\hat{c}_{sd}(\tau_{r})\right| = \frac{1 - \frac{\rho_{12}^{2}(\tau_{p})}{2}(1 + \exp(-\frac{\tau_{p}^{2}}{\tau_{c}^{2}})) + \left|c_{sd}(\tau_{r})\right|^{2}}{2M_{I}}$$

$$\operatorname{cov}[\left|\hat{c}_{dd}(\tau_{r})\right|,]\left|\hat{c}_{sd}(\tau_{r})\right| = \frac{\left|c_{dd}(\tau_{r})\right|\left|c_{sd}(\tau_{r})\right|}{2M_{I}} \bullet$$

$$\frac{\exp(-\frac{\tau_{p}^{2}}{4\tau_{c}^{2}}) - \rho_{12}(\tau_{p})\exp(-\frac{\tau_{p}^{2}}{\tau_{c}^{2}})\cosh(\frac{\tau_{r}\tau_{p}}{\tau_{c}^{2}})}{\exp(-\frac{\tau_{p}^{2}}{2\tau_{c}^{2}}) - \rho_{12}(\tau_{p})\exp(-\frac{\tau_{p}^{2}}{\tau_{c}^{2}})\cosh(\frac{\tau_{r}\tau_{p}}{\tau_{c}^{2}})}$$
(16)

In Eqs. (13a) and (13b), we assume $\tau_1 < \tau_r < \tau_2$

and $\tau_2 - \tau_1 = T_s$.

4. SA radar simulator based on the configuration of the NWRT

We use the Monte Carlo method to simulate the sum and difference signals received by the NWRT. In Monte Carlo simulation, lots of randomly located particles are placed in a scattering volume, and each particle is moving with the three-dimensional wind field. After every pulse repetition time (PRT), their locations are updated corresponding to the 3-dimension wind field and radar collects backscattering wave of each scatter and sums them together. The Monte Carlo method was first used by Holdsworth et al. [1995] to simulate atmospheric radar signals. This method was used by Zhang et al. [1998] to study target detection with the presence of clutter. Capsonni et al. [1998] simulated the radar signal by summing all the backscattered waves of randomly generated scatterers.

The details of the SA radar simulator are as follows: 1. Radar Parameters and Meteorological Parameters Antenna Diameter: 3.65 m.

Transmitting Frequency: 3.2 GHz.

PRT: 800 µs.

(15)

Transmitting one-way 3-dB beamwidth: 1.53°.

Transmitting pulse width (τ): 1.57 μ s.

Receiving one-way 3-dB azimuth beamwidth (left or right side): 2.50°.

Receiving one-way 3-dB azimuth beamwidth (sum pattern): 1.72°.

Position of the center of the resolution volume: y' = 0, z' = 0, x' = 60 km. x' is the radial direction, y' is the baseline wind direction, z' is the cross-baseline wind direction.

Position of the transmitter aperture: x' = 0, y' = 0, z' = 0.

Position of the sum pattern receiver aperture: the same as above.

Position of the virtual left side receiver aperture: y' = 0.61 m, x' = 0, z' = 0.

Position of the virtual right side receiver aperture: y' = -0.61 m, x' = 0, z' = 0.

3-dimensional mean wind velocities: $v_{z}(0) = 0$,

 $v_{x'}(0)$ and $v_{y'}(0)$ are tunable.

Size of the scattering volume: $L_{y'} = 2\phi_6^{(2)}r_0 = 3865 \text{m},$

$$L_{z'} = L_{y'} / 100 = 38.65 \text{m}, \quad L_{x'} = 7L_{z'} = 270.55 \text{m}.$$

 $\phi_6^{(2)}$ is the 6-dB two way azimuth beamwidth in radians (sum pattern 3-dB one way transmitting azimuth beamwidth is 1.53°, virtual left (or right) side 3-dB one way receiving azimuth beamwidth is 2.50°), r₀ is the distance between the center of the resolution volume and the transmitting aperture which is equal to 60km. Because in the simulation we only study the SA in y' direction, thus a smaller size on z' direction is chosen. This is based on the premise that the mean wind velocity on the z' direction is equal to zero.

Number of Scatters in the scattering volume: 1600. Number of pulses: 12500 ($T_d=10s$). Standard Deviation of Turbulence (Turbulence Intensity): It is tunable. Usually it will fall between $0\sim3$ m/s. In the simulation, the turbulence velocity is assumed to be Gaussian distributed.

Position of Particles: Particles are initially randomly uniformly distributed in the scattering volume and moved by mean wind and random turbulence.

2. The motion of scatterers

The positions of scatterers are updated with the mean wind velocity plus the turbulence velocity. After each PRT, the locations of scatterers are updated. If some scatterers move out of the scattering volume, which is quite possible when the mean wind velocity is fast while the scattering volume is small, we put in new scatterers.

For example, if a scatterer moves out +5m from the +X bound of the scattering volume, we put it to +5m plus the coordinate of -X bound of the scattering volume. Thus, the total number of scatterers in the scattering volume maintains the same during the whole dwell time.

3. Gaussian Beams

For the formation of beams that are directly determining the receiving signals, we assume they are Gaussian shaped. The angular weighting function are assumed to be in the following form:

$$A = \exp\left[-\frac{z^{2}(t)}{4r_{0}^{2}\sigma_{\theta T}^{2}} - \frac{(z'(t) - z_{r}')^{2}}{4r_{0}^{2}\sigma_{\theta R}^{2}} - \frac{(y'(t) - y_{r}')^{2}}{4r_{0}^{2}\sigma_{\phi T}^{2}} - \frac{(y'(t) - y_{r}')^{2}}{4r_{0}^{2}\sigma_{\phi R}^{2}}\right]$$
(17)

The range weighting function is in the following form:

$$W = \exp[-\frac{(x' - x_0')^2}{4\sigma_R^2}]$$
 (18)

 $\sigma_{_{ heta T}}$ is the transmitting characteristic one-way 3-dB elevation beamwidth, $\sigma_{_{\phi T}}$ is the transmitting

characteristic one-way 3-dB azimuth beamwidth. $\sigma_{\theta T} = \sigma_{\phi T} = 0.0114$ radian if we assume the array is circular shape. $\sigma_{\theta R}$ is the receiving characteristic one-way 3-dB elevation beamwidth, it is equal to 0.0128 radian; $\sigma_{\phi R}$ is the receiving characteristic one-way 3-dB azimuth beamwidth, it is equal to 0.0186 radian when considering apparent left or right side receiving antenna and 0.0128 when considering sum receiving pattern ; σ_R^2 is the second central moment of the weighting function, it is equal to $(0.35c\tau/2)^2$ (Doviak and Zrnic 2006, section 5.3); x_0' is the projection of the center of the resolution volume on the beam direction; (x'(t)) y'(t) z'(t) is the position of a scatterer; (0,0,0) is the position of the transmitter; $(0, y_r', z_r')$ is the position of the receiver. For the left side virtual receiver, $y_r' = 0.61$ m, $z_r' = 0$; for the right side virtual receiver, $y_r' = -0.61m$, $z_r' = 0$; for the sum pattern receiver, y_r ' =0, z_r ' =0. The received signal is composed of the elemental signals from all the scatterers:

$$S = \sum_{i} A_{i} W_{i} \exp[-j(r_{i} + r_{r})]$$
(19)

In the above equation i denotes the ith scatterer. r_{ti} is the distance from the transmitter to the ith scatterer, r_{ri} is the distance from the receiver to the ith scatterer.

In this simulation, the radar data is generated for signals from a single resolution volume at range r_0 .



Figure 4: The distribution of I channel data from simulation



Figure 5: The distribution of Q channel data from simulation

From the two figures above, we can see that the I and Q data are perfectly Gaussian distributed.

5. Results

In this section, the standard deviations of the apparent baseline wind retrieved using G-FCA, G-CCR, and INT(Σ, Δ_a) method with the simulation will be validated by the theory derived by Doviak et al.[2004] and Eqs. (12a)-(16) in this paper. The meteorological parameters are: $v_{x'}(0) = 0$, $s_{x'} = 0$, $\sigma_{tx'} = 0 \sim 2$ m/s,

 $v_{ay'}$ =20m/s, and $v_{az'}$ =0. The radar parameters are: λ =0.09375m, T_s=800µs, T_d=10s, and Δy_{12} '=1.22m, $\sigma_{e\theta}$ =0.0085rad, $\sigma_{e\phi}$ =0.0097rad. The number of experiment is 100. For the INT method, we use the intersection of $c_{sd}(\tau)$ and $c_{dd}(\tau)$. The results are shown below:



Figure 6: Standard deviations of $V_{ay'}$ obtained from simulation compare with theory.

From Fig. 5, we can see that the simulation results match the theory very well, which means that after we correct the differential phase and attenuations of the NWRT, we can retrieve the apparent baseline wind with the same accuracy as the conventional SA. However, when the turbulence intensity is very small, the simulation results of INT (Σ , Δ_a) method are smaller than the theoretical value. The reason for that is because in deriving the analytical expression for the variance of $\hat{\tau}_i$ we ignore the covariance term of $\hat{\tau}_l$ and $\hat{\tau}_r$. The covariance term cannot be ignored when the turbulence is small. The G-FCA and G-CCR have comparable performance when the turbulence

The I and Q channels are checked, and their distributions are shown below (sum channel):

intensity is bigger than 0.3m/s. INT (Σ, Δ_a) method has very poor performance when the turbulence is larger than 0.5m/s and will bring big estimating error. Thus, INT (Σ, Δ_a) method is only useful when the turbulence is small.

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