## Daily Mean Temperature Formulas

Glen Conner\* Kentucky State Climatologist Emeritus Western Kentucky University, Bowling Green, Kentucky

Stuart Foster State Climatologist for Kentucky Western Kentucky University, Bowling Green, Kentucky

# 1. INTRODUCTION

The mean of twenty-four hourly temperature observations made during a calendar day is accepted as the true daily mean. Few nineteenth century observers could make hourly observations of temperature. Instead, climate networks prescribed surrogate methods to approximate the true daily mean. This paper reports the results of an examination of twenty-five surrogate formulas that were prescribed to calculate the daily mean and quantifies their efficacy of approximating it.

#### 1.1 Background

The National Climatic Data Center's Climate Database Modernization Program has imaged and indexed most of the nineteenth century weather observation forms from the United States held by the National Archives. These were produced by a variety of climate networks (Miller, 1931). Digitization of the data from more than 360 stations has been completed. One of the issues that arose is the variance in observation times. Another issue is the varied methods used to calculate the daily mean temperature.

The United States Army Surgeon General issued instructions in 1818 to his climate network to determine the daily mean by adding the temperature readings at 7 a.m., 2 p.m., and 9 p.m. and dividing the sum by three. The Meteorological Society in Mannheim, Germany first used that formula in 1781 (Dewey, 1857). Subsequent climate networks prescribed formulas that also were believed to approximate the true daily mean temperature. The degree of bias that the networks considered to be acceptable varied (McAdie,1891 and Bigelow, 1909).

The sum of the maximum and minimum temperatures divided by two to produce the daily

*Corresponding author address:* Glen Conner, Department of Geography and Geology, Western Kentucky University, Bowling Green, KY 42101. Email: glen.conner@wku.edu mean has been in use since it was prescribed by the Weather Bureau after 1925. That prescription was made more as a matter of convenience and ease of calculation rather than of need. Weather Bureau stations were manned throughout the day and made hourly observations. Since then, that method of approximating the true daily mean has persisted even as automated observations eliminated convenience as a factor.

Climate change studies incorporating nineteenth century observations are now possible. To facilitate them, this paper identifies the magnitude of bias induced by each formula used in calculating the daily mean and quantifies the magnitude of the bias that resulted in the annual means.

### 2. DEFINITIONS

#### 2.1 True Daily Mean Temperature

The American Meteorological Society's glossary of meteorology (Glickman, 2000) defines mean daily temperature: "Mean of the temperatures observed at 24 equidistant times in the course of a continuous 24-hour period (normally the mean solar day from midnight to midnight according to the zonal time of the station)." In this paper, that definition is used for the true daily mean.

#### 2.2 True Annual Mean Temperature

The American Meteorological Society's glossary of meteorology defines the true annual mean: "... annual mean of air temperature based upon hourly observations at a given place, or on some combination of less frequent observations designed to represent this mean as nearly as possible." That definition is so broad that it encompasses all surrogate means. Therefore, the term "True Annual Mean Temperature" as used in this paper will refer to the annual mean of hourly temperatures.

# 3. Methodology

The original observational records, related metadata, station histories, and supporting documents were examined to identify twenty-five surrogate

formulas that had been used to approximate the true daily mean (Conner, 2008). Techniques used to apply each of those formulas to hourly data were developed using data recorded for one twelve month period at one station of the Kentucky Mesonet (Conner and Foster, 2008). Those proven techniques were applied in this study to hourly temperature data from 2008 from thirty stations (Figure 1).



Figure 1. Hourly data stations used in this study

Surrogate daily means, rounded to two decimal places were calculated in an S-Plus script by applying each formula to hourly data from 2008 that were extracted from each of the thirty stations at the hours prescribed by each formula. From them, monthly and annual means were derived. The resultant surrogate monthly and annual means produced by each formula at each station were compared to the true monthly mean derived from the hourly observations from that station. The derived biases were the basis for subsequent analyses.

#### 4. RESULTS

Researchers previously found there were significant differences in the means produced by different surrogate formulas. Twenty-five of those formulas (F1 through F25) are evaluated here. The formulas used local standard time for the hours of observation (h1 through h24) or the extreme maximum (xmax) and minimum (xmin) temperatures. For this study, they were divided into five groups to assess each formula's bias in approximating the true daily mean. Those groups are discussed in the following sub-paragraphs.

## 4.1 True Monthly Mean Formula

Hourly data used in this study were the average of samples taken by the thirty ASOS stations for the previous five minutes ending with the last observation of each hour. The true daily mean was found by dividing those hourly values by 24. The true monthly mean was found by dividing the sum of the true daily means by the number of days in the month. The annual mean was derived from those twelve values. That annual mean was the baseline against which each formula's mean was compared to determine bias at each station.

## 4.2. Paired Observations Formulas

The means of homonymous observation times were one formula group that was thought to produce an acceptable approximation of the true daily mean. The symmetry was appealing and the pairs offered convenient observation times to the observers.

> F2 = (h10+h22)/2 F9 = (h06+h18)/2 F17= (h07+h19)/2 F19= (h08+h20)/2 F21= (h09+h21)/2

When these formulas used the 2008 hourly data in this study, all five produced annual means that were from 0.29°C to 0.96°C colder than the true mean. The least bias was the one (F2) advocated in England for a 10 a.m. - 10 p.m. pair (Adie, 1831).

The formula (F19) used by the Signal Service and the Weather Bureau for fifty-five years from 1870 through 1925 is of particular interest. It used the 8 a.m. and 8 p.m. pair and produced an annual mean that was 0.96°C colder than the true mean in this study. That was the greatest cold bias of all 25 formulas applied.

# 4.3 Triad Observations Formulas

Twelve Triad formulas that used three observations each day were used to calculate the daily mean during the nineteenth century. In this study, eight of the resultant annual means were from 0.08°C to 1.01°C warmer than the true mean; four were from 0.11°C to 0.36°C colder. Dove's formula used by the New York Academies (F8) in the early 1800's used temperatures at 6 a.m., 2 p.m., and 10 p.m. That formula (F 7) produced the least bias of the triads of 0.08°C warmer than the true annual mean. Between 1818 and 1855, the Surgeon General used the Mannheim method (F15) to add the 7a.m., 2 p.m., and 9 p.m. observations and divide by three for the daily mean. That formula (F 15) produced the second best of the triad group; an annual mean with 0.10°C warmer than the true annual mean.

 $\begin{array}{l} F5 &= (h06+h12+h17)/3\\ F6 &= (h06+h12+h19)/3\\ F7 &= (h06+h14+h21)/3\\ F8 &= (h06+h14+h22)/3\\ F10 &= (h07+h12+h18)/3\\ F11 &= (h07+h13+h21)/3\\ F12 &= (h07+h13+h22)/3\\ F14 &= (h07+h14+h20)/3 \end{array}$ 

F15= (h07+h14+h21)/3
F16= (h07+h15+h23)/3
F18= (h08+h14+h20)/3
F20= (h09+h15+h21)/3

#### 4.4. Combined Observations Formulas

Four of the triad formulas for the daily mean were modified to more closely approximate the true mean. Those modifications produced annual means that were from only  $0.02^{\circ}$ C colder (F3 and F4) to  $0.11^{\circ}$ C (F 1) warmer than the true mean. The most widely used of these was the Smithsonian Institution's that used the F-13 formula (Guyot, 1855) (the sum of 7 a.m., 2 p.m., and two times the 9 p.m. reading, divided by four) from 1850 to 1870. The Surgeon General and the New York Academies also used the Smithsonian formula from 1855 to 1888 (Conner, 2008). In this study, its bias was only  $0.09^{\circ}$ C warmer than the true annual mean.

The Signal Service's use of six observations (F3) at an interval of four hours each day, added and divided by six, also produced an accurate approximation. However, the time demands on observers caused it to be abandoned after one year of use.

> F1 = (h07+h14+h14+h21+h21+h31)/6 F3 = (h03+h07+h11+h15+h19+h23)/6 F4 = (h03+h09+h15+h21)/4F13= (h07+h14+h21+h21)/4

## 4.5 Maximum-Minimum Formula

Maximum and minimum thermometers were first used to capture the extremes of the day. They were recorded and reported but not used in daily mean calculations. A formula that allowed reading of those thermometers just once per day at a time chosen by the observer, offered an irresistible convenience to them. Just add the maximum and minimum and divide by two to approximate the daily mean. That formula (F22) was adopted by the Weather Bureau in 1926 and has been used since that time. In this study, it's annual mean was 0.09° warmer than the true man.

Three other methods were proposed (Hann, 1903) to modify that formula but none were more accurate and none found acceptance. Those decisions were good ones. All three produced colder annual means; from 0.31°C to 0.45°C colder than the true mean.

F22= (xmin+xmax)/2 F23= (xmin+h15])/2 F24= (xmin+xmax+h08+h20)/4 F25= (xmin+xmax+h09+h21)/4

## 5. DISCUSSION

### 5.1 Annual Bias in Formulas' Results

The hourly data from 1 January 2008 through 31 Dec 2008 were used in this study. Data came from thirty ASOS stations. Those data were used to calculate the bias of the annual means and standard deviations of each of the twenty-five formulas. In Figure 2, each formula is plotted by its number with reference to its mean and standard deviation.



Figure 2. Annual means and standard deviations of the biases of each of the twenty-five formulas

The formula (F3) used by the Signal Service required six observations per day at four-hour intervals. The observation frequency produced the least bias  $(0.02^{\circ}C \text{ colder})$  with the least standard deviation  $(0.03^{\circ}C)$ . The frequency of observation also caused it to be abandoned after a very short time.

Of the major climate networks, the least biased formula (F-15) was the one used by the earliest network, the Army Surgeon General's, from 1818 to 1840. The bias of its mean was just  $0.10^{\circ}$ C warmer than true annual mean with a standard deviation of only  $0.08^{\circ}$ C (Figure 3).



#### Figure 3. Annual mean of formula 15 used by the Surgeon General

The paired formulas were the only formula group to be consistent in their bias; all were colder than the true annual mean. The one (F 19) used by

the Signal Service and Weather Bureau from 1870 to 1925 was the coldest of the group at 0.96°C colder.

Of the triad formula group, eight of the twelve were warmer than the true mean. The variance in the bias ranged widely from 1.01°C warmer to 0.36°C colder. Generally, earlier morning and afternoon observations caused warmer readings; later evening readings caused colder ones. The least bias came from the Surgeon General and Signal Service formula (F15), only 0.10°C warmer.

The combination formula group had the least variation in biases. Only one of them was used by a major network. That one (F 13), used by the Smithsonian, produced a mean that was only 0.09°C warmer than the true annual mean.

Four formulas used the maximum and minimum temperatures. Three were from 0.31°C to 0.45°C colder than the true annual mean. In our evaluation, the formula (F 22) used by the Weather Bureau and the National Weather Service since 1925 produced an annual mean that was just 0.09°C) warmer than the true mean. Previous contemporaneous studies in the nineteenth century had found that its formula (sum of maximum and minimum divided by two) would produce annual means about 0.3°C warmer than the true mean (Buchan, 1868).

# 6. CONCLUSIONS

Climate networks prescribed times for their observers to make temperature measurements. Those times were accompanied by a formula for calculating the daily mean. Times and formulas combined to produce an approximation of the true daily mean.

The digital data now being accumulated through the Climate Database Modernization Program offers the first opportunity for climatologists to understand nineteenth century climate using actual observations. To avoid misunderstanding and misinterpreting the results, each station's data should be evaluated considering the times of observation and the formula it used at that location.

This study confirms that the times of observation and the formula used introduces a bias into the mean annual temperature. For the major climate networks, that bias is minimal if the formula prescribed by the network is used to calculate it. The technique used herein should be applied to other stations that have or had hourly temperature data for comparison with their ancestor stations that have nineteenth century temperature data. The study is continuing and will discover if any spatial or seasonal variance in the biases can be identified.

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