3.3 INVERTING SURFACE OBSERVATIONS TO FIND BOUNDARY LAYER DEPTH

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1. Motivation

Traditional methods to retrieve the depth of the convective boundary layer, z_i, include measurements from lidar, sodar, radiosondes, and instruments mounted on aircraft, balloon, etc (Stull 1988). In terms of man-power and cost, these methods are expensive. Therefore, it would be convenient if cheap and more efficient method was available to determine z_i. In the convective boundary layer, the largest eddies or the boundary layer spanning eddies (BLSE) extend from the surface to the stably stratified capping inversion layer. Therefore, surface observations are influenced by the eddies, and because these eddies are influenced by z_i, the boundary layer depth also has an impact on surface layer observations. The influence of the boundary layer depth on surface layer observations is observed in standard deviation of the along-wind and cross-wind velocity components (Panofsky et al. 1977).

$$\sigma_{u,v} = u_*\left(12 - 0.5\frac{z_i}{L}\right)^{\frac{1}{3}} \quad (1)$$

where L is the Monin-Obukhov length, u_* is the friction velocity, and $\sigma_{u,v}$ is the standard deviation of the along-wind and cross-wind component. This equation shows that $\sigma_{u,v}$ does not obey either Monin-Obukhov or mixed-layer similarity theory because these variables are not dependent on $\frac{z}{L}$ in the surface layer or $\frac{z}{z_i}$ in the mixed layer. Instead, $\sigma_{u,v}$ is approximately constant in the vertical direction, and is a function of the stability parameter $\frac{z_i}{L}$; this is a result of the BLSEs. Solving (1) for z_i yields

$$z_i = \left(12 - \left(\frac{\sigma_{u,v}}{u_*}\right)^3\right)2L \quad (2)$$

Therefore, if the Monin-Obukhov length, the friction velocity, and the standard deviations of the horizontal wind are known, then it is possible to solve for the boundary layer depth. These three variables will be determined from surface layer observations

2. Determining L, $\sigma_{u,v}$, and u_* From Surface Layer Observations

We developed two methods to determine these variables. The first method requires wind, or wind and temperature measurements, at several different vertical levels in the surface layer, and the second method requires calculation of relevant fluxes.

a. a. Optimization Method

For the optimization method, an optimization technique determines L and u_* by matching a vertical profile of averaged wind or temperature observations with vertical wind and temperature predictions from similarity theory. The equation for the mean wind as a function of height in the surface layer is given by (Panofsky and Dutton 1984)

$$U(z) = \frac{u_*}{k}\left(\ln\left(\frac{z}{z_o}\right) - \Psi_u\left(\frac{z}{L}\right)\right) \quad (3)$$

where k is the von Karman constant, z_o is the unknown roughness length and $\Psi_u\left(\frac{z}{L}\right)$ is a function that decreases the convexity of $U(z)$ in unstable conditions and represents how the BLSEs change the mean wind profile. If a time series of wind observations are available at several vertical levels in the atmospheric surface layer, then we can average these observations and equate them with (3) plus an error

$$U_{ob,n}(z_n) = \frac{u_*}{k}\left(\ln\left(\frac{z_n}{z_o}\right) - \Psi_u\left(\frac{z_n}{L}\right)\right) + E \quad (4)$$

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where $U_{ob,n}(z_n)$ is the mean wind at a height z_n, n represents the observation number, and E describes the error of the wind prediction. From (4), we can form a cost function that an optimization technique will minimize

$$\left(\sum_{n=1}^{N_{ob}}\left(U_{ob,n}(z_n)-\frac{u_*}{k}\left(\ln\left(\frac{z_n}{z_o}\right)-\Psi_u\left(\frac{z_n}{L}\right)\right)\right)^2\right)^{\frac{1}{2}}=E_u' \qquad (5)$$

where N_{obs} denotes the number of observation levels and E' is an alternate form of the error. The goal of the optimization technique is to minimize E_u' by estimating proper values of L, z_o, and u_*. Similarly, if multi-level temperature observations are available instead of wind we can do the same process with the Monin-Obukhov similarity equation for mean temperature. Here, we have

$$T(z)=T_o+\frac{T_*}{k}\left(\ln\left(\frac{z}{z_o}\right)-\Psi_T\left(\frac{z}{L}\right)\right) \qquad (6)$$

where $T(z)$ is the predicted vertical profile of the mean temperature, T_o the surface temperature, T_* is given by $\frac{\overline{u'T'}}{u_*}$ where u' and T' are the turbulent components for streamwise velocity and temperature respectively, and $\Psi_T\left(\frac{z}{L}\right)$ is analogous to $\Psi_u\left(\frac{z}{L}\right)$ except for temperature (Panofsky and Dutton 1984). Similarly the cost function has the form

$$\left(\sum_{n=1}^{N_{ob}}\left(T_{ob,n}(z_n)-\left(T_o+\frac{T_*}{k}\left(\ln\left(\frac{z_n}{z_o}\right)-\Psi_T\left(\frac{z_n}{L}\right)\right)\right)\right)^2\right)^{\frac{1}{2}}=E_T' \qquad (7)$$

For this formulation, the optimization technique must search for T_o, T_*, z_o, and L. Equation (2) requires the friction velocity, therefore we solve for this variable via the relation

$$u_*=\left(\frac{(L\times T_*\times k\times g)}{\overline{T}}\right)^{\frac{1}{2}} \qquad (8)$$

where $\overline{T}$ is the mean temperature and g is the gravity. Now that we have estimates of the Obukhov length and the friction velocity, only a measurement of the standard deviation of the horizontal wind is necessary. This variable is determined from the observed wind.

b. Flux Method

The variables required to solve for z_i in (2) can be determined if measurements of atmospheric fluxes are available. Therefore, instead of estimating these variables via numerical optimization, they can be directly computed from surface layer measurements. The definition of L is given by

$$L=-\frac{{u_*}^3}{k\frac{g}{\overline{\theta_v}}\overline{w'\theta_v'}} \qquad (9)$$

where $\frac{g}{\overline{\theta_v}}\overline{w'\theta_v'}$ is the buoyancy flux, w' is the turbulent vertical velocity, θ_v' is the turbulent virtual potential temperature and $\overline{\theta_v}$ is the mean virtual potential temperature. If observations of these variables are taken, then it is possible to compute the Monin-Obukhov length. Similarly, the definition of u_* is given by

$$u_*=\left(\left(\overline{u'w'}\right)^2+\left(\overline{v'w'}\right)^2\right)^{\frac{1}{4}} \qquad (10)$$

where u' and v' are the along-wind and cross-wind turbulent velocity components respectively. If the momentum fluxes $\left(\overline{u'w'}\right)$ and $\left(\overline{v'w'}\right)$ are computed from atmospheric data, then an observation of u_* is available. Finally, if $\sigma_{u,v}$ is also calculated is a similar manner as above, we have all the variables necessary in (2) to calculate z_i. The flux method should be more accurate than the optimization method because more of the unknown variables are determined via atmospheric measurements

2. Results

To test both of these methods, data was generously provided from Howard University whom participated in the D.C. PBL variability experiment. This data included surface layer observations of temperature at 8 vertical levels, and surface layer winds recorded at 3 vertical levels. Further, turbulent fluxes were provided by this data set. Participants in the PBL study at Howard University also launched radiosondes to determine PBL depth at 1300, 1500, 1700, 1900 and 2100 UTC on four days (September 14-15 & 19-20). These data were more than sufficient to test the surface layer z_i calculation methods.

a. Optimization Method

Success of the optimization method is dependent on accurately determining the convexity of the temperature and wind profiles, because the convexity is determined by L and z_o. Because the data provided has more temperature data than wind data we use the temperature data for the fit. This ample dataset should give the algorithm enough information to determine the convexity of the temperature profile, allowing successful determination of the unknown variables. The temperature data are averaged over a time period of one hour, which may not be a sufficient averaging time for (6) to be a valid model of vertical temperature. Although, the data is averaged, it may contain both sampling noise and calibration errors. To combat these errors, we employ a hybrid Genetic Algorithm (GA), a powerful optimization technique, to provide a best estimate of the unknown variables (Haupt and Haupt 2004). The GA process is explained thoroughly in Haupt and Haupt (2004), thus, an explanation of the GA is omitted here. The GA minimizes (7) until the prescribed number of iterations is exceeded or convergence occurs. As in Long et al (2010), a Nelder Meade Downhill Simplex (NMDS) algorithm then uses the GA solution to provide a better estimate of the unknown variables. Therefore, the GA is used to find the appropriate solution basin and the NMDS cascades down that basin to obtain the global minimum in the cost surface.

Results from the optimization method are shown in Figure (1). Preliminary research shows that this method did not accurately estimate the depth of the convective boundary layer. In fact, the estimates of z_i are scattered, and there exist time periods where no estimate of z_i is available. The latter occurs because the algorithm determined that the observed temperature profile is characteristic of a neutral atmosphere, wherein L approaches infinity and (1) is not valid. Although there are some accurate estimates of the boundary layer depth, the scatter over all estimates is too large. Therefore, the optimization technique was not able to consistently determine the unknown variables.

b. Flux Method

The data set provided by Howard was sufficient to calculate all of the unknown variables in (1) via observations. The results of this method are shown in Figure (2). The figure shows that although the flux method was much more successful than the optimization method, it still did not provide accurate estimates of the boundary layer depth. Interestingly, the results for September (14-15) display a lot of scatter, while results for September (19-20) are more consistent. A likely cause for the scatter is due to non-stationarity of the atmosphere. Another interesting feature is that predictions of boundary layer depth on September (19-20) generally under-predict z_i. Consistency of the predictions for these two days is promising; however, the estimates are consistently below the observed boundary layer depth.

3. Conclusions

The above methods to calculate z_i did not provide accurate estimates of the depth of the atmospheric boundary layer. The failure of these methods could be attributed to several causes. First, implicit in the assumption of similarity theory is stationarity of the turbulence. Any deviations from this assumption will cause this theory, and thus our formulations, to break down. A second culprit is observational noise. Because $\sigma_{u,v}$ and u_* are both cubed in (2), any instrumental error in these variables causes a much larger error in z_i. Further, for the optimization method, scatter in the mean temperature or mean wind profiles (although more-so the temperature than the wind) makes it difficult to determine the convexity of these profiles, and hence it is difficult to accurately estimate L. Likewise, any calibration issues could adversely impact the profile curvature on which the optimization method depends. A third error may come from the equation for $\sigma_{u,v}$ given by (1). This equation may need to be re-evaluated because of the large scatter in the original observations from which it was developed. A fourth and final error may be caused by considering measurements at only a single point. Averaging observations over several points can give a more accurate representation of the mean properties of the turbulent ABL. Unfortunately, the above methods lose their efficiency if several observations stations are necessary. Therefore, the verdict does not look promising for determining z_i from surface layer observations, but there is still some hope. In future work, we will also investigate other methods of determining z_i from surface layer observations.

4. Acknowledgements

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6. Figures

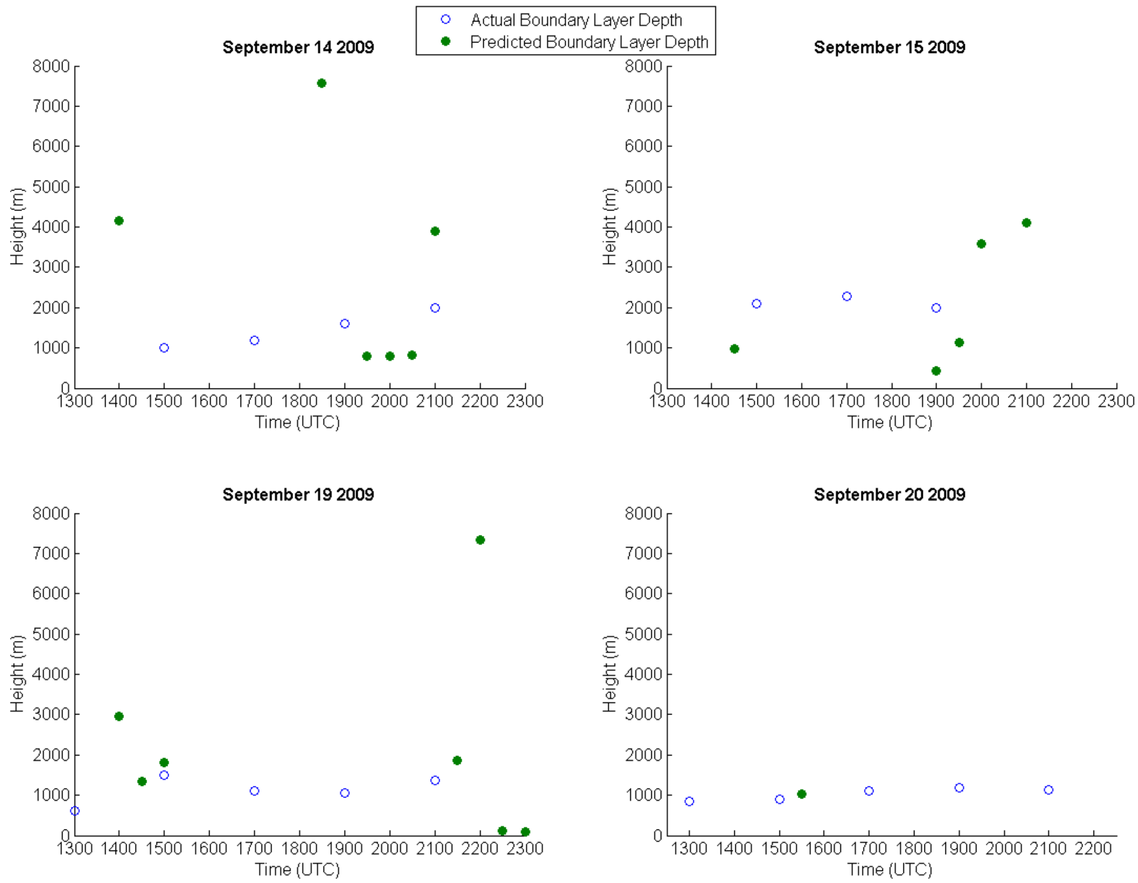

Figure 1: Boundary Layer Depth Predictions from the Hybrid GA Optimization Method

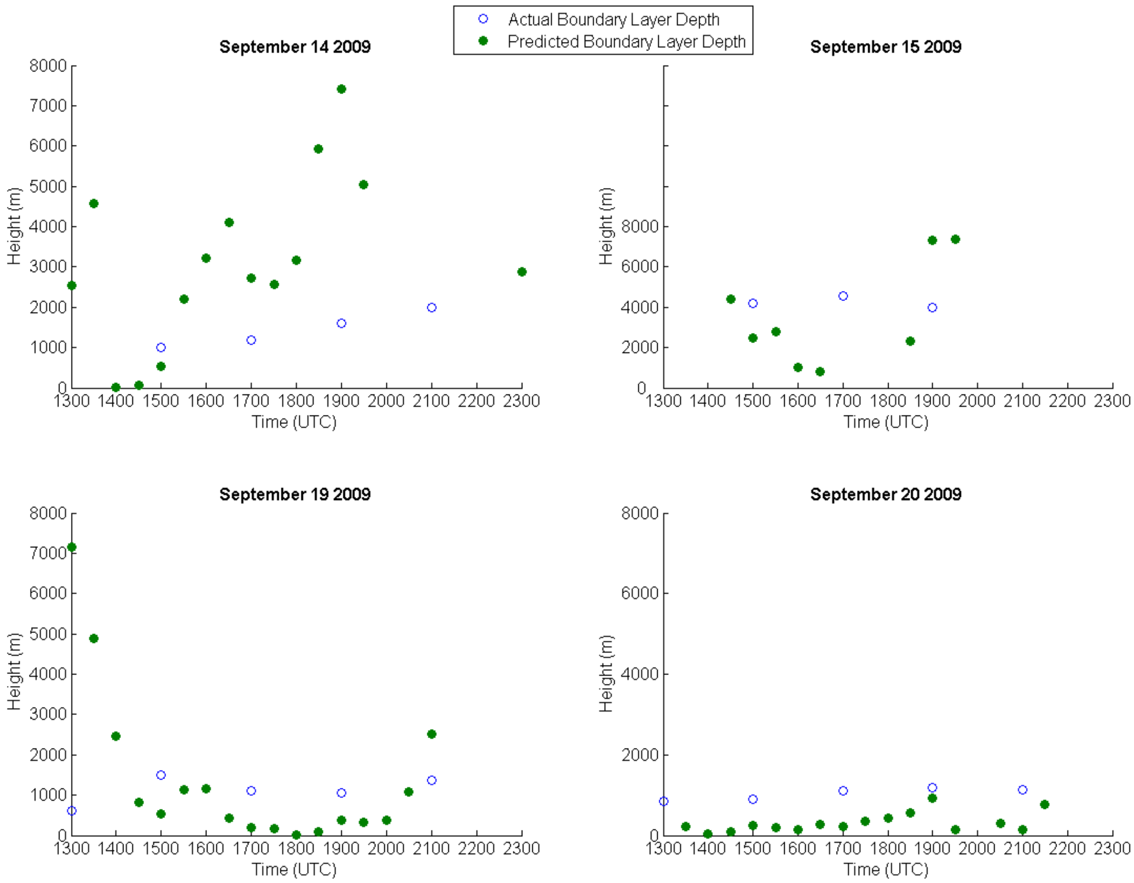

Figure 2: Boundary layer depth predictions from the Flux Method