

# Nonlinear Optical Flow for Verification

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**Introduction:** An image processing method, called optical flow, is employed for performing verification of spatial forecasts of sea-level pressure. Many variations of the method have been examined in the literature (Keil and Craig MWR 2007; Marzban and Sandgathe 3<sup>rd</sup> international workshop on verification, 2007). Some methods are purely data-driven where no assumptions are made regarding the relationship between a forecast field and an observed field. The method presented here is a highly parametric method which does impose assumptions on the relationship. The method is also nonlinear, and so, is sufficiently flexible to make for a useful verification method. Its parametric structure allows for “explanations” which would otherwise be impossible.

## Background:

Let  $I_o(x,y)$ ,  $I_f(x,y)$  = intensity of observed and forecast fields (e.g., sea-level pressure). Traditional optical flow assumes:

$$I_o(x,y) \sim I_f(x+dx, y+dy)$$

$$I_o(x,y) \sim I_f + \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy$$

Given two fields  $I_o$  and  $I_f$ , and their spatial derivatives, one has data on the corresponding terms in the above equation. The parameters  $(dx,dy)$  can be estimated (as regression coefficients).

**Optical flow field** = all pairs  $(dx,dy)$ , one per grid point, each estimated from data surrounding that grid point in a window of size  $W$ .

**Method:** Relax the optical flow assumption to allow for a change in intensity, and retain higher-order terms in Taylor expansion:

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More details available at  
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$$I_o(x,y) \sim A(x,y) + I_f + \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy + \\ + \frac{\partial^2 I}{\partial x^2} dx^2 + \frac{\partial^2 I}{\partial y^2} dy^2 + \frac{\partial^2 I}{\partial x \partial y} dxdy$$

Find maximum likelihood estimates of 3 parameters (error components):

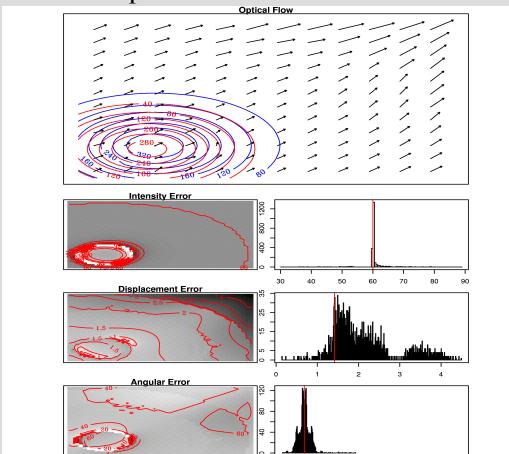
$(dx,dy)$  = displacement error

$A(x,y)$  = intensity error,

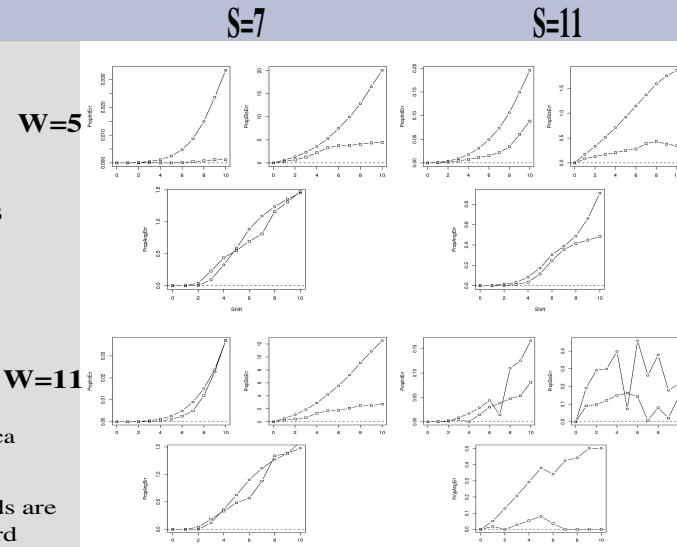
Also show  $(dx,dy)$  as magnitude of displacement error and angular error.

Details of this optical flow method are at  
<http://faculty.washington.edu/marzban/optica1.pdf>

**Simulation:** Observed and forecast fields are taken to be gaussian humps with standard deviation  $S$ , and they are shifted from 1 to 10 grid lengths apart. For each shift, the proportion of error is computed for the three error components.



Two gaussian humps with standard deviation  $S=7$ , the flow field (top), the intensity error (middle), and magnitude and angle of displacement error.  $W=5$

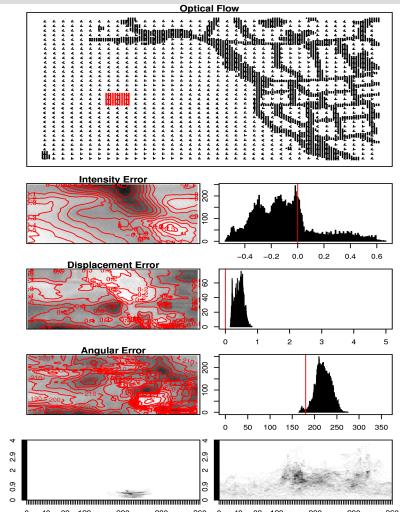


Proportion of Error in intensity (upper-left), in displacement (upper right), and angle (bottom) as a function of shift between gaussian humps with  $S=7$  &  $S=11$ , with  $W=5$  and  $W=11$ .

Circle = linear,  
Square = nonlinear..

**Conclusion 1:** For relatively large objects, nonlinear optical flow performs better than linear optical flow, in terms of all three measures of performance (intensity error, and magnitude and direction of displacement error) regardless of window size  $W$ . For smaller objects the nonlinear model does better only in terms of intensity and magnitude of displacement error; in terms of the direction of the displacement error, the two methods perform comparably.

**Real Data:** SLP forecasts from the University of Washington WRF-ARW model executed at 36km resolution using NCEP GFS initial and boundary conditions are assessed in the nonlinear optical flow approach. The analysis is performed over 273 days (4/2/08-3/31/09).



**Conclusion 2:** Intensity errors (Obs - forecast) are found to be negatively biased, about 0.2 hPa. They are mostly contained to the Canadian Rockies. The magnitude of displ. errors is small (< grid length), but there is a small directional bias.

A comparison with MM5-GFS (UW) and COAMPS-NOGAPS (APL) suggests that no single model is better than the other two in terms of all three measures of performance.