1. INTRODUCTION

Predictions of air pollutant dispersion over urban areas is a challenging problem because of the complexity of the topography, the number of physical factors governing the phenomenon, and their inherent uncertainty.

We focus on the mean concentration field at ground level, which is a fundamental risk assessment factor. In general, the consensus is that the maximum ground-level mean concentration $C$ as a function of the alongwind distance from the source $x$ decays according to a $C \propto x^{-2}$ power law. Robins and Cheng (2003) present an empirical relationship establishing an upper bound to the observed concentrations:

$$\frac{CU}{Q} = K_0 \frac{x^2}{x^2}$$

where $U$ is the mean wind speed at building height, $Q$ is the mass release rate, and the constant $K_0 = 25$. This relationship was subsequently tested against several data sets, proving to be a satisfactory first approximation.

Hanna et al. (2007) present field data for the evolution of concentration with distance for the field studies at Oklahoma City (JU2003), Salt Lake City (URBAN 2000) and London (DAPPLE). There was approximate agreement with a $x^{-2}$ power law evolution in accord with Eq. (1) up to a distance of 1000 m downwind of the source. However, the value of the constant $K_0$ varied with the location and time of day. A value of $K_0 = 10$ provided a good fit to the nighttime data at Salt Lake City and Oklahoma City. Daytime data at Oklahoma City and London yielded a value of $K_0 = 3$. In other words, the concentration data show a marked difference of about a factor of three or four between day and night releases.

We propose a dispersion model based on a Gaussian formulation of dispersion, where the horizontal diffusion coefficient is determined by the theory of Taylor (1921), and the vertical diffusion coefficient by the theory of Hunt and Weber (1979).

2. DISPERSION MODEL

We assume that the mean concentration field $c$ of a tracer emitted from a continuous source can be approximated by a reflected Gaussian plume model, which for ground-level releases takes the form

$$c = \frac{Q}{\pi U \sigma_y \sigma_z} \exp \left( - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2} \right)$$

where $y$ indicates the crosswind direction, $z$ the vertical direction, $\sigma_y$ and $\sigma_z$ are the standard deviations of the crosswind and vertical distributions of concentration, respectively, and the source is located at $y = 0$.

The coefficient $\sigma_y$ is calculated according to the theory of Taylor (1921), which for an exponential velocity autocorrelation function (Neumann, 1978; Tennekes, 1979) gives

$$\sigma_y^2 = \sigma_{y0}^2 + 2\sigma_y^2 \tau_y \left[ \frac{t}{T_y} + \exp \left( - \frac{t}{T_y} \right) \right]$$

where $\sigma_{y0}$ is the plume crosswind standard deviation at the source, $\sigma_y^2 = \langle v^2 \rangle$ is the variance of the Lagrangian crosswind velocity $v$, and $T_y$ is the turbulence crosswind decorrelation time scale. The coefficient $\sigma_z$ is described by the theory of Hunt and Weber (1979), which was originally developed to model dispersion from ground level sources in neutral atmosphere. The original formulation was modified to include a vertical limit as follows:

$$\sigma_z^2 = \sigma_{z0}^2 + \frac{b^2 \sigma_y^2 \tau_y^2}{1 + b^2 \sigma_z^2 \tau_y^2} \left( \frac{2L_z}{\pi} \right)$$

where $\sigma_{z0}$ is the plume vertical standard deviation at the source, $\sigma_z^2 = \langle w^2 \rangle$ is the variance of the Lagrangian vertical velocity $w$, $L_z$ is the turbulence vertical length scale, and $b$ is an empirical constant. We used $b = 0.5$ for nighttime atmosphere, and $b = 1$ for daytime atmosphere.

3. EXPERIMENTS

The model is tested with data from four urban dispersion experiments: i) Oklahoma City (JU2003) (Allwine et al., 2004; Clawson et al., 2005; Hanna et al., 2007; Dugway Proving Ground, 2005); ii) Salt Lake City (URBAN 2000) (Allwine et al., 2002; Hanna et al., 2003; Hanna et al., 2007); iii) London (DAPPLE, 2002; Neophytou and Britter, 2004; Hanna et al., 2007); and iv) St. Louis (McElroy and Pooler, 1968).

The horizontal and vertical turbulence time scales were estimated as $T_y = L_y/\sigma_y$ and $T_z = L_z/\sigma_z$, respectively. $L_y$ is the horizontal turbulence length scale, and $L_z$ was assumed to equal the height of the boundary layer or the capping inversion. Because no direct measurements of $L_y$ and $L_z$ are available, the same values of $L_y$ and $L_z$ were used for all the experiments: we assumed $L_y = 2000$ m and $L_z = 800$ m for daytime conditions; $L_y = 1000$ m and $L_z = 200$ m for nighttime conditions. Also, where turbulent velocity variance observations were not available, the following similarity relationships were used:

$$\sigma_y = 1.9 u_*; \quad \sigma_z = 1.3 u_*$$

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Table 1: Summary of turbulence and flow characteristics in the experiments. \( \overline{U} \) is the advection velocity averaged over all experiment trials.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Stability</th>
<th>( \overline{U} ) [ms(^{-1})]</th>
<th>( \sigma_v ) [ms(^{-1})]</th>
<th>( \sigma_w ) [ms(^{-1})]</th>
<th>( T_y ) [s]</th>
<th>( T_z ) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt Lake City</td>
<td>nighttime</td>
<td>0.49</td>
<td>0.25</td>
<td>0.16</td>
<td>4082</td>
<td>1237</td>
</tr>
<tr>
<td></td>
<td>daytime</td>
<td>1.03</td>
<td>0.52</td>
<td>0.34</td>
<td>3883</td>
<td>2354</td>
</tr>
<tr>
<td>Oklahoma City</td>
<td>nighttime</td>
<td>2.08</td>
<td>0.99</td>
<td>0.68</td>
<td>1010</td>
<td>294</td>
</tr>
<tr>
<td></td>
<td>daytime</td>
<td>2.13</td>
<td>1.09</td>
<td>0.70</td>
<td>1835</td>
<td>1143</td>
</tr>
<tr>
<td>St. Louis</td>
<td>nighttime</td>
<td>2.72</td>
<td>0.45</td>
<td>0.30</td>
<td>2208</td>
<td>669</td>
</tr>
<tr>
<td></td>
<td>daytime</td>
<td>2.79</td>
<td>1.76</td>
<td>1.20</td>
<td>1137</td>
<td>665</td>
</tr>
<tr>
<td>London</td>
<td>daytime</td>
<td>3.00</td>
<td>1.08</td>
<td>0.72</td>
<td>1845</td>
<td>1118</td>
</tr>
</tbody>
</table>

Table 2: Error measures between observed and modeled data

<table>
<thead>
<tr>
<th></th>
<th>Corr</th>
<th>FB</th>
<th>NMSE</th>
<th>VG</th>
<th>Fac2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nighttime</td>
<td>0.83</td>
<td>0.34</td>
<td>1.62</td>
<td>1.92</td>
<td>63.64</td>
</tr>
<tr>
<td>Daytime</td>
<td>0.84</td>
<td>0.50</td>
<td>2.19</td>
<td>1.82</td>
<td>64.71</td>
</tr>
<tr>
<td>All data</td>
<td>0.73</td>
<td>0.07</td>
<td>1.78</td>
<td>1.87</td>
<td>64.13</td>
</tr>
</tbody>
</table>

where \( u_* \) is the friction velocity. The relevant turbulence and flow characteristics for the above experiments are reported in Table 1, where the advection velocity averaged over all the experiment trials is indicated by \( \overline{U} \).

4. DATA ANALYSIS AND COMPARISON

A finite source size corresponding to \( \sigma_{yo} = \sigma_{zo} = 3 \) m was used in the model. Because the source size affects the modeled concentration only at short distance from the source, its effects are practically negligible at the range of distances included in our analysis.

Figure 1 shows the predicted versus observed values of \( \frac{C}{Q} \) for the experiments conducted in nighttime conditions (left panel), daytime (center panel), and combined daytime and nighttime data (right panel). The nighttime observations include 121 data points spanning over about three decades of concentration values, collected over distances from the source ranging about two orders of magnitudes. The daytime observations include 102 data points spanning about four decades of concentration values, collected over about three decades of distances.

The accuracy of the prediction has been assessed by several error measures including correlation (Corr), fractional bias (FB), normalized mean square error (NMSE), geometric variance (VG) and percentage of data within a factor 2 from the observations (Fac2), all of which are reported in Table 2.

5. DISCUSSION

We proposed a model to describe dispersion in urban areas based on a simple Gaussian formulation, where the diffusion coefficients are determined by the theories of Taylor (1921) and Hunt and Weber (1979). The model applies to both daytime and nighttime cases.

To validate the model, we have analyzed field data from dispersion experiments conducted in Oklahoma City, Salt Lake City, London, and St. Louis. The data cover about three decades of concentration for the nighttime cases, and about four decades for the daytime cases.

The agreement between data and theory is good for both daytime and nighttime conditions. Because the same Lagrangian similarity theory hypotheses were used for daytime and nighttime atmosphere, the good agreement suggests that nocturnal stratification in urban areas is weak enough not to alter the dispersion mechanisms observed in neutral atmosphere. Nevertheless, the stratification effects on dispersion are not negligible, as shown by a smaller vertical dispersion coefficient.

6. ACKNOWLEDGEMENTS

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REFERENCES


FIG. 1: Predicted versus observed concentration data scaled by the release rate $Q$ for all experiments. Left: nighttime conditions; center: daytime conditions; right: night- and day-time experiments combined.


Neophytou, M. K. and R. E. Britter, 2004: Comparison of predictions from the ASUDM (Another Simple Urban Dispersion Model) with results from the DAPPLE tracer field experiment. *DAPPLE Cambridge Note 4*.


