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TORNADOES, THOMSON, AND TURBULENCE: AN ANALOGOUS PERSPECTIVE ON TORNADOGENESIS AND ATMOSPHERIC COHERENT STRUCTURE

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1. INTRODUCTION

Theoretical development regarding atmospheric turbulence has been one of the most challenging scientific problems over the last century. Given the breadth of application (e.g. wind energy, tornadogenesis, and climate change), it is clear why there is such an interest in the subject. However, while incremental advances have been made in recent years (with the help of exponentially improving computational resources) there has not been a clear paradigm shift in some time. Even so, there is a slowly growing body of literature utilizing the potentially synergistic relationship between the fields of electro-magnetism (EM) and hydrodynamics (HD). This synergy is based on the similarity between the equations that govern fluid dynamics and the equations that govern electro-magnetism. Thomson (1931) was one of the first to employ this type of "analogous thinking" when developing a visual framework for electromagnetism. A table of analogous variables has been compiled from several sources (*Belevich*, 2008; *Marmanis*, 1998, *Pinhiero*, 2009) in Fig. 1. Exploring and exploiting this analogous relationship may be a catalyst for a radical shift in how coherent structure in fluids is studied.

2. VORTEX DYNAMICS: METHOD AND DIAGNOSTICS

One of the main struggles in the numerical modeling of tornadoes is the tendency for over-production of turbulent diffusion in the vicinity of the vortex. Several methods have been developed to combat this numerical loss of kinetic energy, including "vortex confinement", (e.g. *Steinhoff and Underhill*, 1994) and other antidiffusion methods. However, it seems desirable to find physical reasoning for why a tornado does not break down into turbulence, given the extreme deformation fields near the vortex. Observations show that there can be coherent organization, subsequent breakdown, and reorganization of the parent vortex and surrounding vortex filaments (e.g. *Rotunno*, 1984).

In order for vortex filaments to merge into the parent vortex, they must (1) be aligned with the parent vortex and (2) be advected toward (or into) the core of the parent vortex. Using the Lamb vector and some of the relationships well-known in electromagnetism, we examined the behavior and self-organization potential of vortex filaments surrounding a tornado in a high-resolution simulation.

We use the University of Wisconsin Nonhydrostatic Modeling System (UWNMS: see *Tripoli*, 1992), with up to 5 nested grids, obtaining horizontal and vertical resolution below 25 meters. An idealized vertical sounding, based on a tornadic supercell environment, is used to initialize the meteorological fields.

One can imagine randomly oriented small-scale vortices (or angular moments) embedded within the larger scale rotation of the tornado. By decomposing the Lamb vector into mean and perturbation components, we find only one combination (using the mean vorticity and perturbation velocity) will geometrically yield a torque across a segment of a perturbation vortex filament embedded within a large-scale vorticity field

$$\frac{\partial \mathbf{u}'}{\partial t} \approx -\ \bar{\boldsymbol{\omega}} \times \mathbf{u}' \ \equiv -\mathbf{l}_{\mathbf{a}}' \qquad \text{(1a)}$$

where $\mathbf{u}'$ is the perturbation velocity, $\bar{\boldsymbol{\omega}}$ is the mean vorticity, and $\mathbf{l}_{\mathbf{a}}'$ is this component of the Lamb vector. The magnitude of the curl of this term, $\left|\nabla \times \mathbf{l}_{\mathbf{a}}\right|$, is used to diagnose "gyroscopic alignment torque", which is the HD analog to the torque on a magnetic dipole moment immersed in a large-scale magnetic induction:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \qquad \text{(1b)}$$

where $\mathbf{B}$ is the large-scale magnetic induction, and $\boldsymbol{\mu}$ is the magnetic moment of the small dipole. These parameters are illustrated in Fig. 2.

Regarding the alignment of vortex loops, Stokes Theorem states that vortex lines cannot just 'end' in the middle of a fluid; they must either 'loop back on themselves, or terminate at a surface. That does not preclude alignment of vortex segments, however. One can envision a poloidal configuration of vortex lines aligning themselves within the tornado.

We investigate another EM-analogous parameter which is related to a mechanism in terms of hurricane prediction (known as the "beta effect"). We term this new

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parameter (which is a *vector*) the "vortex beta force", $\boldsymbol{\beta}$. Hurricanes (small vortices) are known to be drawn upgradient toward higher same-signed vorticity (as the Coriolis parameter increases with higher latitude) through the conventional "beta-effect". This is absolutely consistent with this new $\boldsymbol{\beta}$ parameter, and it is directly analogous to the same mechanism responsible for "magnetic shielding" of charged particles: the force on the particle in proportional to the gradient of the configuration of vorticity aligned with the particle's magnetic moment:

$$\mathbf{F} = \nabla(\boldsymbol{\mu} \bullet \mathbf{B}) \tag{2a}$$

Substituting in the analogous variables, one obtains a formula for the force on a small local moment, which is generally aligned with the large-scale rotation:

$$\mathbf{F} = \nabla(\boldsymbol{\mu} \bullet \bar{\boldsymbol{\omega}}) = \boldsymbol{\beta} \tag{2b}$$

We investigate these two analogous formulations of vortex interaction and discuss them in the following section.

3. "EM-LIKE" TORQUING AND MERGING: RESULTS AND DISCUSSION

3.1 Gyroscopic torque

To simplify the gyroscopic torque formulation, we looked at the mathematical formulation for the electro-magnetic torque, and applied this analogously to the vortex interaction problem. To calculate this new parameter, information was needed about the LOCALLY defined angular momentum. Since the pressure gradient force is the only force (besides friction) that accelerates the flow in a Lagrangian sense, the normal component of pressure gradient acceleration to velocity was computed, and from this a local radius of curvature was obtained. Obviously, there is a strong correlation of local angular momentum and the local curvature radius:

$$\boldsymbol{\mu} = \mathbf{r} \times \rho \mathbf{u} \tag{3}$$

Where $\boldsymbol{\mu}$ is the angular moment, $\mathbf{r}$ is the radius, ρ is density, and $\mathbf{u}$ is velocity.

Again, substituting in the analogous variables, one obtains a formula for this torque on the local moment:

$$\boldsymbol{\tau} = \left|\boldsymbol{\mu} \times \bar{\boldsymbol{\omega}}\right| \tag{4}$$

While there were reservations and questions regarding the use of a variable point of reference for the local angular momentum calculation, there was overwhelming agreement with the Lamb vector formulation (Fig. 3). Given this information, plus given the calculated curvature radius near the tornado strongly agreed with the 'actual' radius (~ 500m) we have confidence that the data are meaningful.

We put this type of interaction in the context of vortex loops generated by a pulsing rear-flanking downdraft near a mesocyclone. The gyroscopic torque preferentially rotates the vortex loop into the 'correct' configuration, where this vorticity reconnects with the mesocyclone and brings the funnel to the ground (Fig. 3, right side).

3.2 Merging parameter

Once the surrounding vortices are aligned with the parent vortex, another mechanism is needed to draw the small-scale vortex upgradient. This "vortex beta" parameter is shown in Figure 4, where an isosurface of 3-D vorticity magnitude (the main feature *is* the tornado) is colored by the intensity of this vortex-beta parameter. Yellow values indicate high levels of the vortex-beta force.

This vortex-beta force competes against the so-called Magnus effect, which acts through the Lamb-vector term using the mean velocity and the perturbation vorticity:

$$\frac{\partial \mathbf{u}'}{\partial t} \approx - \ \boldsymbol{\omega}' \times \bar{\mathbf{u}} \ \ \equiv -\mathbf{l}'_{\mathbf{b}} \tag{5}$$

This force will act to *eject* like-signed embedded vortices *away* from the center of large-scale rotation. Thus, there will be a *competition* between the beta force (strongly dependent upon the *gradient* of like-signed large-scale vorticity) and the Magnus effect in the final merger process. These two forces can define a fundamental balance relationship, and formulated into a "net merging force" :

$$\boldsymbol{\alpha} = \boldsymbol{\beta} + \mathbf{l}'_{\mathbf{b}} = \nabla(\boldsymbol{\mu} \bullet \bar{\boldsymbol{\omega}}) + \ \boldsymbol{\omega}' \times \bar{\mathbf{u}} \tag{6}$$

This upgradient force on like-signed vortical structure is referenced in several places in the literature, even including motion in the Great Red Spot on Jupiter (*Marcus*, 2000), where it is argued that it is energetically favorable for 'prograde' (like-signed) vorticity to be drawn upgradient, while expelling 'adverse' vortical structures.

4. CONCLUSION AND FUTURE WORK

We have illustrated the concept of utilizing the EM-HD analogy in terms of vortex interaction during tornadogenesis, highlighting two self-organizational mechanisms of reorientation and upgradient advection of local angular moments embedded within a large-scale rotation. Self-alignment and merging of small-scale moments in the presence of a large-scale field is ubiquitous in both EM and HD. It is likely that the reason is linked to the tendency of a physical system to seek the lowest energy state. We plan to investigate other vortex interaction problems (e.g. hurricanes and boundary-layer turbulence) using this "analogous thinking" in upcoming work.

5. REFERENCES

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Turbulent hydrodynamics		Electromagnetism (after Marmanis(1998), Belevich(2008) and Pinheiro (2009))		Analogous Variables	
Navier-Stokes	$\frac{\partial \mathbf{u}}{\partial t} = -(\boldsymbol{\omega}\times\mathbf{u}) - \nabla\left(\frac{p}{\rho}+\frac{u^2}{2}\right)+\nu\nabla^2\mathbf{u}$	$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E}-\nabla\phi$ $\mathbf{E}$ = electric field	Vector and scalar potential	$\mathbf{u}$ $\mathbf{l}$	$\mathbf{A}$ $\mathbf{E}$
Lamb vector and vorticity	$\mathbf{l}\equiv(\boldsymbol{\omega}\times\mathbf{u})$ $\nabla\bullet\boldsymbol{\omega}=0$ $\boldsymbol{\omega}=\nabla\times\mathbf{u}$	$\mathbf{B}=\nabla\times\mathbf{A}$ magnetic induction $\nabla\bullet\mathbf{B}=0$	Coloumb Thomson	$\boldsymbol{\omega}$	$\mathbf{B}$
vorticity tendency	$\frac{\partial\boldsymbol{\omega}}{\partial t}=-\nabla\times\mathbf{l}^{*}+\nu\nabla^2\boldsymbol{\omega}$ * viscous term removed	$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$	Faraday's Law		
Lamb vector tendency and turbulent current ($\mathbf{j}$)	$\frac{\partial\mathbf{l}}{\partial t}=\nabla\times\boldsymbol{\eta}-\mathbf{j}$	$\frac{\partial(\varepsilon_0\mathbf{E})}{\partial t}=c^2\nabla\times\mathbf{H}-\mu_0\mathbf{J}$ (zero polarization)	Ampere's Law	$\boldsymbol{\eta}$	$\mathbf{H}$
vorticity field strength ($\boldsymbol{\eta}$) and magnetization	$\boldsymbol{\eta}=\left\|u^2\right\|\boldsymbol{\omega}-\mathbf{M}$ $\mathbf{M}=\mathbf{u}(\mathbf{u}\bullet\boldsymbol{\omega})+\nu\nabla^2\mathbf{u}$	$\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$	magnetic field strength and magnetization		
turbulent charge density	$\nabla\bullet\mathbf{l}=\mathbf{u}\bullet\nabla\times\boldsymbol{\omega}-\left\|\omega^2\right\|\equiv\rho_n$	$\nabla\bullet(\varepsilon_0\mathbf{E})=\rho_e$	electric charge density	ρ_n	ρ_e

Figure 1. A table showing the analogous mathematical structure and variables between the equations of fluid dynamics and electromagnetism. Variables are color-coded to highlight the analogies.

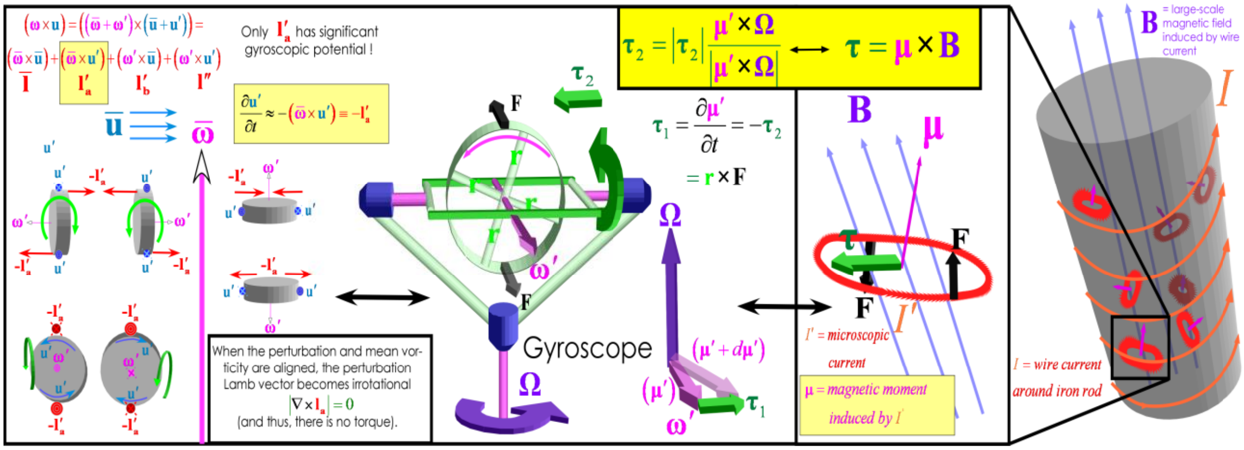

Figure 2. The left side of the diagram shows the geometric configuration of a segment of perturbation vorticity ($\boldsymbol{\omega}'$) embedded within a larger scale vortical field ($\bar{\boldsymbol{\omega}}$). The curl of the second term in the Lamb vector decomposition ($\bar{\boldsymbol{\omega}}\times\mathbf{u}' \equiv \mathbf{l}'_a$) describes the same type of "counter-torque" yielding alignment of a flywheel in a gimbal gyroscope. This quantity lies in the direction of the cross product of the local moment and the mean rotation, mathematically identical in form to the torque on a current loop or magnetic dipole placed in an external magnetic field (right side of diagram).

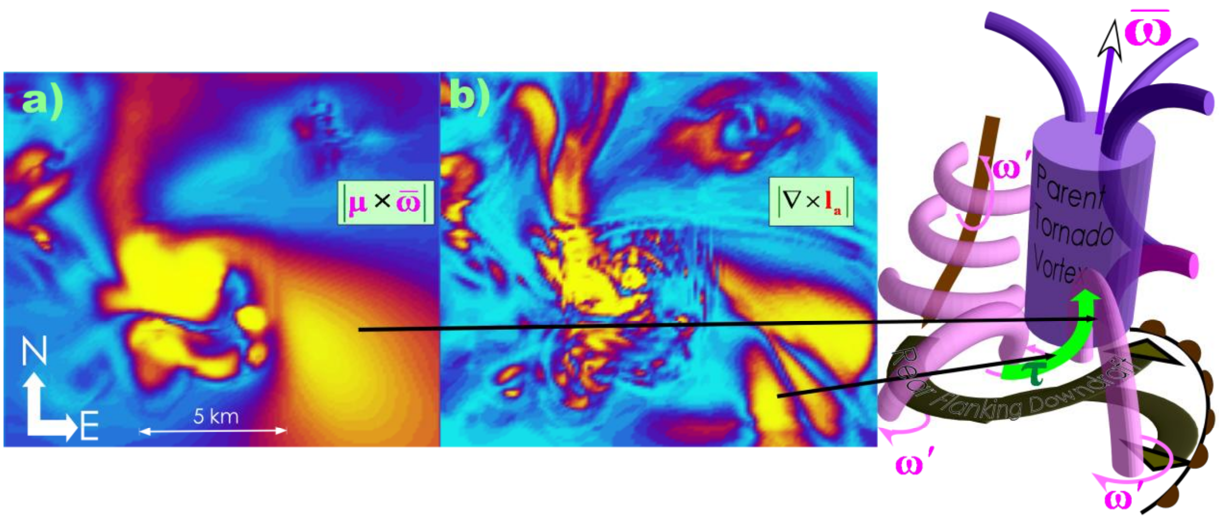

Figure 3. There is remarkable comparison of the EM-formulated gryoscopic torque (left) and the Lamb vector formulation (right). The bright yellow patches indicate strong torque. The area in the southeast part of the domain is the region of strong vorticity associated with the advancing gust front associated with the rear flanking downdraft. Also note the reduction of numerical noise in the EM-formulation. In the far right portion of the figure, a schematic of the tornadic vortex evolution is given. For example, a new downdraft pulse is also forming north of the mesocylone.

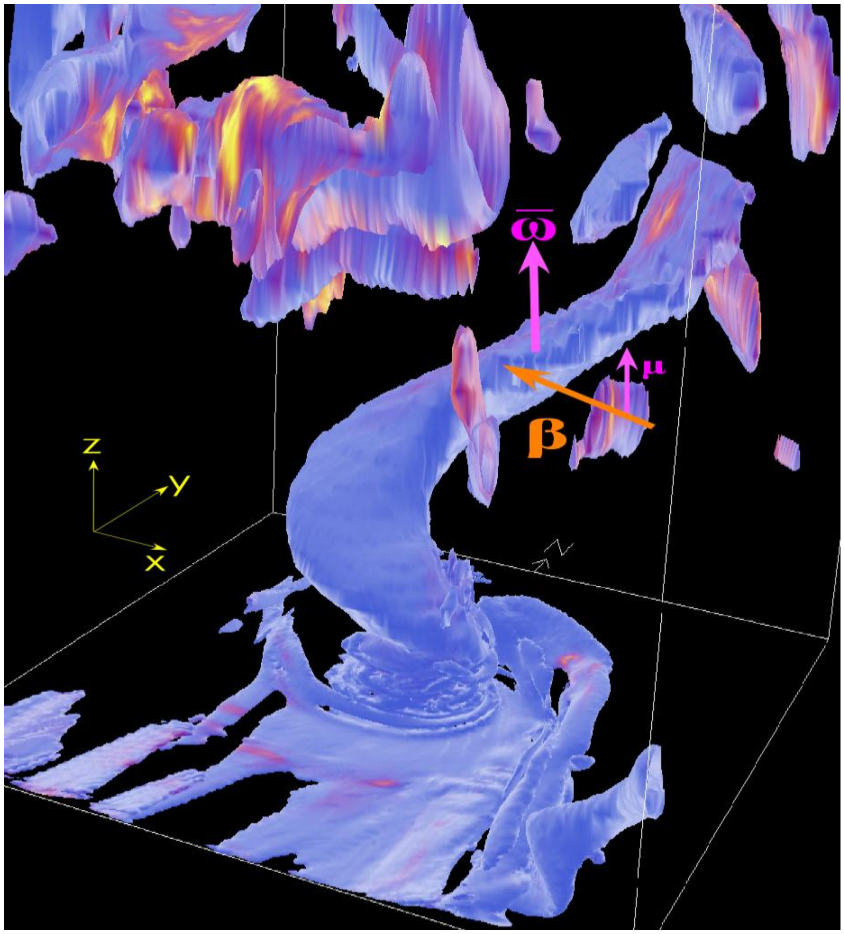

Figure 4. Isosurface of vorticity magnitude, colored by the magnitude of the “3-D vortex beta vector”. Orange arrow indicates direction of vortex beta force.