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Deficiencies of Slab Models of the Hurricane Boundary Layer

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1. Introduction

Diagnostic models of the tropical cyclone boundary layer have important practical uses, including for engineering design and climatological risk assessment studies, and as components of tropical cyclone potential intensity models. A widely used class of such models has been slab models, in which the governing equations are depth-averaged. Here, a slab model is compared to one which fully resolves height, and it is shown that the vertical averaging leads to substantial differences in the simulations. The slab model produces excessively strong inflow and too great departure of the boundary-layer mean winds from gradient balance. Given the considerable impact of the vertical averaging in slab models on the simulated flow in the tropical cyclone boundary layer, it is difficult to recommend their further use for applications where quantitative accuracy is important. Other applications will require care to ensure that the results are not unduly affected by the depth-averaging.

2. Model Formulation

Two diagnostic models of the TCBL are used here, a slab model and a height-resolving model. Each diagnoses the boundary-layer flow as the response to a specified, optionally translating, pressure field representative of a tropical cyclone. Thus each model can be provided with identical forcing, thereby isolating the effects of the boundary-layer representation from the rest of the storm. The slab model is depth-averaged, while the height-resolving model solves the full three-dimensional equations of motion with a simple parameterisation of turbulent diffusion. Both use the same parameterisation of surface drag and, as far as is possible, boundary conditions. The thermodynamics of the boundary layer will not be studied, not because it is unimportant, but because the focus is on getting the flow correct, which is a necessary first step to calculating the flux and advection terms in the thermodynamic budgets. Derivations of slab models can be found, for example, in Shapiro (1983) or Smith (2003), while the height-resolving model formulation is given in Kepert and Wang (2001). Details of the exact model formulations as used in this study are in Kepert (2010a).

3. Results

3.1 *Flow in the height-resolving model*

Figure 1 shows the boundary-layer flow in a stationary, axisymmetric cyclone with maximum gradient wind of 40 m s^{-1} at a radius of 40 km, according to the height-resolving model. The forcing vortex is the same as in Smith and Vogl (2008) for ease of comparison with their slab model results. The depth of the inflow layer decreases rapidly with radius, from about 2 km at 300-km radius to below 400 m in the eye. This decrease is consistent with observations (e.g. Frank 1984; Kepert 2006a,b; Bell and Montgomery 2008), and linear models and scaling arguments that show that the boundary layer depth

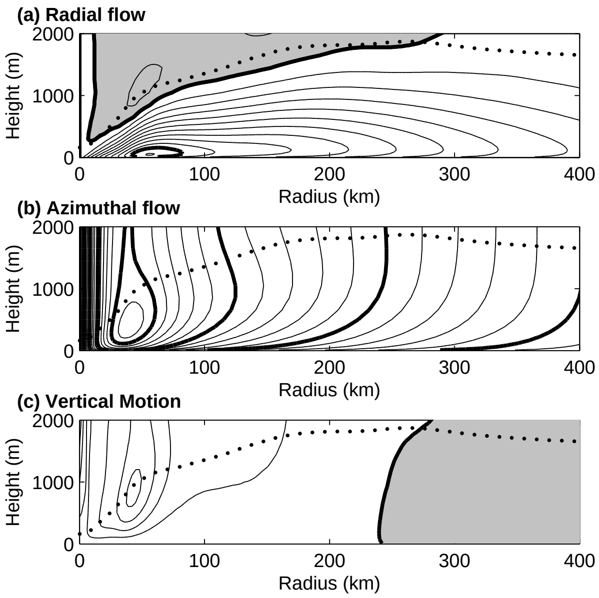

Figure 1: The boundary layer flow in a stationary storm simulated by the height-resolving model. (a) radius-height section of the radial wind, contour interval 1 m s^{-1}, multiples of 10 m s^{-1} shown bold, positive values shaded. (b) radius-height section of azimuthal wind, contour interval 2 m s^{-1}, multiples of 10 m s^{-1} shown bold. (c) radius-height section of the vertical velocity, contour interval 0.05 m s^{-1}, zero line shown bold, subsidence shaded. The dotted line in each panel indicates the level at which the stress magnitude reduces to 20% of its surface value.

in the core of the storm scales as $I^{-1/2}$, where I is the inertial stability (e.g Rosenthal 1962; Eliassen and Lystad 1977; Kepert 2001). The maximum azimuthal wind is 43.2 m s^{-1} at a height of 400 m, and is about 8% supergradient. The supergradient flow is mostly within the inflow layer, but does extend upwards into the outflow layer at the top of the boundary layer, and was extensively analysed by Kepert and Wang (2001).

The turbulent stress has maximum magnitude at the surface and decreases monotonically with height in this and similar simulations. The dotted lines in Fig. 1 show the height at which the momentum flux magnitude falls to 0.2 of its surface value; the value 0.2 was chosen as it roughly coincides with the top of the inflow layer. Clearly the turbulent transport of momentum is a significant part of the dynamics of the outflow layer, consistent with the discussion in Kepert and Wang (2001).

3.2 *Comparison of boundary-layer mean flow in slab and height-resolved models*

The flows from the slab and height-resolving models are compared in Fig. 2. This comparison uses the same forcing vortex and surface drag parameterisation in both models; the flow from the height-resolving model is averaged over the same height range as the prescribed boundary layer depth in the slab model. This height is less than the boundary-layer depth except in the inner core, but as can be inferred from Fig. 1, other reasonable choices will not dramatically change the results. The slab model has the stronger inflow except within the eye, most markedly so at and immediately outside of the radius of maximum winds (RMW). Thus the eyewall updraft is very much stronger in the slab model. The frictionally-forced updrafts outside of 250 km radius are more similar, because there the stronger $\partial u/\partial r$ term in the continuity equation compensates for the stronger inflow in the slab model. The height-resolving model has the height-mean azimuthal flow slightly subgradient except in the vicinity of the RMW, where it is slightly supergradient. This situation is in strong contrast to the slab model, which has much larger departures from gradient balance through most of the storm. Observations show that the azimuthal-mean surface inflow angle in tropical cyclones over the sea is usually in the range 20 – $25°$. For example, Hurricane Frederic (1979) had an azimuthal-mean surface inflow angle of 21 – $22°$, according to the over-water composite analysis of Powell (1980). The height-resolving model simulation shown in Fig. 2b has a surface inflow angle of 20 – $25°$ over most of the domain, reducing to smaller values inside of radius 70 km. Observations of the depth-averaged inflow angle are seldom reported, but can safely be assumed to be less than the surface value. The slab model inflow angle exceeds $20°$

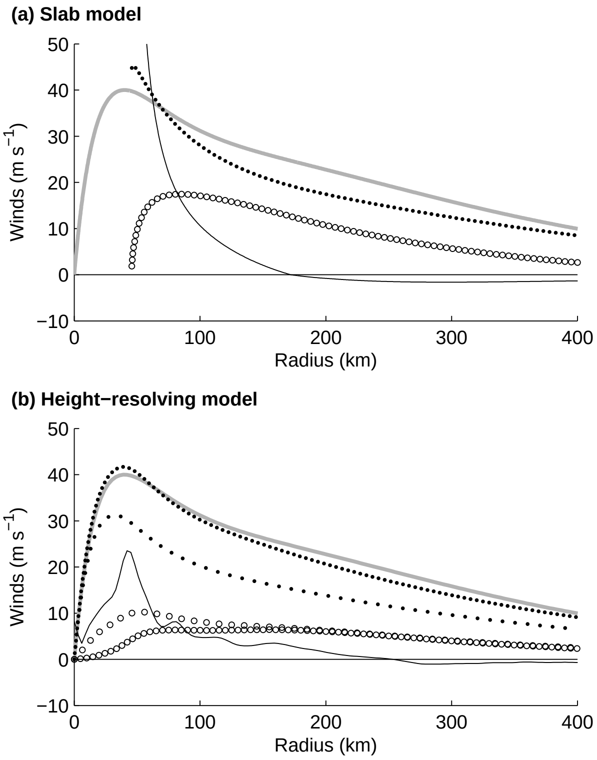

Figure 2: (a) Axisymmetric boundary layer flow according to the slab model. Gradient wind (thick grey), boundary-layer mean azimuthal (dots), inwards (open circles) and upwards (thin black, multiplied by 100) flow components. Parameter values are as in Smith and Montgomery (2008), including the boundary-layer height which is fixed at $h = 800$ m. (b) Simulation of the same vortex as in (a), except by the height-resolving model, as already shown in Fig. 1. Curves with closely-spaced symbols are averaged over the lower 800 m, while those with less dense symbols show the flow at 10-m height. The vertical velocity is at 800-m height.

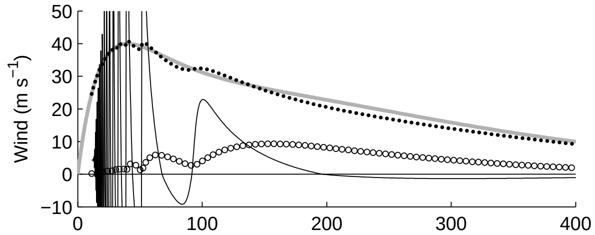

Figure 3: Boundary-layer flow simulated by the slab model as in Fig. 2a, except with C_D halved.

between 70 and 360 km, and exceeds $30°$ from 90 to 220 km radius, which is unrealistically large.

One might suspect that the excess inflow in the slab model is because the surface drag there is calculated from the boundary-layer mean wind, whereas the height-resolving model uses the 10-m wind. One can crudely correct for this by reducing the wind speeds in the surface stress calculation by a factor of, say, 0.7 – 0.9, to better represent the surface wind (see e.g. Powell and Black 1990; Kepert and Wang 2001; Franklin et al. 2003, and references therein regarding the choice of constant). Vickery and Twisdale (1995) reduce their surface drag coefficient by 50% for this reason. This adjustment reduces the departure of the boundary-layer flow from the gradient flow at large radii (Fig. 3). However, the solution now displays marked oscillations inwards of about 150 km radius, similar to those analysed by Smith and Vogl (2008, section 4.1) but beginning at much larger radius than they reported.

3.3 *Further results from axisymmetric slab models*

The slab model was tested on a variety of vortex radial profiles of differing sizes, intensities and structures, and found to produce unphysical results when applied to some vortex profiles. An example of especially pathological behaviour is shown in Fig. 4, where the model is forced with the gradient wind radial profile fitted to aircraft and dropsonde observations in Hurricane Georges by Kepert (2006a). The oscillations in the height-mean radial and azimuthal wind components that are apparent inwards of $r = 150$ km are similar to those analysed near the RMW by Smith (2003) and Smith and Vogl (2008),

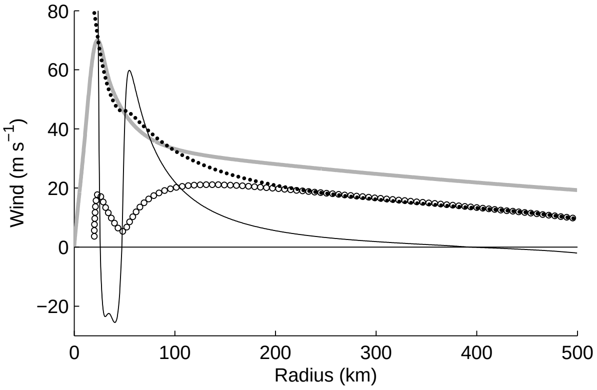

Figure 4: Simulated axisymetric flow in the boundary layer of Hurricane Georges on 19 Sept 1998, according to the slab model. Gradient wind (thick grey), boundary-layer mean azimuthal (dots), inwards (open circles) and upwards (thin line, multiplied by 100) flow components. Note the bimodal structure of the inflow and the consequent strong downdraft near radius 50 km. Model parameter settings were as in Fig. 2a.

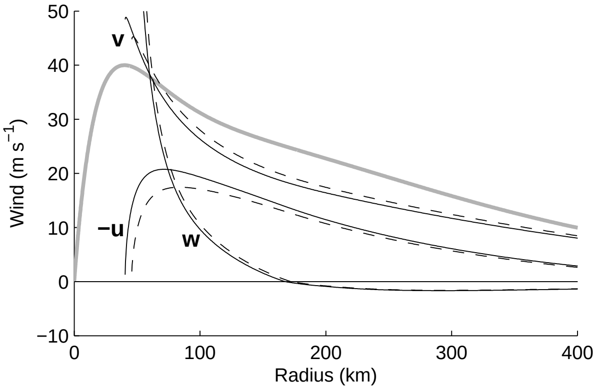

Figure 5: Axisymmetric boundary layer flow according to the slab model. Thick grey: gradient wind. Black curves: boundary-layer mean azimuthal, inwards and upwards (multiplied by 100) flow components as labelled. Dashed curves are for $f = 5 \times 10^{-5}$ s^{-1} and are identical to those in Fig. 2, solid curves use $f = 3.77 \times 10^{-5}$ s^{-1}.

and also shown here in Fig. 3, but have not previously been reported except near and within the RMW. They produce a strong oscillation in the vertical motion, to the extent that the frictionally-forced vertical motion near 50 km, or only twice the RMW, is actually downwards. In contrast, the modelled flow from the simulation in Kepert (2006a, Fig. 23a), which used the full 3-dimensional boundary layer model of Kepert and Wang (2001) and was shown to agree reasonably well with the observations, does not display this bizarre character. The case of Hurricane Georges is particularly interesting, since analysis of dropsonde observations showed that the upper boundary layer flow was not supergradient, and simulation with the height-resolving model showed only very slightly supergradient flow (Kepert 2006a). In contrast, Smith and Vogl (2008) emphasise that supergradient flow is ubiquitous in their slab model.

The slab model is unphysically sensitive to the choice of Coriolis parameter. The Rossby number $Ro = v_{gr}/(rf)$ exceeds unity inwards of about 250 km in the simulation in Fig. 2, and is 16 at the RMW. This would normally imply that the Coriolis force should have a diminishing effect on the solution in this region. Instead, it is clear from comparing the dashed and solid curves in Fig. 5 that changing f from 5×10^{-5} s^{-1} to 3.77×10^{-5} s^{-1}, corresponding to $20°$ and $15°$ degrees latitude respectively, results in a change of up to 10% in the modelled boundary-layer flow. Such strong sensitivity is markedly at variance with our expectation from scaling arguments. In contrast, the height-resolving model is almost insensitive to the Coriolis parameter in this region (not shown).

Most published slab model applications use constant h, although the height-resolving model, linear models and observations show a marked reduction in boundary-layer depth towards the storm centre (section 3.1). Smith and Vogl (2008) present two calculations with such a variation, although they choose boundary-layer depths which are arguably too small, being around 100 m at the RMW. The simulation they presented used quite a large value of their shallow convection velocity scale, which controls the flux through the top of the boundary layer due to parameterised shallow convection, $w_{sc} = -0.057$ m s^{-1}. Fig. 6 presents a comparison of this simulation (dashed lines) to one with $w_{sc} = 0$ (thin solid lines), from

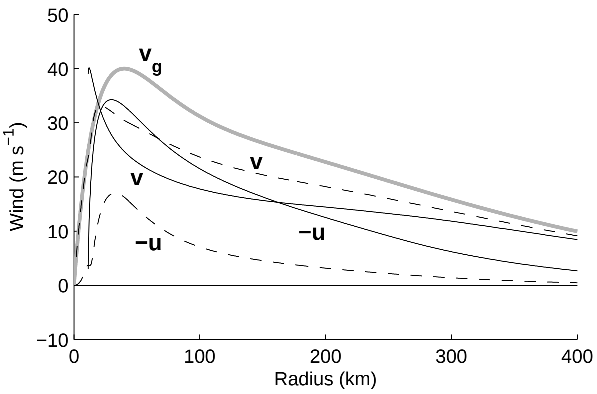

Figure 6: Simulations with the slab model with radially varying h. The depth h varies as $I^{-1/2}$, and is $800\,\mathrm{m}$ at $r = 500\,\mathrm{km}$. The dashed curves show the simulated flow with $w_s c = -0.057\,\mathrm{m\,s^{-1}}$ as in Smith and Vogl (2008), while the thin solid lines have $w_{sc} = 0$, both with azimuthal and inflow components as marked. The thick grey curve is the gradient wind.

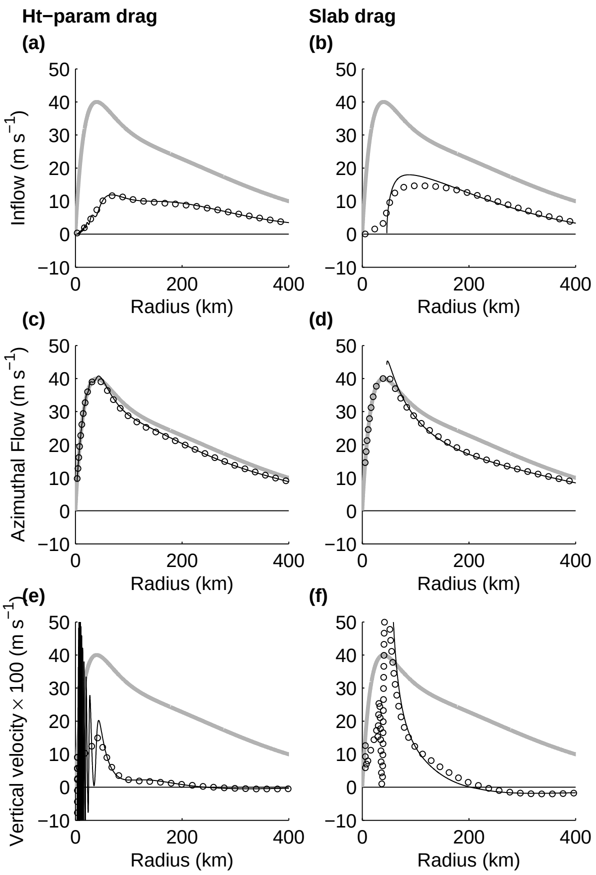

Figure 7: Boundary-layer flow simulated by four models. The left column is for the height-parameterised model and for the hybrid model with the same surface drag condition. The right column is for the slab model and for the hybrid model with the same surface drag condition. (a) Height-mean inflow for the height-parameterised (thin black curve) and hybrid model (ii) (circles), together with gradient wind (thick grey curve). (c) The height-mean azimuthal wind, models and line-styles as in (a). (e) The vertical motion in the limit $z \to \infty$, models and line-styles as in (a). (b) Height-mean inflow for the hybrid model (i) (thin black curve) and for the slab model (circles). (d) The height-mean azimuthal wind, models and line-styles as in (b). (e) The vertical motion in the limit $z \to \infty$, models and line-styles as in (b). The slab model simulation here omits vertical advection through the upper boundary, but is otherwise the same as shown in Fig. 2a.

which it is clear that omitting w_{sc} produces grossly excessive inflow. It not clear that it is physically reasonable to allow shallow convection to have such a large influence on the boundary-layer flow in the core of a tropical cyclone. This excessive inflow can also be controlled by reducing the ratio C_D/h, but this leads to an oscillating solution similar to that in Fig. 3. Attempts to discover a satisfactory set of parameters for the slab model with radially varying h were unsuccessful. A wide range of settings was tested, but none of the simulations was regarded as particularly satisfactory, since they are all unrealistic in one or more of the following aspects: overly large influence of shallow convection, too shallow boundary layer depth, too large inflow and departure from gradient balance, or extensive oscillations.

3.4 *Why is the slab model inaccurate?*

It has been shown that the slab model is inaccurate, when measured against the height-resolved model. A further model, intermediate between the slab and height-resolved models, has been developed (Kepert 2010b). In this model, the vertical structure of the flow is parameterised by an Ekman-like spiral with two free parameters, so this model is called the height-parameterised model. Differential equations are derived for these parameters, and the model is solved by integrating them in from large radius. In contrast to the slab model, the more realistic vertical profile allows the application of the surface drag to the surface wind instead of the boundary-layer mean wind, and more accurate treatment of nonlinear terms. This model is considerably more accurate than the slab model, when measured against simulations from the height-resolving model. In addition, two further models that are hybrids of the slab and height-parameterised models were developed. Specifically, (i) a model with the slab model's treatment of the surface drag and height-parameterised model's treatment of the other nonlinear terms, and (ii) a model with the height-parameterised model's treatment of the surface drag and the slab model's treatment of the other nonlinear terms, are examined. Full details of these models are in Kepert (2010b).

Solutions of these models with the same parent vortex as before are shown in Fig. 7. The left column is for the models with the height-parameterised surface drag condition, while the right column is for those models which apply the surface drag to the boundary-layer mean wind. The open circles indicate that the nonlinear terms are calculated as in the height-parameterised model, while the thin black lines have slab-model style advection. Outside of about 200-km radius, the differences between the simulations are dominated by the method used for the surface drag, with the slab-model method leading to stronger inflow and weaker azimuthal flow. Inwards of about 200-km radius, the two simulations with slab-model surface drag diverge (right column), with the height-parameterised method of calculating the nonlinear terms leading to weaker inflow, the azimuthal flow being approximately in gradient balance, and the elimination of the singularity which terminated the slab model integration near the RMW. In the left column, the solution is nearly independent of how the nonlinear terms are calculated, except inside the RMW, where the slab-model method leads to high-frequency oscillations in w. The difference between the curves shows up at smaller radius than in the right column, because the flow is not so far from gradient balance and so the nonlinear terms are smaller and therefore less sensitive to their method of computation. These differences are representative of those found in other simulations. Slab-model drag leads to excessive inflow and departures from gradient balance, and slab-model style calculation of the nonlinear terms greatly increases the tendency of the solution to become singular or to oscillate.

4. Discussion and Conclusions

Marked differences in the boundary-layer flow occur between that predicted by a simple slab model and that predicted by a height-resolving model. In addition, the slab model was shown to be capable of quite pathological behaviour for some reasonable parameter settings, and has an unphysical sensitivity to f. Analysis of the reasons for these properties shows that two factors are responsible:

1. the calculation of the surface drag using the boundary-layer mean wind rather than the surface wind, and

2. the inaccurate treatment of the nonlinear terms in the depth-averaging.

The first of these is problematic at all radii, while the second becomes significant in the inner core. There is some uncertainty in what values of physical parameters should be applied, and arguably a smaller value of C_D can be justified in the slab model since the drag is being applied to the boundary-layer mean wind. This adjustment reduces the excess inflow and subgradient flow in the slab model, but can trigger the quasi-inertial oscillation, so cannot be regarded as an improvement. These results confirm and help explain the recent TCBL model intercomparison by Khare et al. (2009), who found that the slab model was significantly less accurate than the linear model of Kepert (2001) when compared to observational analyses.

The slab model is known to be subject to quasi-inertial oscillations (Smith 2003; Smith and Vogl 2008). These oscillations were shown to be due to the inaccurate treatment of the nonlinear terms. This point is important, since Smith and Vogl (2008) have argued that these oscillations are an artefact of prescribing the pressure gradient at the top of the boundary layer in regions of outflow. Their argument, if correct, would preclude the use of diagnostic models of the boundary layer, including those of Rosenthal (1962), Smith (1968), Leslie and Smith (1970), Bode and Smith (1975), Kuo (1971, 1982), Shapiro (1983), Thompson and Cardone (1996), Kepert (2001), Kepert and Wang (2001), Vickery et al. (2000, 2009), Smith (2003), Smith and Vogl (2008), Powell et al. (2005), and Foster (2009), in the most important part of the storm. Fortunately, the argument of Smith and Vogl (2008) is incorrect. It is not the prescribed pressure gradient that is responsible for the oscillations in the slab model, but rather the inaccurate treatment of the nonlinear advection terms. A more reasonable treatment, as in the height-parameterised model, greatly reduces the propensity to oscillate, while extensive experience with the height-resolved model suggests that extra degrees of freedom in the vertical completely eliminates this problem.

Simplified models of the TCBL are useful for a number of purposes, with major applications including climatological risk assessment and engineering design. The considerable inaccuracies demonstrated here implies that considerable caution must be applied in future if using slab models for quantitative prediction. Such applications have demonstrated satisfactory agreement between model and observations (Vickery and Twisdale 1995; Thompson and Cardone 1996; Vickery et al. 2000, 2009), but the slab model output has in such cases been rather empirically adjusted before comparison with observations. Moreover, most such verifications have been of wind speed, where the biases in radial and azimuthal components will partly cancel, rather than of the wind vector. While these authors are to be commended for their validation efforts, it appears that the tuning of these adjustments has concealed fundamental deficiencies in the model.

Another important application of simplified models has been as a component of tropical cyclone potential intensity (PI) models. Recently, Smith and Montgomery (2008) have shown that further approximations within a slab model, including those made in Emanuel's PI model, can produce large changes in the flow. Those differences are of similar magnitude to the differences between slab and height-resolved models demonstrated here. The results in this paper support the conclusion of Smith and Montgomery (2008) as to the need to improve the boundary-layer component within existing PI models. However, it is very clear that simply relaxing some approximations but remaining with the slab model approach would be replacing one inaccurate model with another. A better solution could be an extension of the height-parameterised model presented here, to include prediction of the thermodynamic parameters. Research is continuing to develop such a model.

Recently, Smith et al. (2009) have argued that boundary layer dynamics play a crucial role in tropical cyclone intensification. Their arguments are strongly influenced by the slab model results of Smith and Vogl (2008), so the fact that slab models overestimate the depth-mean inflow and the strength of the supergradient flow may be cause to doubt their reasoning. Indeed, while Smith and Vogl (2008) emphasise the ubiquity of supergradient flow in the slab model, analysis of observations shows that not all storms contain boundary layer supergradient flow, consistent with simulation of these storms by the height-resolving model (Kepert 2006a; Schwendike and Kepert 2008). Further, Smith et al. (2009) do not give the mechanism by which the boundary layer dynamics and supergradient flow leads to an adjustment in the cyclone's mass field, necessary for intensification. Schubert et al. (1980) studied geostrophic adjustment of the first internal mode in initially balanced vortices and showed that wind forcing can lead to a significant adjustment of the mass field provided that the scale of the forcing is less than the Rossby radius of deformation, $L_R = NH/I$, where N is the Brunt-Väisälä frequency, H is the vertical scale, and

I is the inertial stability. In the cyclone core, L_R for the first internal mode is similar to or less than the RMW, so the mass field will adjust to the wind field for deep imbalances. However, for shallow regions of imbalance, as in the supergradient flow at the top of the boundary layer, L_R is much less, and most of the kinetic energy in the imbalance will be lost as inertia-gravity waves. In this context, the outflow immediately above the supergradient wind maximum (Fig. 1) can be regarded as a continuously-forced inertia wave that adjusts the wind back to the mass field, with little if any impact on the mass. (This is not an exact analogy, since diffusion is non-negligible in this layer.) In general, simplified models are valuable because their use may lead to understanding. However, such use requires care to ensure that the conclusions reached are not an artefact of the simplifications in the model. It is hoped that the analysis of the deficiencies of slab models of the TCBL presented here will facilitate such caution in the future.

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